Using Response Times to Refine and Augment Estimates of Latent Constructs

Seth Sanders[1] and Lynne Steuerle Schofield[2]

[1]Cornell University

[2]Swarthmore College

October 15, 2024

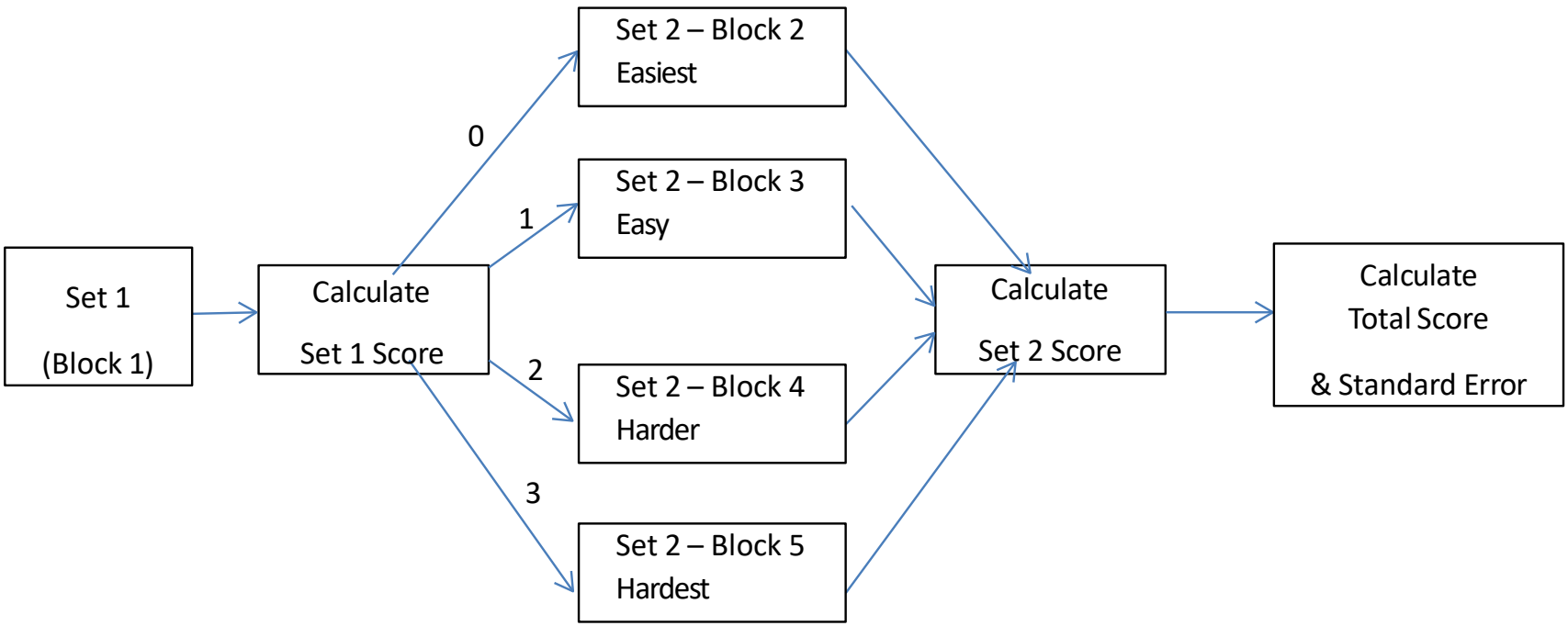
- Ubiquitous to include tests and assessments meant to measure cognitive and/or non-cognitive skills on surveys
- Most tests are scored using either Classical Test Theory (CTT) or Item Response Theory (IRT)
- Some surveys are now providing "response times" (RT)
  - RTs measure how long it takes for the respondent to answer each item.
  - RTs are collected as a by-product of normal survey operations; can be derived from time stamps on CAPI systems
  - Traditionally, have been used to monitor data quality and survey efficiency
- Cognitive neuroscientists note that RTs can be used to measure a form of cognition.

1. Does including survey-based RT in IRT models provide a more refined measure of θ, the latent construct being measured?
2. Does the survey-based RT on its own give us information about other possible outcomes?

PSID data

- We use the 2016 Well-being and Daily Life Supplement (Freedman, 2017)
  - Section J: Quantitative Reasoning; drawn from the 2012 Health and Retirement Study (HRS; Fisher et al. 2014).
  - Section K: Financial Literacy; measures the use of math skills in daily life
  - Section H: Verbal Reasoning.; a series of sentence completion questions,
  - Parts of Sections D: Conscientiousness,
- From the main survey we use:
  - Education
  - Gender
  - How people took the survey (we only include the $n = 6182$ individuals who took the survey via the web)

Figure 1: Number Series flow in PSID Wellbeing and Daily Life Supplement



	Item	Answer	n	Percent Correct	Mean Time	Diff Param
		Block 1				
Q1	8 __ 12 14	10	6182	98.4%	27.2	-4.86
Q2	23 26 30 35 __	41	6182	87.6%	25.3	-2.20
Q3	18 17 15 __ 8	12	6182	83.1%	35.6	-1.51
		Block 2				
Q1	6 7 __ 9	8	71	61.4%	12.6	-3.85
Q2	6 __ 4 3	5	71	56.0%	13.5	-3.38
Q3	5 8 11 __	14	71	40.3%	24.1	-2.46
		Block 3				
Q1	__ 4 6 8	2	544	96.3%	9.2	-5.23
Q2	1 3 3 5 7 7 __	9	544	56.2%	28.4	-2.55
Q3	18 10 6 __ 3	4	544	37.5%	38.5	-1.77
		Block 4				
Q1	17 __ 12 8	15	945	46.2%	46.8	-1.32
Q2	10 __ 3 1	6	945	30.9%	36.9	-0.83
Q3	17 16 14 10 __	2, 3	945	16.6%	47.4	0.10
		Block 5				
Q1	__ 20 26 38 62	17	4622	75.1%	84.7	-0.35
Q2	5 __ 11 19 35	7	4622	73.5%	78.9	-0.27
Q3	70 __ __ 84	72, 76, 78, 82	4622	35.8%	218.3	1.28

Table: Tasks on the Quantitative Reasoning Test

	Item	Answer	n	Percent Correct	Mean Time	Diff Param
		Block 1				
Q1	8 __ 12 14	10	6182	98.4%	27.2	-4.86
Q2	23 26 30 35 __	41	6182	87.6%	25.3	-2.20
Q3	18 17 15 __ 8	12	6182	83.1%	35.6	-1.51
		Block 2				
Q1	6 7 __ 9	8	71	61.4%	12.6	-3.85
Q2	6 __ 4 3	5	71	56.0%	13.5	-3.38
Q3	5 8 11 __	14	71	40.3%	24.1	-2.46
		Block 3				
Q1	__ 4 6 8	2	544	96.3%	9.2	-5.23
Q2	1 3 3 5 7 7 __	9	544	56.2%	28.4	-2.55
Q3	18 10 6 __ 3	4	544	37.5%	38.5	-1.77
		Block 4				
Q1	17 __ 12 8	15	945	46.2%	46.8	-1.32
Q2	10 __ 3 1	6	945	30.9%	36.9	-0.83
Q3	17 16 14 10 __	2, 3	945	16.6%	47.4	0.10
		Block 5				
Q1	__ 20 26 38 62	17	4622	75.1%	84.7	-0.35
Q2	5 __ 11 19 35	7	4622	73.5%	78.9	-0.27
Q3	70 __ __ 84	72, 76, 78, 82	4622	35.8%	218.3	1.28

Table: Tasks on the Quantitative Reasoning Test

	Item	Answer	n	Percent Correct	Mean Time	Diff Param
		Block 1				
Q1	8 __ 12 14	10	6182	98.4%	27.2	-4.86
Q2	23 26 30 35 __	41	6182	87.6%	25.3	-2.20
Q3	18 17 15 __ 8	12	6182	83.1%	35.6	-1.51
		Block 2				
Q1	6 7 __ 9	8	71	61.4%	12.6	-3.85
Q2	6 __ 4 3	5	71	56.0%	13.5	-3.38
Q3	5 8 11 __	14	71	40.3%	24.1	-2.46
		Block 3				
Q1	__ 4 6 8	2	544	96.3%	9.2	-5.23
Q2	1 3 3 5 7 7 __	9	544	56.2%	28.4	-2.55
Q3	18 10 6 __ 3	4	544	37.5%	38.5	-1.77
		Block 4				
Q1	17 __ 12 8	15	945	46.2%	46.8	-1.32
Q2	10 __ 3 1	6	945	30.9%	36.9	-0.83
Q3	17 16 14 10 __	2, 3	945	16.6%	47.4	0.10
		Block 5				
Q1	__ 20 26 38 62	17	4622	75.1%	84.7	-0.35
Q2	5 __ 11 19 35	7	4622	73.5%	78.9	-0.27
Q3	70 __ __ 84	72, 76, 78, 82	4622	35.8%	218.3	1.28

Table: Tasks on the Quantitative Reasoning Test

	Item	Answer	n	Percent Correct	Mean Time	Diff Param
		Block 1				
Q1	8 __ 12 14	10	6182	98.4%	27.2	-4.86
Q2	23 26 30 35 __	41	6182	87.6%	25.3	-2.20
Q3	18 17 15 __ 8	12	6182	83.1%	35.6	-1.51
		Block 2				
Q1	6 7 __ 9	8	71	61.4%	12.6	-3.85
Q2	6 __ 4 3	5	71	56.0%	13.5	-3.38
Q3	5 8 11 __	14	71	40.3%	24.1	-2.46
		Block 3				
Q1	__ 4 6 8	2	544	96.3%	9.2	-5.23
Q2	1 3 3 5 7 7 __	9	544	56.2%	28.4	-2.55
Q3	18 10 6 __ 3	4	544	37.5%	38.5	-1.77
		Block 4				
Q1	17 __ 12 8	15	945	46.2%	46.8	-1.32
Q2	10 __ 3 1	6	945	30.9%	36.9	-0.83
Q3	17 16 14 10 __	2, 3	945	16.6%	47.4	0.10
		Block 5				
Q1	__ 20 26 38 62	17	4622	75.1%	84.7	-0.35
Q2	5 __ 11 19 35	7	4622	73.5%	78.9	-0.27
Q3	70 __ __ 84	72, 76, 78, 82	4622	35.8%	218.3	1.28

Table: Tasks on the Quantitative Reasoning Test

- Basic idea in IRT is that an individual's score is a function of both the individual's "ability" and characteristics of the items themselves. (Lord and Novick, 1968)
  - So it matters both how many items are answered correctly and which items were answered correctly
  - IRT makes it possible to score people on the same scale even when they answer different questions.
- A standard unidimensional model is the 2-parameter ogive (2-PO) model (Johnson and Albert, 1999)

$$P(X_{ij} = 1|\theta_i, a_j, b_j) = \Phi(a_j(\theta_i - b_j)) \quad (1)$$

  - $X_{ij} = 1$ when individual i answers item j correctly and is 0 otherwise.
  - $a_j =$ "discrimination" parameter; acts as a weight determining how much to count item j
  - $b_j =$ "difficulty" parameter; as b_j increases, the probability that most examinees will answer the item correctly decreases.

- Neuroscientists and psychologists use RTs as a signal of cognitive functioning
- Mathematical models formalize how the neural circuitry operates (Ratcliff 1978; Smith 2000).
- RT distributions are often skewed; so we'll use a log normal distribution (van der Linden, 2006).
- A standard RT model for an item is

$$\log(RT_{ij}) = \phi_j(\tau_i - \lambda_j) + \varepsilon_{ij}. \tag{2}$$

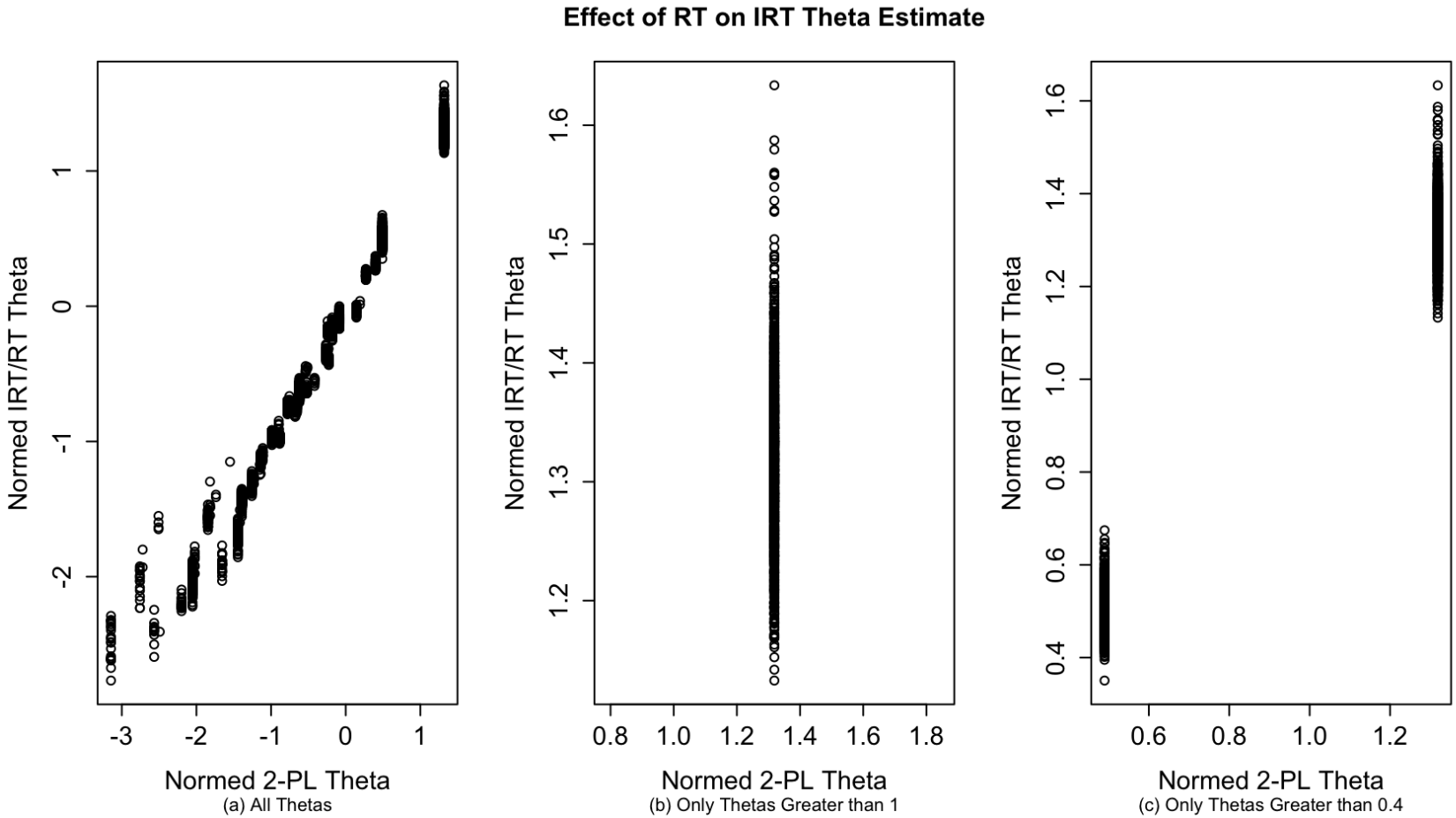
  - τ_i is a respondent's latent "working speed"
  - $\phi_j =$ "time discrimination" parameter; models how sensitive the item is at differentiating different speed-levels of the test takers
  - $\lambda_j =$ "time intensity" parameter; measures the average time needed to complete the item.

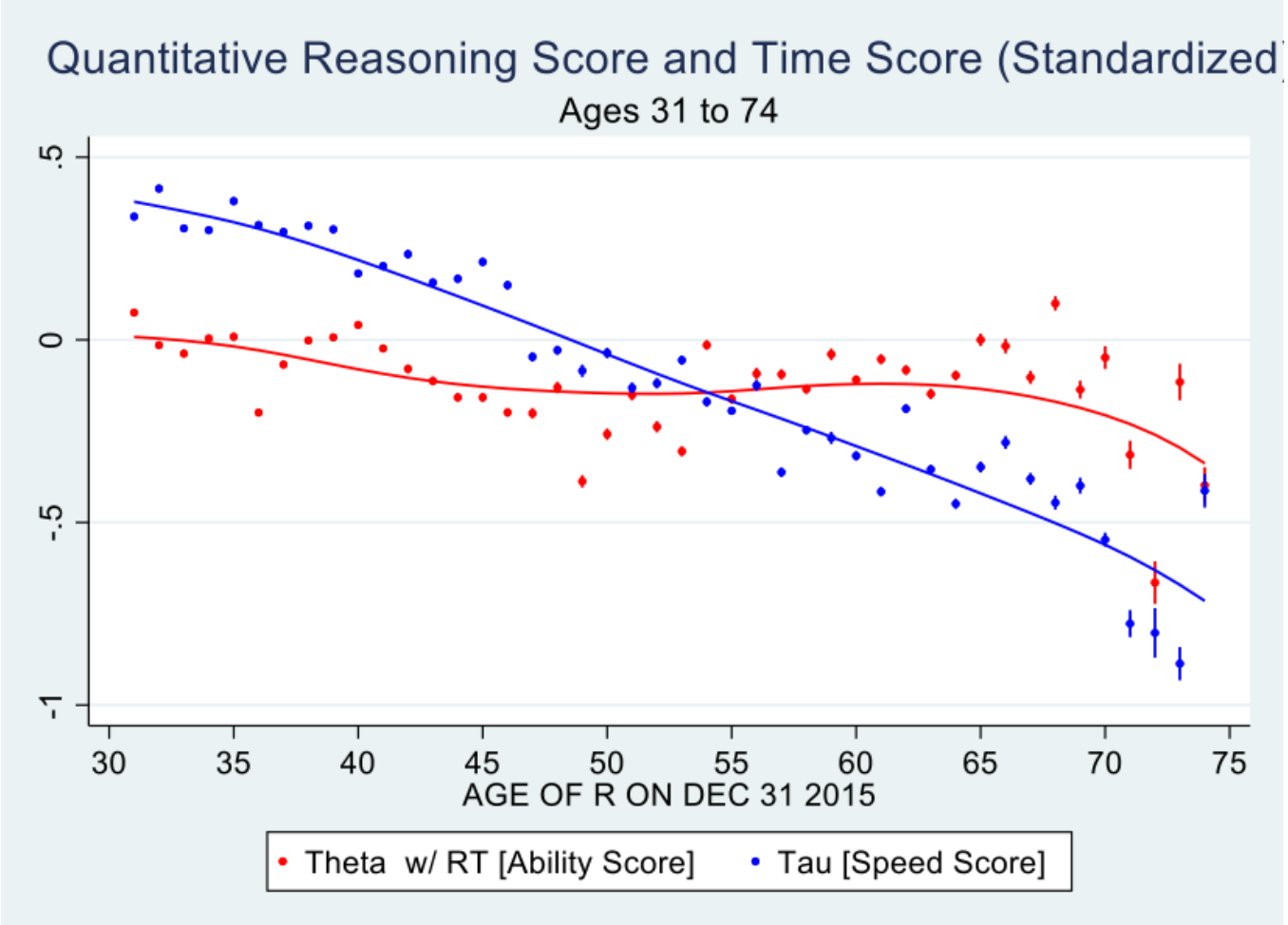
- We use R's LNIRT package (Fox, Klotzke, and Simsek, 2023) to simultaneously estimate τ and θ using a hierarchical model.
  - First level: Separate IRT (2-PO) and RT (log normal) models
  - Second level: priors on τ, θ, and the item parameters

$$
\begin{aligned}
(\theta_i, \tau_i) &\sim N(\mu_P, \Sigma_P) \\
\mu_P &= (\mu_\theta, \mu_\tau) \\
\Sigma_P &= \begin{pmatrix} \sigma_\theta^2 & \rho \\ \rho & \sigma_\tau^2 \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&(a_j, b_j, \phi_j, \lambda_j) \sim N(\mu_J, \Sigma_J) \\
&\mu_J = (\mu_a, \mu_b, \mu_\phi, \mu_\lambda) \\
&\Sigma_J = \begin{pmatrix} \sigma_a^2 & \sigma_{ab} & \sigma_{a\phi} & \sigma_{a\lambda} \\ \sigma_{ab} & \sigma_b^2 & \sigma_{b\phi} & \sigma_{b\lambda} \\ \sigma_{a\phi} & \sigma_{b\phi} & \sigma_\phi^2 & \sigma\phi\lambda \\ \sigma_{a\lambda} & \sigma_{b\lambda} & \sigma_{\phi\lambda} & \sigma_\lambda^2 \end{pmatrix}
\end{aligned}
$$

- $\rho =$ the covariance between θ and τ; mathematically accounts for the possible speed-accuracy trade-off.
- $a_j > 0$ and $\phi_j > 0$


Effect of RT on IRT Theta Estimate
Normed IRT/RT Theta
Normed 2-PL Theta
(a) All Thetas
(b) Only Thetas Greater than 1
(c) Only Thetas Greater than 0.4


Quantitative Reasoning Score and Time Score (Standardized)
Ages 31 to 74
.5
0
-.5
-1
30
35
40
45
50
55
60
65
70
75
AGE OF R ON DEC 31 2015
Theta w/ RT [Ability Score]
Tau [Speed Score]

Theta Quartile	Tau Quartile	Financial Lit Coef (SE)	Financial Lit P-value vs Q1	Verbal Coef (SE)	Verbal P-value vs Q1	Conscientiousness Coef (SE)	Conscientiousness P-value vs Q1
1	1	0		0		0	
1	2	-0.209***	0.003	-0.255***	0.000	-0.00083	0.995
		(0.071)		(0.070)		(0.121)	
1	3	-0.224***	0.010	-0.450***	0.000	-0.289*	0.052
		(0.087)		(0.085)		(0.149)	
1	4	-0.872***	0.000	-0.831***	0.000	-0.264	0.105
		(0.095)		(0.094)		(0.162)	
2	1	0.870***		0.569***		0.0327	
		(0.077)		(0.076)		(0.132)	
2	2	0.659***	0.019	0.239***	0.000	-0.0576	0.558
		(0.080)		(0.079)		(0.137)	
2	3	0.338***	0.000	-0.0155	0.000	-0.0295	0.668
		(0.074)		(0.073)		(0.137)	
2	4	0.246***	0.000	-0.203***	0.000	-0.00845	0.767
		(0.070)		(0.069)		(0.119)	
3	1	1.040***		0.510***		-0.0184	
		(0.079)		(0.0780)		(0.135)	
3	2	0.904***	0.111	0.608***	0.240	0.163	0.210
		(0.072)		(0.071)		(0.124)	
3	3	1.154***	0.176	0.541***	0.711	-0.0272	0.951
		(0.070)		(0.069)		(0.120)	
3	4	1.178***	0.130	0.631***	0.177	-0.203	0.231
		(0.079)		(0.078)		(0.135)	
4	1	1.394***		0.822***		-0.113	
		(0.089)		(0.088)		(0.153)	
4	2	1.574***	0.066	0.832***	0.918	-0.140	0.871
		(0.077)		(0.076)		(0.132)	
4	3	1.563***	0.074	0.789***	0.724	-0.119	0.972
		(0.073)		(0.072)		(0.124)	
4	4	1.641***	0.007	0.797***	0.781	-0.112	0.996
		(0.068)		(0.068)		(0.117)	
	Mean	4.48		4.17		12.12	

Table: Predicting Other Measures by $\hat{\theta}$ Quartile and $\hat{\tau}$ Quartile

Theta Quartile	Tau Quartile	Financial Lit Coef (SE)	Financial Lit P-value vs Q1	Verbal Coef (SE)	Verbal P-value vs Q1	Conscientiousness Coef (SE)	Conscientiousness P-value vs Q1
1	1	0		0		0	
1	2	-0.209*** (0.071)	0.003	-0.255*** (0.070)	0.000	-0.00083 (0.121)	0.995
1	3	-0.224*** (0.087)	0.010	-0.450*** (0.085)	0.000	-0.289* (0.149)	0.052
1	4	-0.872*** (0.095)	0.000	-0.831*** (0.094)	0.000	-0.264 (0.162)	0.105
2	1	0.870*** (0.077)		0.569*** (0.076)		0.0327 (0.132)	
2	2	0.659*** (0.080)	0.019	0.239*** (0.079)	0.000	-0.0576 (0.137)	0.558
2	3	0.338*** (0.074)	0.000	-0.0155 (0.073)	0.000	-0.0295 (0.137)	0.668
2	4	0.246*** (0.070)	0.000	-0.203*** (0.069)	0.000	-0.00845 (0.119)	0.767
3	1	1.040*** (0.079)		0.510*** (0.0780)		-0.0184 (0.135)	
3	2	0.904*** (0.072)	0.111	0.608*** (0.071)	0.240	0.163 (0.124)	0.210
3	3	1.154*** (0.070)	0.176	0.541*** (0.069)	0.711	-0.0272 (0.120)	0.951
3	4	1.178*** (0.079)	0.130	0.631*** (0.078)	0.177	-0.203 (0.135)	0.231
4	1	1.394*** (0.089)		0.822*** (0.088)		-0.113 (0.153)	
4	2	1.574*** (0.077)	0.066	0.832*** (0.076)	0.918	-0.140 (0.132)	0.871
4	3	1.563*** (0.073)	0.074	0.789*** (0.072)	0.724	-0.119 (0.124)	0.972
4	4	1.641*** (0.068)	0.007	0.797*** (0.068)	0.781	-0.112 (0.117)	0.996
	Mean	4.48		4.17		12.12	

Table: Predicting Other Measures by $\hat{\theta}$ Quartile and $\hat{\tau}$ Quartile

Theta Quartile	Tau Quartile	Financial Lit Coef (SE)	Financial Lit P-value vs Q1	Verbal Coef (SE)	Verbal P-value vs Q1	Conscientiousness Coef (SE)	Conscientiousness P-value vs Q1
1	1	0		0		0	
1	2	-0.209***	0.003	-0.255***	0.000	-0.00083	0.995
		(0.071)		(0.070)		(0.121)	
1	3	-0.224***	0.010	-0.450***	0.000	-0.289*	0.052
		(0.087)		(0.085)		(0.149)	
1	4	-0.872***	0.000	-0.831***	0.000	-0.264	0.105
		(0.095)		(0.094)		(0.162)	
2	1	0.870***		0.569***		0.0327	
		(0.077)		(0.076)		(0.132)	
2	2	0.659***	0.019	0.239***	0.000	-0.0576	0.558
		(0.080)		(0.079)		(0.137)	
2	3	0.338***	0.000	-0.0155	0.000	-0.0295	0.668
		(0.074)		(0.073)		(0.137)	
2	4	0.246***	0.000	-0.203***	0.000	-0.00845	0.767
		(0.070)		(0.069)		(0.119)	
3	1	1.040***		0.510***		-0.0184	
		(0.079)		(0.0780)		(0.135)	
3	2	0.904***	0.111	0.608***	0.240	0.163	0.210
		(0.072)		(0.071)		(0.124)	
3	3	1.154***	0.176	0.541***	0.711	-0.0272	0.951
		(0.070)		(0.069)		(0.120)	
3	4	1.178***	0.130	0.631***	0.177	-0.203	0.231
		(0.079)		(0.078)		(0.135)	
4	1	1.394***		0.822***		-0.113	
		(0.089)		(0.088)		(0.153)	
4	2	1.574***	0.066	0.832***	0.918	-0.140	0.871
		(0.077)		(0.076)		(0.132)	
4	3	1.563***	0.074	0.789***	0.724	-0.119	0.972
		(0.073)		(0.072)		(0.124)	
4	4	1.641***	0.007	0.797***	0.781	-0.112	0.996
		(0.068)		(0.068)		(0.117)	
	Mean	4.48		4.17		12.12	

Table: Predicting Other Measures by $\hat{\theta}$ Quartile and $\hat{\tau}$ Quartile

Theta Quartile	Tau Quartile	Financial Lit Coef (SE)	Financial Lit P-value vs Q1	Verbal Coef (SE)	Verbal P-value vs Q1	Conscientiousness Coef (SE)	Conscientiousness P-value vs Q1
1	1	0		0		0	
1	2	-0.209***	0.003	-0.255***	0.000	-0.00083	0.995
		(0.071)		(0.070)		(0.121)	
1	3	-0.224***	0.010	-0.450***	0.000	-0.289*	0.052
		(0.087)		(0.085)		(0.149)	
1	4	-0.872***	0.000	-0.831***	0.000	-0.264	0.105
		(0.095)		(0.094)		(0.162)	
2	1	0.870***		0.569***		0.0327	
		(0.077)		(0.076)		(0.132)	
2	2	0.659***	0.019	0.239***	0.000	-0.0576	0.558
		(0.080)		(0.079)		(0.137)	
2	3	0.338***	0.000	-0.0155	0.000	-0.0295	0.668
		(0.074)		(0.073)		(0.137)	
2	4	0.246***	0.000	-0.203***	0.000	-0.00845	0.767
		(0.070)		(0.069)		(0.119)	
3	1	1.040***		0.510***		-0.0184	
		(0.079)		(0.0780)		(0.135)	
3	2	0.904***	0.111	0.608***	0.240	0.163	0.210
		(0.072)		(0.071)		(0.124)	
3	3	1.154***	0.176	0.541***	0.711	-0.0272	0.951
		(0.070)		(0.069)		(0.120)	
3	4	1.178***	0.130	0.631***	0.177	-0.203	0.231
		(0.079)		(0.078)		(0.135)	
4	1	1.394***		0.822***		-0.113	
		(0.089)		(0.088)		(0.153)	
4	2	1.574***	0.066	0.832***	0.918	-0.140	0.871
		(0.077)		(0.076)		(0.132)	
4	3	1.563***	0.074	0.789***	0.724	-0.119	0.972
		(0.073)		(0.072)		(0.124)	
4	4	1.641***	0.007	0.797***	0.781	-0.112	0.996
		(0.068)		(0.068)		(0.117)	
	Mean	4.48		4.17		12.12	

Table: Predicting Other Measures by $\hat{\theta}$ Quartile and $\hat{\tau}$ Quartile

Theta Quartile	Tau Quartile	Financial Lit Coef (SE)	Financial Lit P-value vs Q1	Verbal Coef (SE)	Verbal P-value vs Q1	Conscientiousness Coef (SE)	Conscientiousness P-value vs Q1
1	1	0		0		0	
1	2	-0.209***	0.003	-0.255***	0.000	-0.00083	0.995
		(0.071)		(0.070)		(0.121)	
1	3	-0.224***	0.010	-0.450***	0.000	-0.289*	0.052
		(0.087)		(0.085)		(0.149)	
1	4	-0.872***	0.000	-0.831***	0.000	-0.264	0.105
		(0.095)		(0.094)		(0.162)	
2	1	0.870***		0.569***		0.0327	
		(0.077)		(0.076)		(0.132)	
2	2	0.659***	0.019	0.239***	0.000	-0.0576	0.558
		(0.080)		(0.079)		(0.137)	
2	3	0.338***	0.000	-0.0155	0.000	-0.0295	0.668
		(0.074)		(0.073)		(0.137)	
2	4	0.246***	0.000	-0.203***	0.000	-0.00845	0.767
		(0.070)		(0.069)		(0.119)	
3	1	1.040***		0.510***		-0.0184	
		(0.079)		(0.0780)		(0.135)	
3	2	0.904***	0.111	0.608***	0.240	0.163	0.210
		(0.072)		(0.071)		(0.124)	
3	3	1.154***	0.176	0.541***	0.711	-0.0272	0.951
		(0.070)		(0.069)		(0.120)	
3	4	1.178***	0.130	0.631***	0.177	-0.203	0.231
		(0.079)		(0.078)		(0.135)	
4	1	1.394***		0.822***		-0.113	
		(0.089)		(0.088)		(0.153)	
4	2	1.574***	0.066	0.832***	0.918	-0.140	0.871
		(0.077)		(0.076)		(0.132)	
4	3	1.563***	0.074	0.789***	0.724	-0.119	0.972
		(0.073)		(0.072)		(0.124)	
4	4	1.641***	0.007	0.797***	0.781	-0.112	0.996
		(0.068)		(0.068)		(0.117)	
	Mean	4.48		4.17		12.12	

Table: Predicting Other Measures by $\hat{\theta}$ Quartile and $\hat{\tau}$ Quartile

Theta Quartile	Tau Quartile	Financial Lit Coef (SE)	Financial Lit P-value vs Q1	Verbal Coef (SE)	Verbal P-value vs Q1	Conscientiousness Coef (SE)	Conscientiousness P-value vs Q1
1	1	0		0		0	
1	2	-0.209***	0.003	-0.255***	0.000	-0.00083	0.995
		(0.071)		(0.070)		(0.121)	
1	3	-0.224***	0.010	-0.450***	0.000	-0.289*	0.052
		(0.087)		(0.085)		(0.149)	
1	4	-0.872***	0.000	-0.831***	0.000	-0.264	0.105
		(0.095)		(0.094)		(0.162)	
2	1	0.870***		0.569***		0.0327	
		(0.077)		(0.076)		(0.132)	
2	2	0.659***	0.019	0.239***	0.000	-0.0576	0.558
		(0.080)		(0.079)		(0.137)	
2	3	0.338***	0.000	-0.0155	0.000	-0.0295	0.668
		(0.074)		(0.073)		(0.137)	
2	4	0.246***	0.000	-0.203***	0.000	-0.00845	0.767
		(0.070)		(0.069)		(0.119)	
3	1	1.040***		0.510***		-0.0184	
		(0.079)		(0.0780)		(0.135)	
3	2	0.904***	0.111	0.608***	0.240	0.163	0.210
		(0.072)		(0.071)		(0.124)	
3	3	1.154***	0.176	0.541***	0.711	-0.0272	0.951
		(0.070)		(0.069)		(0.120)	
3	4	1.178***	0.130	0.631***	0.177	-0.203	0.231
		(0.079)		(0.078)		(0.135)	
4	1	1.394***		0.822***		-0.113	
		(0.089)		(0.088)		(0.153)	
4	2	1.574***	0.066	0.832***	0.918	-0.140	0.871
		(0.077)		(0.076)		(0.132)	
4	3	1.563***	0.074	0.789***	0.724	-0.119	0.972
		(0.073)		(0.072)		(0.124)	
4	4	1.641***	0.007	0.797***	0.781	-0.112	0.996
		(0.068)		(0.068)		(0.117)	
	Mean	4.48		4.17		12.12	

Table: Predicting Other Measures by $\hat{\theta}$ Quartile and $\hat{\tau}$ Quartile

Theta Quartile	Tau Quartile	Financial Lit Coef (SE)	Financial Lit P-value vs Q1	Verbal Coef (SE)	Verbal P-value vs Q1	Conscientiousness Coef (SE)	Conscientiousness P-value vs Q1
1	1	0		0		0	
1	2	-0.209***	0.003	-0.255***	0.000	-0.00083	0.995
		(0.071)		(0.070)		(0.121)	
1	3	-0.224***	0.010	-0.450***	0.000	-0.289*	0.052
		(0.087)		(0.085)		(0.149)	
1	4	-0.872***	0.000	-0.831***	0.000	-0.264	0.105
		(0.095)		(0.094)		(0.162)	
2	1	0.870***		0.569***		0.0327	
		(0.077)		(0.076)		(0.132)	
2	2	0.659***	0.019	0.239***	0.000	-0.0576	0.558
		(0.080)		(0.079)		(0.137)	
2	3	0.338***	0.000	-0.0155	0.000	-0.0295	0.668
		(0.074)		(0.073)		(0.137)	
2	4	0.246***	0.000	-0.203***	0.000	-0.00845	0.767
		(0.070)		(0.069)		(0.119)	
3	1	1.040***		0.510***		-0.0184	
		(0.079)		(0.0780)		(0.135)	
3	2	0.904***	0.111	0.608***	0.240	0.163	0.210
		(0.072)		(0.071)		(0.124)	
3	3	1.154***	0.176	0.541***	0.711	-0.0272	0.951
		(0.070)		(0.069)		(0.120)	
3	4	1.178***	0.130	0.631***	0.177	-0.203	0.231
		(0.079)		(0.078)		(0.135)	
4	1	1.394***		0.822***		-0.113	
		(0.089)		(0.088)		(0.153)	
4	2	1.574***	0.066	0.832***	0.918	-0.140	0.871
		(0.077)		(0.076)		(0.132)	
4	3	1.563***	0.074	0.789***	0.724	-0.119	0.972
		(0.073)		(0.072)		(0.124)	
4	4	1.641***	0.007	0.797***	0.781	-0.112	0.996
		(0.068)		(0.068)		(0.117)	
	Mean	4.48		4.17		12.12	

Table: Predicting Other Measures by $\hat{\theta}$ Quartile and $\hat{\tau}$ Quartile

Theta Quartile	Tau Quartile	Financial Lit Coef (SE)	Financial Lit P-value vs Q1	Verbal Coef (SE)	Verbal P-value vs Q1	Conscientiousness Coef (SE)	Conscientiousness P-value vs Q1
1	1	0		0		0	
1	2	-0.209*** (0.071)	0.003	-0.255*** (0.070)	0.000	-0.00083 (0.121)	0.995
1	3	-0.224*** (0.087)	0.010	-0.450*** (0.085)	0.000	-0.289* (0.149)	0.052
1	4	-0.872*** (0.095)	0.000	-0.831*** (0.094)	0.000	-0.264 (0.162)	0.105
2	1	0.870*** (0.077)		0.569*** (0.076)		0.0327 (0.132)	
2	2	0.659*** (0.080)	0.019	0.239*** (0.079)	0.000	-0.0576 (0.137)	0.558
2	3	0.338*** (0.074)	0.000	-0.0155 (0.073)	0.000	-0.0295 (0.137)	0.668
2	4	0.246*** (0.070)	0.000	-0.203*** (0.069)	0.000	-0.00845 (0.119)	0.767
3	1	1.040*** (0.079)		0.510*** (0.0780)		-0.0184 (0.135)	
3	2	0.904*** (0.072)	0.111	0.608*** (0.071)	0.240	0.163 (0.124)	0.210
3	3	1.154*** (0.070)	0.176	0.541*** (0.069)	0.711	-0.0272 (0.120)	0.951
3	4	1.178*** (0.079)	0.130	0.631*** (0.078)	0.177	-0.203 (0.135)	0.231
4	1	1.394*** (0.089)		0.822*** (0.088)		-0.113 (0.153)	
4	2	1.574*** (0.077)	0.066	0.832*** (0.076)	0.918	-0.140 (0.132)	0.871
4	3	1.563*** (0.073)	0.074	0.789*** (0.072)	0.724	-0.119 (0.124)	0.972
4	4	1.641*** (0.068)	0.007	0.797*** (0.068)	0.781	-0.112 (0.117)	0.996
	Mean	4.48		4.17		12.12	

Table: Predicting Other Measures by $\hat{\theta}$ Quartile and $\hat{\tau}$ Quartile

	Everyone			
	N	$<$ 12 Years	12 to 15 Years	16+ Years
Quartile 1 of θ	1757	8.95%	65.25%	25.80%
Quartile 2 of θ	1559	3.29%	55.21%	41.50%
Quartile 3 of θ	1410	1.14%	47.08%	51.78%
Quartile 4 of θ	1410	1.07%	33.11%	65.81%
	Quartile 1 of $\hat{\theta}$			
	N	$<$ 12 Years	12 to 15 Years	16+ Years
Quartile 1 of τ	727	7.87%	64.09%	28.03%
Quartile 2 of τ	521	10.27%	68.73%	21.00%
Quartile 3 of τ	287	6.17%	61.91%	31.93%
Quartile 4 of τ	222	13.10%	65.12%	21.78%
	Quartile 4 of $\hat{\theta}$			
	N	$<$ 12 Years	12 to 15 Years	16+ Years
Quartile 1 of τ	198	1.99%	39.68%	58.33%
Quartile 2 of τ	312	0.51%	42.38%	57.11%
Quartile 3 of τ	394	1.01%	27.93%	71.06%
Quartile 4 of τ	506	1.11%	28.32%	70.57%

Table: Education Status by $\hat{\theta}$ Quartile and $\hat{\tau}$ Quartile

	Everyone			
	N	$<$ 12 Years	12 to 15 Years	16+ Years
Quartile 1 of θ	1757	8.95%	65.25%	25.80%
Quartile 2 of θ	1559	3.29%	55.21%	41.50%
Quartile 3 of θ	1410	1.14%	47.08%	51.78%
Quartile 4 of θ	1410	1.07%	33.11%	65.81%
	Quartile 1 of $\hat{\theta}$			
	N	$<$ 12 Years	12 to 15 Years	16+ Years
Quartile 1 of τ	727	7.87%	64.09%	28.03%
Quartile 2 of τ	521	10.27%	68.73%	21.00%
Quartile 3 of τ	287	6.17%	61.91%	31.93%
Quartile 4 of τ	222	13.10%	65.12%	21.78%
	Quartile 4 of $\hat{\theta}$			
	N	$<$ 12 Years	12 to 15 Years	16+ Years
Quartile 1 of τ	198	1.99%	39.68%	58.33%
Quartile 2 of τ	312	0.51%	42.38%	57.11%
Quartile 3 of τ	394	1.01%	27.93%	71.06%
Quartile 4 of τ	506	1.11%	28.32%	70.57%

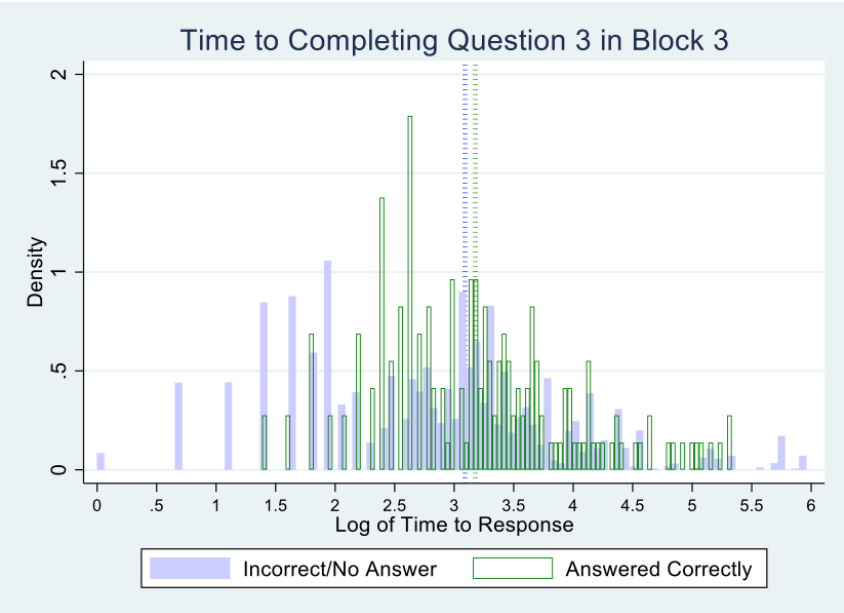
Table: Education Status by $\hat{\theta}$ Quartile and $\hat{\tau}$ Quartile

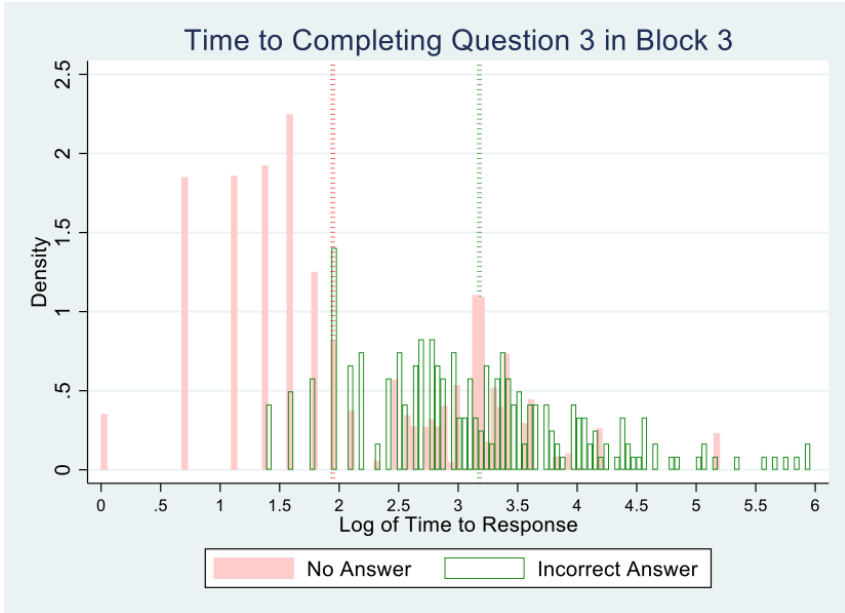
Conclusions and Further work:

- Jointly modeling accuracy and speed produces two independent measures that are related to outcomes in complicated but understandable ways
  - Being faster suggests an additional measure of mathematical ability for higher ability people
  - *But* being slower suggests more human capital for lower ability people
- Our model is technically wrong – we *should* be accounting for the error in the latent traits and modeling the estimates of θ and τ and the structural equation simultaneously (see Schofield, Junker, Taylor, and Black, 2014)

Ideas for survey developers and administrators:

- This work is only possible with item level data – both the accuracy and the RT for each item.
  - Providing both accuracy and RT item level data would be enormously helpful for all tests and assessment; to our knowledge most surveys provide timing data at the survey "block" level.
- Borrowing from neuroscience, surveys could includes tasks that are designed so that speed intentional has meaning
- Further work should investigate if including RT measures in our IRT models might allow for more efficient survey administration.
  - All else being equal, more items tends to produce more accurate estimates of θ, but RT also provides more accuracy.





Illustrative Example of an Item's Time by Type of Answer

Theta Quartile	Tau Quartile	No Education Control Coef (SE)	No Education Control P-value vs Q1	Yes Education Control Coef (SE)	Yes Education Control P-value vs Q1
1	1	0		0	
1	2	-0.209*** (0.0709)	0.0032	-0.174** (0.0695)	0.0121
1	3	-0.224*** (0.0865)	0.0097	-0.267*** (0.0851)	0.0017
1	4	-0.872*** (0.0953)	0.0000	-0.861*** (0.0939)	0.0000
2	1	0.870*** (0.0772)		0.761*** (0.0760)	
2	2	0.659*** (0.0799)	0.0194	0.576*** (0.0787)	0.0370
2	3	0.338*** (0.0737)	0.0000	0.250*** (0.0726)	0.0000
2	4	0.246*** (0.0696)	0.0000	0.160** (0.0685)	0.0000
3	1	1.040*** (0.0790)		0.938*** (0.0780)	
3	2	0.904*** (0.0723)	0.1105	0.791*** (0.0714)	0.0785
3	3	1.154*** (0.0704)	0.1759	1.011** (0.0697)	0.3797
3	4	1.178*** (0.0791)	0.1300	0.994*** (0.0783)	0.5302
4	1	1.394*** (0.0892)		1.225*** (0.0881)	
4	2	1.574*** (0.0771)	0.0658	1.408*** (0.0763)	0.0579
4	3	1.563*** (0.0727)	0.0742	1.338*** (0.0726)	0.2236
4	4	1.641*** (0.0684)	0.0069	1.419*** (0.0686)	0.0313

Table: Regressions of Theta and Tau on Financial Literacy with and without Ed