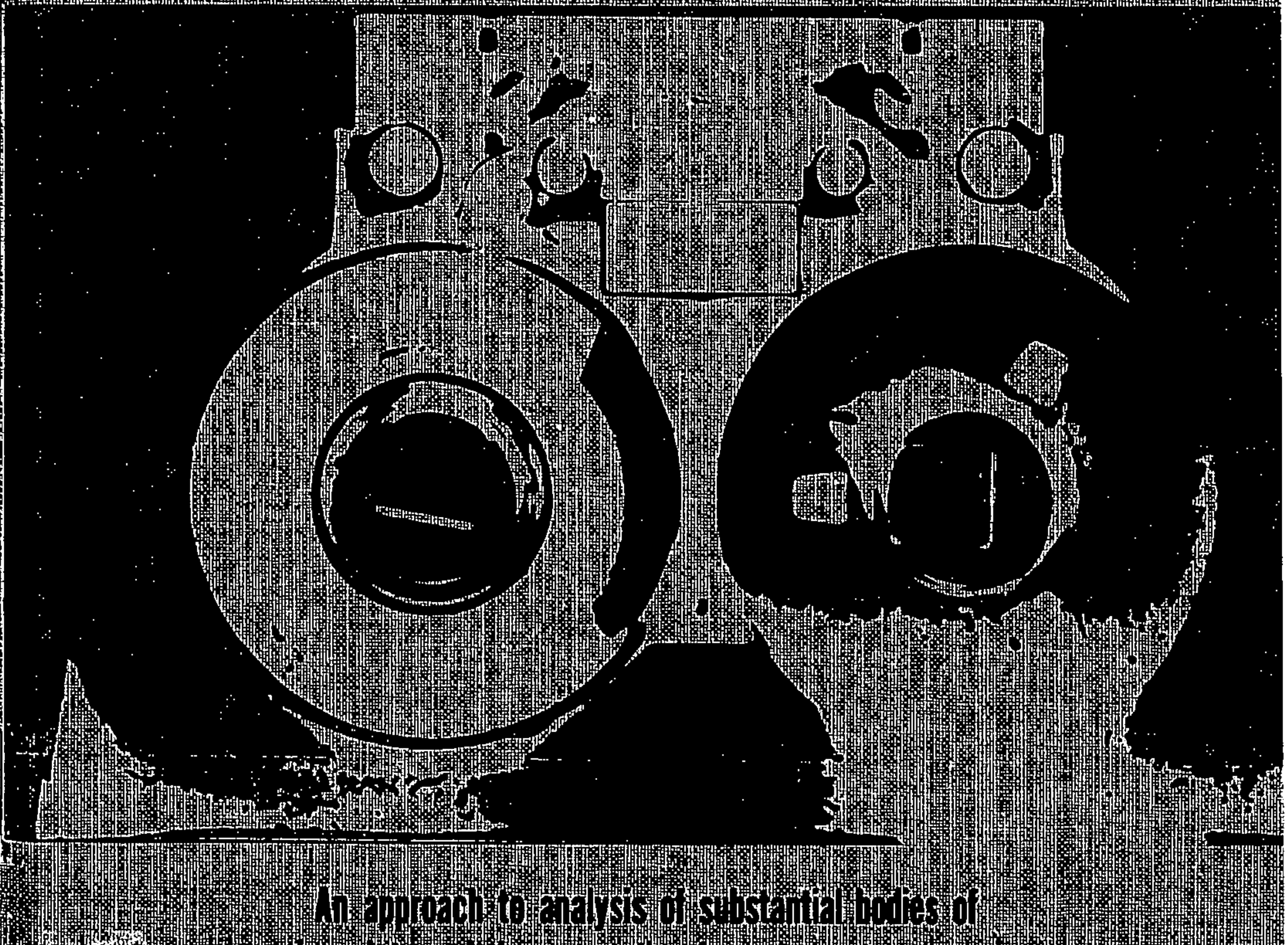
*Revised Edition*

# SEARCHING FOR STRUCTURE

John A. Sonquist
Elizabeth Lauh Baker
James N. Morgan



An approach to analysis of substantial bodies of micro-data and documentation for a computer program

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# Preface

Some years ago, in rebellion against the restrictive assumptions of conventional multivariate techniques and the cumbersome inconvenience of ransacking sets of data in other ways, we produced a computer program entitled The Automatic Interaction Detector.[1] This program simulated what researchers had been doing with data for many years but with prestated strategy and in a reproducible way.

The Structure-Search program described here, termed AID3, is a new and elaborated version of the original AID algorithm. This manual is intended as a technical guide to using the program. In order to make the documentation more complete, parts of the original AID monograph, "The Detection of Interaction Effects," have been incorporated into the text.

This version of AID3 was designed and implemented by the authors in cooperation with other members of ISR's Survey Research Center Computer Support Group. The advice and help of Judith Rattenbury, Neal Van Eck, Laura Klem, Duane Thomas and Robert Messenger are especially acknowledged. Tecla Schrader aided in developing the final program by testing and retesting many combinations of options on several data sets. William Haney provided valuable editorial suggestions. Joan Brinser cleared up the worst obscurities. Maryon Wells, Tracie Brooks, Nancy Mayer and Alice Sano helped with the typing. Priscilla Hildebrandt and Ellen Bronson typed the completed manuscript.

The financial support of the National Science Foundation and the Shell Oil Company is gratefully acknowledged. Mr. V. Hwang, Mr. J. Viladas and Mr. A.

[1] The support of the National Science Foundation for development of both the original "AID" program and the new AID3 documented here is gratefully acknowledged.

Armitage provided valuable suggestions.

The data computation upon which this paper is based employed the OSIRIS computer software system, which was jointly developed by the component Centers of the Institute for Social Research, The University of Michigan, using funds from the NSF, the Inter-university Consortium for Political Research and other sources.

# I

# Introduction to the Program

## 1.1 Program Overview

In many social science research situations the problem in the data analysis is to determine which of the variables are related to the phenomenon in question (under what conditions and through what intervening processes) but may not necessarily involve the exact testing of specific hypotheses.

Data analysis consists of searching for the best model, combining theory and examination of data in the process, and then assessing the best model (or two) by well-known processes of statistical inference. The pure theory of statistical inference requires that the second step be done on a fresh set of data, not those used to select the best model. It also assumes that the model is properly specified. The choice among several competing and probably misspecified models creates unsolved problems in statistics.

The present program focuses on the first step--the searching of data for an optimal model. Theory is involved in the selection, explanatory variables, their hierarchical ranking, and in the interpretation of the results. The likelihood that another sample would give the same results can be estimated by looking at the competitive possibilities at each split, but the probability of replicating the full process is usually negligible, and a test of the final results requires a fresh, independent set of data. Hence no significance tests are provided in this program--they are inappropriate.[1]

The general principle of the AID3 program is an application of a prestated, if complex, strategy simulating the procedures of a good researcher in searching for the predictors that increase his power to account for the variance of the

[1] See J. Morgan and F. Andrews (1973).

dependent variable. Thus the basic principle of least squares is followed, and the focus is on power in reducing error, i.e., on importance rather than on significance. In place of restrictive assumptions, reliance is on a prearranged procedure which starts with the most stable and dependable finding (division of the data set on that predictor which reduces the variance of the dependent variable the most) and works down to less and less dependable and powerful findings on smaller and smaller subgroups.

The data-model to which the procedure is applicable may be termed a "sample survey model," in which values of a set of predictors $X_1$, $X_2$, ... $X_p$, and a dependent variable Y, have been obtained over a set of observations, or units of analysis, $U_1$; $U_2$, ... $U_\alpha$ ... $U_n$ . A weight, $W_\alpha$, may also be established for $U_\alpha$ if the sample is not representative and self-weighting, or if one observation is considered to be more reliable than another. Data may be considered "missing" or undefined on any of the $X_i$.· In particular, this analysis situation is defined to be one in which the $X_i$ are a mixture of nominal and/or ordinal scales (or code intervals of an equal-interval scale) and Y is a continuous, or equal-interval scale. The $X_i$ variables may consist of a mixture of "independent variables" and also "specifiers" (conditions) and "elaborators" (intervening variables).

The question "what dichotomous split on which single predictor variable will give us a maximum improvement in our ability to predict values of the dependent variable?" embedded in an iterative scheme is the basis for the algorithm used in this program. The program divides the sample, through a series of binary splits, into a mutually exclusive series of subgroups. Every observation is a member of exactly one of these subgroups. They are chosen so that at each step in the procedure, the two new means account for more of the total sum of squares (reduce the predictive error more) than the means of any other pair of subgroups.

A major advantage of this procedure is the transparency of the process and the results. At each decision point, the printed output allows one to examine all the alternative divisions of the data set. If several predictors were similar in importance, clearly another set of data might have produced different results. At the end of the process, what one has is a set of subgroups whose definition (pedigree) is clearly and easily defined by the process by which they were isolated and whose characteristics (mean and variance of the dependent variable) are simple statistics.

It is always easy to explain any process by describing it in relation to something else. But this process is <u>not</u> like stepwise regression, factor analysis , or even analysis of variance. The only thing with which it is really comparable is the activity of a researcher investigating a body of data with a basic theory about what variables are important. Stepwise regression adds predictors, but every one has its effect measured over the whole data set. This new procedure measures the effect of each predictor on each subgroup. Variance analysis asks how much of the variance is accounted for by each predictor and by each interaction effect, but it insists that effects, main or interaction, are to be measured over the whole sample. It thus assumes what is often not true. In any case, variance analysis runs into statistical problems with survey data which are not orthogonal since a factorial design with equal numbers in all the n-dimensioned subcells is not possible. The basic additivity of the variances does not hold anyway with such real data.

The variance analysis in the present program is a sequential one-way analysis of variance that is simple, robust, and easy to understand. Factor analysis or smallest-space analyses ignore any dependent variable and merely attempt to reduce a set of things to a smaller set. Factor analysis serves a different purpose and may be necessary to develop a dependent or criterion variable for analysis. With the kind of flexibility in the use of predictors provided by the present program, however, the utility gained by reducing the number or dimensionality of predictors is questionable, particularly since those methods ignore the dependent variable and make a number of unnecessary assumptions of measurability, linearity and additivity. Indeed, one of the things that comes out of analysis using the present program is a new set of complex variables (defining subgroups) which have high explanatory power, and should lead to improved theory as well.

Finally, multiple discriminant functions, canonical correlation, and other "multivariate" procedures all impose restrictive assumptions, e.g. additivity, linearity. Of course, once the best set of non-linearities and non-additivities is decided upon, a linear model can be designed to include them, and fit to a fresh set of data for testing.

A warning to potential users of this program: Data sets with a thousand cases or more are necessary; otherwise the power of the search processes must be restricted drastically or those processes will carry one into a never-never land of idiosyncratic results. A well-behaved dependent variable without extreme cases or severe bimodalities is also assumed. A dichotomous dependent "variable" is usable if it takes one of its values more than 20 and less than 80 percent of the time. The predictors should be classifications, where each of the classes is in a single dimension; otherwise one really should make dichotomies out of each of the categories. Finally, some theory must be applied, if only in the selection of the predictors. If all of them are at the same level in the causal process, they can be used simultaneously; but if they are at different levels, a more complex strategy must be used.

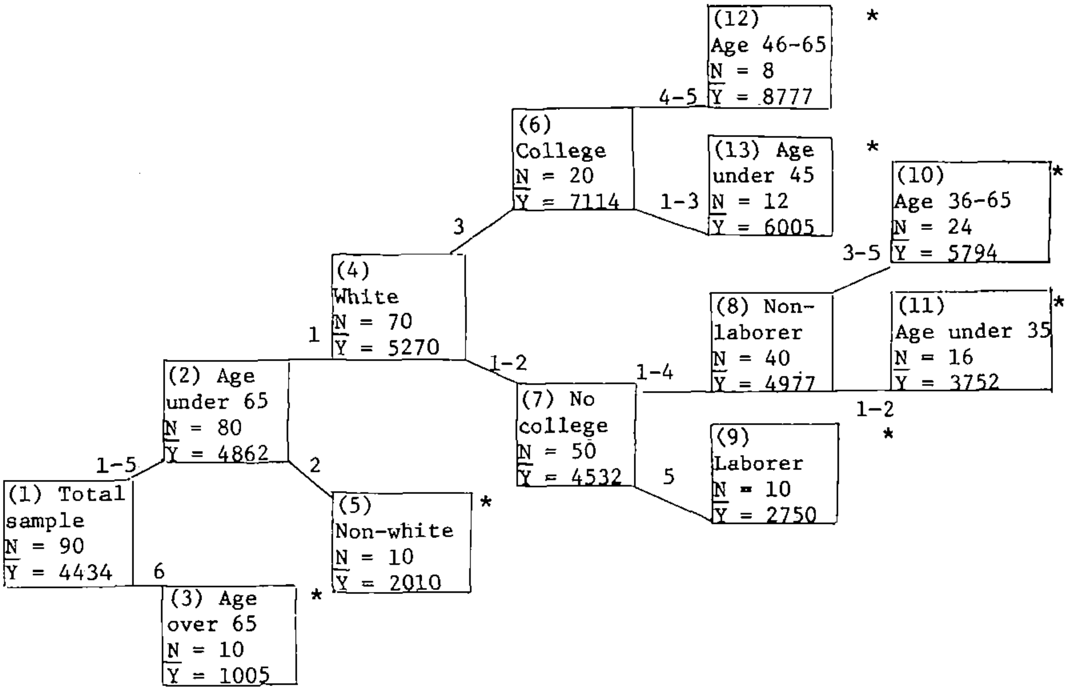
## 1.2 Output Illustration for the Original Algorithm

The following results, contrived, but realistic, will illustrate the basic output of the procedure. Suppose that Age, Race, Education, Occupation, and Length of Time in Present Job, are used in an analysis to predict Income. Age is an ordered series of categories represented by the numbers [1,2, ...,6]. Race is coded [1 or 2], Occupation is coded [1,2, ..., 5], Education is coded [1,2,3], and Time on Job is coded [1,2, ..., 5]. We find the following mutually exclusive groups whose means may be used to predict the income of observations falling into that group:

| Group | Type | N | Mean Income | σ |
|---|---|---|---|---|
| 12 | Age 46-65, white, college | 8 | $8777 | $773 |
| 13 | Age under 45, white, college | 12 | 6005 | 812 |
| 10 | Age 36-65, white, no college, nonlaborer | 24 | 5794 | 487 |
| 11 | Age under 35, white, no college, nonlaborer | 16 | 3752 | 559 |
| 9 | Age under 65, white, no college, laborer | 10 | 2750 | 250 |
| 5 | Age under 65, nonwhite | 10 | 2010 | 10 |
| 3 | Age over 65 | 10 | 1005 | 5 |
| Total | | 90 | 4434 | 2263 |

A one-way analysis of variance over these seven groups would account for 95 per cent of the variation in income.

These results are arrived at by the following procedure, as represented by the tree of binary splits:



When the total sample (group 1) is examined, the maximum reduction in the unexplained sum of squares is obtained by splitting the sample into two new groups, "age under 65" (classes 1-5 on age) and "age 65 and over" (those coded 6 on age). Note that each group may contain some nonwhites and varying education and occupation groups. Group 2, the "under-65" people are then split into "white" and "nonwhite." Note that group 5, the "nonwhites" are all under age 65. Similarly the "white, under age 65" group is further divided, into college and non-college individuals, etc. A group which can no longer be split is marked with an asterisk and constitutes one of the above final groups. The variable "Length of Time in Present Job" has not been used. At each step there existed another variable which proved more useful in explaining the variance remaining in that particular group.

The predicted value $Y_\alpha$ for any individual $\alpha$ is the mean, $\bar{Y}_i$, of his final group. Thus $Y = \bar{Y}_i + \varepsilon$, where $\varepsilon$ is an error term. Prediction of income on the basis of age, education, occupation and race would provide a considerable reduction in error. Variables which "work" are, of course, the most logical candidates for inclusion in a theoretical framework.

## 1.3 Capabilities and Differences from Previous Versions

The AID3 program is a generalized data analysis system, based on modifications of the original AID algorithm, and incorporating highly flexible capabilities for selecting subsets of variables and analyzing segments of the user's data file. It provides capabilities for controlling the way in which the variables are used in the automated sequential analysis, and provides for improved user intervention in this sequential partitioning process.

The analysis of variance model implemented in the original AID algorithm has been extended to include a covariate. Thus, an analysis can now be set up to maximize differences in group means, differences in the slopes of the regression of the dependent variable on the covariate, or differences in explained sums of squares due to regression (means and slopes). The sequential partitioning process of the procedure has undergone extensive modifications to make it more sophisticated in its search. The algorithm can now be set to examine the explanatory power of several sequences of prospective partitions before the choice of the first one is actually made. Thus, present explanatory power can be sacrificed temporarily in favor of even greater potential gains in subsequent partitions.

User controls over the behavior of the algorithm have been significantly improved, facilitating the exploration of interesting findings revealed in the course of the analysis. The analyst can request that the automated search procedure start from a particular point in a pre-specified partial tree structure. He can specify that certain predictors be used first in the partitioning process, or he can insist that statistics be computed for certain predictors, but that they not be actually used as the basis for a partition. The entire set of predictors can even be replaced in successive stages of a run. The analyst can use these capabilities to impose limitations on the partitioning process that are consistent with the kinds of causal explanatory assumptions he is willing to make about his data.

A further improvement is the capability to compute the potential explanatory power contained in the entire subset of predictors chosen for the analysis. This "configuration" rating is the upper bound of the statistical "usefulness" of this set of predictors, and represents the amount of variation that could be explained if the predictors were to be used in a model containing all possible interaction terms and main effects. It is essentially an analysis of variance using all the subgroup means in a k-way table if there are k different predictors.

The type of input acceptable to the program is basically the same as that acceptable to the original version, but with two important restrictions lifted. First, extremely powerful recoding and variable generation capabilities have now been provided. Thus, it is no longer necessary for the user to make expensive and time-consuming preliminary runs on other programs to recode or alter his input variables. He may even alter the coding scheme of his dependent variable and predictors in the course of one run, or he may generate new forms of any of his input variables. Secondly, this capability is used to provide an extremely powerful facility for selecting subsets of the input file for analysis, and for repeating analyses over several such groups. In addition, this recoding facility provides improved user control over the handling of missing data, either by assignment, exclusion, or randomization.

Improvements have also been made in the types of output supplied to the analyst. Information formerly scattered over many pages has been gathered together in more concise tabular form. The analyst may choose to receive an output file containing any or all of his input and generated variables as well as predicted values and residuals for each stage of his analysis.

It has been our experience that it takes time to examine the output, rethink over strategy, and that batch processing rather than an interactive mode is satisfactory. Indeed, the general principle of a prestated strategy rather than artistic ad hoc revisions of strategy at each local decision point appeals to us.

Two types of program operation modes are available: parameter definition (or redefinition), and execution. Three types of program functions can be requested in any sequence desired by the user. These are data input, computation, and output. A "run" on the program (to use computer batch-processing terminology) consists of an ordered sequence of parameter definitions and requests for execution of functions using then current values of the parameters. This series of macro instructions is executed in the order defined by the analyst in submitting his control information stream to the computer. After the execution of each function a query is made by the program to this control stream for information as to what function is to be performed next and what parameters are to be re-defined.

# II
# Analysis Strategy

## 2.1 Basic Procedure

The AID3 algorithm uses a repeated one-way analysis of variance technique to explain as much of the variance of a dependent variable as possible.

The simple conceptualizations given below should aid one's understanding of the program. If one thinks of the error in predicting the value of some variable in a small data set and its progressive reduction by knowing things about each case, the following holds:

(1) If one knows absolutely nothing about the variable, not even the sign, and can only predict 0:

Error variance is the sum of squares of the Y's = $\Sigma Y^2$

(2) If one knows only the overall average, one predicts that for each case:

Error variance is $\Sigma(Y-\bar{Y})^2 = \Sigma Y^2 - \frac{(\Sigma Y)^2}{N} = \Sigma Y^2 - N\bar{Y}^2$

(3) If one knows the average $(\bar{Y})$ for each of two groups, and for each case knows which group it is in:

Error variance is: $\Sigma Y_1{}^2 - N_1\bar{Y}_1^2 + \Sigma Y_2^2 - N_2\bar{Y}_2^2 = \Sigma Y^2 - N_1\bar{Y}_1^2 - N_2\bar{Y}_2^2$

which is less than (2) by:

$$N_1\bar{Y}_1^2 + N_2\bar{Y}_2^2 - N\bar{Y}^2$$

Put another way, with one overall average, one explains $N\bar{Y}^2$ of the variance. With averages for two groups, one explains $N_1\bar{Y}_1^2 + N_2\bar{Y}_2^2$ of the variance.

Clearly the two group means must be different and the two groups not too different in size (both of some appreciable size) for maximum further reduction in error by knowing in which group individuals belong.

The present version of the program allows reduction in error not solely by using means but alternatively by using simple regressions for each group. By knowing which of two groups a case is in, one can do better if the two have either different means or different regression slopes (on the single covariate allowed) or both. The program allows either one or both sources of error reduction to be applied. (See Section 2.3 below.)

On the continuum between testing one pre-specified model (set of hypotheses) and completely flexible artistic data-searching, the approaches facilitated by this program fall in the middle. The program operates sequentially, imposes a minimum of assumptions on the data (selection of predictors, and the mode of classifying those with much detail), does a great deal of searching; but it does pre-specify the strategy of the search process so that it is reproducible. A rerun using the same specification for both the dependent variable and the predictors on the same data set will produce identical results. A similar run on another data set will probably produce something similar, at least for the first few steps.

The most common restrictive assumptions made by statisticians for easing the computational and analytical burden are those of linearity and additivity. With large data sets these restrictions are unnecessary. The use of categorical predictors representing subclasses of predictors (dummy variables) with multiple regression can deal with nonlinearities in the relationships quite adequately; hence it is the additivity assumption that is a problem. Additivity means the absence of any interaction effects, the effects of $X_1$ on Y being pervasive and independent of the levels of any other factor. It is cumbersome to handle interactions using regression. Either one runs separate regressions for subgroups, ending up with no overall relationships, or one introduces "interaction terms" which are themselves restrictive and limited. Suppose there are two predictors, each with three levels. Any one of the nine combinations of the two may reveal some non-additive effects, higher if the two are complements or lower if they are substitutes. Should one introduce separate terms (dummy variables) for each possibility, omitting the main effects, or what? If one omitted one level of each of the predictors in order to use usual regression programs (since the membership in a third class is a linear function of membership in the other two -- if you are in neither of them you must be in the third), then the cross-product terms would specify only four of the nine combinations, which particular four depending on which level of each predictor was omitted! The goal is not exhausting all the information in the predictors but discovering how they work.

Once one allows for higher-order interactions, the possibility of introducing variables for all of them in a simultaneous analysis dims rapidly. The real world, however, is full of examples of higher-order interaction effects. There are things which are substitutes for one another--any one of several handicaps can make a family poor. And there are some results which require that a combination of things be right. An interesting, if somewhat usual, example of this is the result of an analysis of time spent on do-it-yourself activities, shown in Figure 1. It indicates that any one of several things inhibit such activity or, put another way, only a combination of several favorable factors leads to a substantial amount of such activity. Read the right-hand boxes down the page.

There is so much confusion between interaction effects and intercorrelation among predictors that it may pay to distinguish them.[1] Multiple regression handles the problem of intercorrelation among predictors, so long as it is not too extreme. When the predictors are categorical, it simply means that one does not have a factorial design, i.e., equal numbers of cases for each of the possible combinations. The weighted means in any one dimension, as a result, reflect both the effect of that dimension and the hidden spurious effects of other factors disproportionately represented in the groups. In Figure 2 the intercorrelation shows up in the distribution of cases (Part A) and in the weighted means of Part B. With reasonably small errors around the subgroup means given in Part B, a dummy-variable regression will uncover the $4,000 education differential and the $2,000 age differential using only the intercorrelation data from Part A and the weighted means outside Part B. The investigator will then report that the negative correlation between age and education had hidden part of the effects of each--the simple weighted means differing by less than the true effects, i.e., by only $2,800 for education and $400 for age.

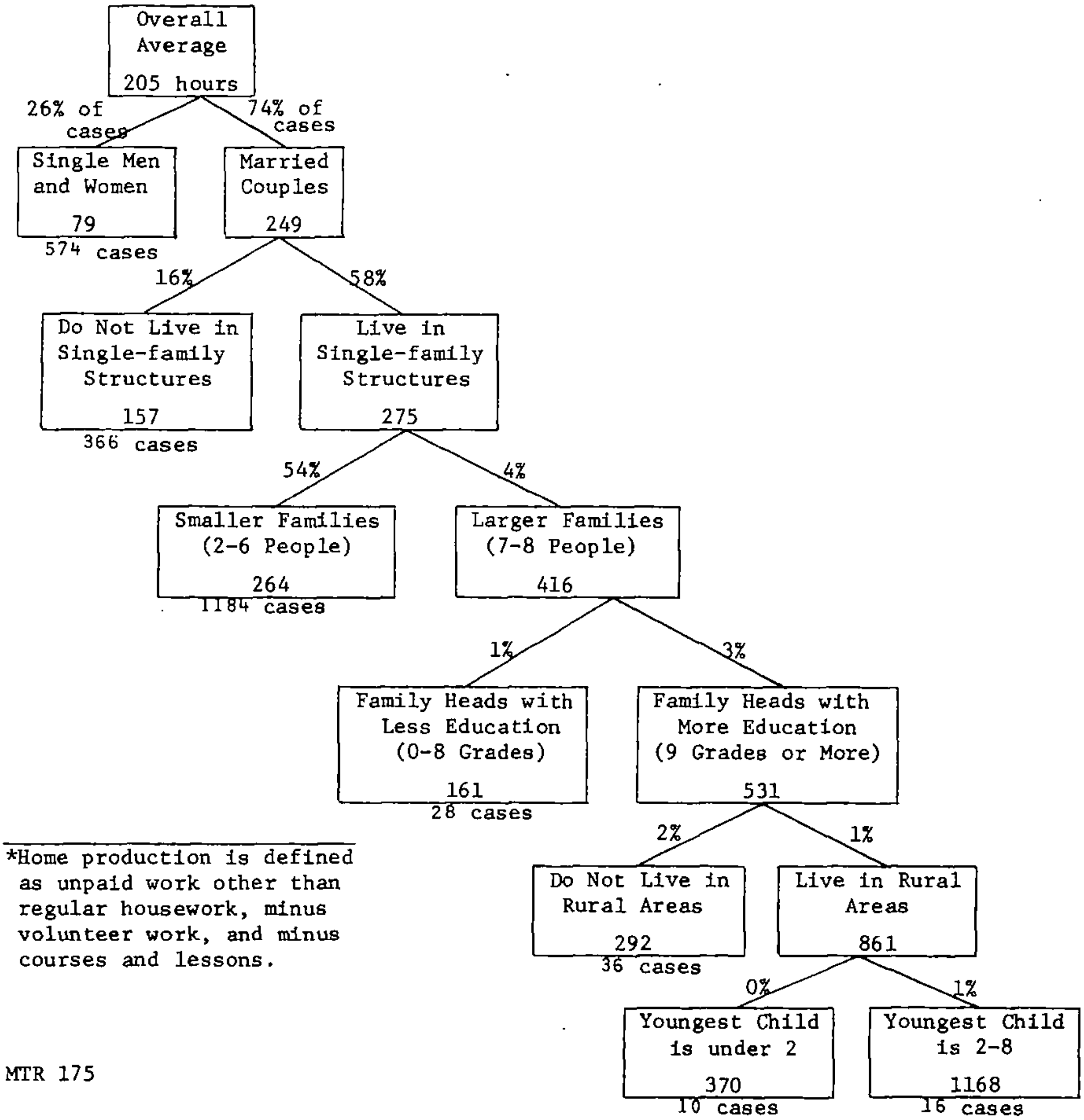
But if there is, in addition to the direct effects, an added $1,000 a year bonus to people with both education and experience (as shown in Part C), a regression would indicate somewhat larger effects of both age and education. Adding a cross-product term would locate the $1,000 interaction effect, but only if one happened to define it in the one way out of four possibilities that hit the correct corner. And with more complex combinations, and/or with more levels, the

[1] See also Appendix VII, James N. Morgan and John A. Sonquist, "Problems in the Analysis of Survey Data, and a Proposal," Journal of the American Statistical Association 58 (June 1963), 415-435, reprinted from Sonquist and Morgan, The Detection of Interaction Effects, Institute for Social Research, The University of Michigan, Ann Arbor, Michigan, 1964.

Or see John B. Lansing and James N. Morgan, Economic Survey Methods, Institute for Social Research, The University of Michigan, Ann Arbor, Michigan, 1971.

Figure 1

Hours of Home Production Done in 1964 by Heads of Families and Wives*
(For 2214 families)


Overall Average
205 hours
26% of cases
74% of cases
Single Men and Women
79
574 cases
Married Couples
249
16%
58%
Do Not Live in Single-family Structures
157
366 cases
Live in Single-family Structures
275
54%
4%
Smaller Families (2-6 People)
264
1184 cases
Larger Families (7-8 People)
416
1%
3%
Family Heads with Less Education (0-8 Grades)
161
28 cases
Family Heads with More Education (9 Grades or More)
531
2%
1%
Do Not Live in Rural Areas
292
36 cases
Live in Rural Areas
861
0%
1%
Youngest Child is under 2
370
10 cases
Youngest Child is 2-8
1168
16 cases

*Home production is defined as unpaid work other than regular housework, minus volunteer work, and minus courses and lessons.

MTR 175

Source: J. Morgan, I. Sirageldin and N. Baerwaldt, Productive Americans, Survey Research Center, Ann Arbor, Michigan, 1965, p 128.

Figure 2

A Simple Example of Intercorrelation and Interaction

Part A.

| | Uneducated | Educated | |
|---|---|---|---|
| Youth | 10 | 40 | 50 |
| Old | 40 | 10 | 50 |
| | 50 | 50 | 100% of Sample |

Intercorrelation
(Negative)

Part B.

| | Uneducated | Educated | Means* |
|---|---|---|---|
| Youth | $3000 | $7000 | $6200 |
| Old | $5000 | $9000 | $5800 |
| | $4600 | $7400 | $6000 |

No Interaction Effect (But Weighted Means are Affected by Intercorrelation)

Part C.

| | Uneducated | Educated | |
|---|---|---|---|
| Youth | $3000 | $7000 | $6200 |
| Old | $5000 | $10,000 | $6000 |
| | $4600 | $7600 | $6100 |

Interaction Effect Added

*All means outside boxes are weighted.

likelihood of finding interactions rapidly diminishes.

If one took the weighted means, one factor at a time, and used them to predict the interior of the table, one would see the large positive deviation of actual from expected in the lower right corner.

How one describes an interaction effect is always arbitrary. In the present example one might say that the effect of education on earnings depended on one's age, or that the effect of age on earnings depended on education. Either may be true, but choice of a statement implying causal direction must be based on other considerations, not the data themselves.

The crucial point is that not only are main effects not necessarily the same or even present in all parts of the sample (or population), but interaction effects themselves may be of various complex kinds affecting only some subgroups.

The theoretical importance of these considerations should be kept in mind. Many theories of human behavior, whether from economics, psychology, sociology or the new political science, deal with behavior of those who have a choice to make, i.e., not dominated by other less interesting forces or constraints. Hypotheses are built on what affects those at the margin, to use the economist's phrase. But if many people are not free to make choices or are dominated by other forces (which may not change over time, or be subject to policy, or even be interesting), then the data may show that the overall effect of some important theoretical variable is insignificant, when in fact it is quite powerful for the relevant subgroup--something that this program will reveal.

If this program handles non-additivities better than regression, does it handle intercorrelations among predictors as well? The answer is that it handles them differently. In regression a simultaneous estimate is made of the effects of each of two correlated predictors, each effect adjusted for the fact that those in a class on one predictor are distributed differently (from the rest of the sample) over classes of the other predictor. The nature of this simultaneous solution can be seen best when it is described as an iterative adjustment process.[1]

In contrast, the AID program divides the sample on the most powerful of two correlated predictors, and searches the two subgroups to see whether the other still matters. If the two are largely correlated and have similar effects on the dependent variable, then the second usually loses most or all of its power and may well never appear in the branching diagram. Since rather few groups

[1] See F. Andrews, J. Sonquist and J. Morgan, Multiple Classification Analysis, Ann Arbor, Survey Research Center.

can exhaust the explanatory power of any one predictor, the pre-emption by one predictor is often dramatic. But this merely dramatizes the problem of intercorrelation. These may be situations where one wants a simultaneous estimation--then a regression model is required.

The program formalizes and makes explicit the exploratory nature of one's analysis so that it can be judged, repeated, and tested on other data sets. Most behavioral-science investigators have only a rudimentary theory, particularly in terms of the measured variables, as distinct from the theoretical constructs they hopefully represent. Even those who start with one model and test it usually end up testing several other alternatives, frequently by segregating subgroups for separate analysis.

To use the present program one must specify a dependent variable, a set of predicting characteristics, and some strategy parameters, which are discussed below. It examines the full data set using each predictor, and with each searches for the best single division according to that predictor. "Best" means the largest reduction in predictive error from knowing to which of two subgroups on that predictor each case belongs (and the means or simple regressions of those subgroups). The criterion is one of importance in reducing error, not statistical significance.

Where a predictor has a natural order (e.g., age) that order can be preserved, or the order can be left unspecified, in which case the categories are reordered according to the level of the subgroup means on the dependent variable. In either case, with k subgroups, there are k-1 possible ways to form two groups, and the division that makes the largest contribution to error reduction ("between sum of squares") is retained.[1] Among these best-for-that-predictor splits, the one (over all the predictors) which reduces the error variance the most is used to divide the data set into two groups.

Needless to say, the same predictor may be used again to divide the sample into several subgroups. Indeed, if there is one extremely powerful predictor, it may dominate the process and it suggests turning to a covariance search process.

A brief description of the splitting process is as follows:

(A) Choose the unsplit group L which has the largest sum of squares

$$SS_L = \sum_{\alpha=1}^{N_L} (Y_{L\alpha} - \overline{Y}_L)^2.$$

The total input sample is the first group, i.e. L=1.

[1] The first versus the k-1 others, the first two versus the k-2 others, etc.

(B) For each predictor $P_i$ find that division of the classes in $P_i$ that provides the largest reduction in the unexplained sum of squares. That is, split L (the "parent" group) into 2 non-overlapping groups (or "children") L1 and L2 so as to maximize the between sum of squares. For example, in a means analysis:

$$BSS_i = N_{L1}\overline{Y}^2_{L1} + N_{L2}\overline{Y}^2{}_{L2} - N_L\overline{Y}^2_L$$

where $N_{L1}+N_{L2}=N_L$, and $N_{L1}$, $N_{L2} \geq$ NMIN; NMIN is a minimum group size requirement (see (2) below). Note that if the order of a predictor with k classes is maintained only k-1 possible splits are checked. If not, one of k! possible orders is selected first (the one in order according to the mean of the dependent variable), then k-1 computations made. To avoid undue chances for idiosyncratic findings, it is wise to maintain the order of each predictor, or, if that is impossible, to convert it to a set of dichotomies.

(C) Select that predictor $P_j$ such that $BSS_j \geq BSS_i$ , $j \neq i$, and if $BSS_j \geq p_e SS_1$ split L into the 2 groups L1 and L2 defined for the predictor $P_j$. The parameter $p_e$ is an eligibility criterion (see (1) below). If $BSS_j < p_e SS_1$ then L is deemed a final group and is not split.

(D) Return to step (A).

The process stops when one or more of the several criteria below are met:

(1) The marginal (added) reduction in error variance if a split occurred would be less than some prestated fraction of the original variance around the mean; often the value .006 (0.6%) is chosen. This is the best criterion to use.

(2) If a split on a group were to occur, one or both would have fewer than some prestated number of cases (e.g., 25) and the mean would be unreliable. This is usually a dangerous rule, since (a) the least squares criterion being used is very sensitive to extreme cases, (b) cases in subgroups can appear extreme even if they don't in the full sample, and (c) the program can alert the researcher to their presence (and damage) by isolating a group of one or two cases that account for a substantial fraction of the variance if this criterion is not used.[1]

(3) The total number of splits has already reached some prestated maximum

[1] See Section 2.8 for one method of treating extreme cases.

(e.g., 30), meaning that there are already that many final groups plus one and more than twice that many groups altogether. This is a useful secondary safeguard to prevent generating too many groups through inadvertence, e.g., in setting the first, main criterion too low. These criteria insure that the process stops before unreliable reduction in error variance occurs.

It should be obvious that if there are M different predictors of K subclasses each, even if all are maintained in some logical order, each split looks at M (K-1) possibilities and by the time twenty-five such splits have been decided upon, the program has selected from among 25M (K-1). With twenty predictors of ten classes each, this is 4,500. If any reordering of scales is allowed, the number explodes. Hence there is no point asking about statistical significance or degrees of freedom.

What can we say about the stability of the process? Each division selected is on the basis of an estimated "between sum of squares," a variance, as compared with similar measures for competing alternatives. The sampling stability (likelihood of producing the identical split on another sample) is clearly dependent on the sampling variance of the difference between the best reduction in variance and the reduction in variance occasioned by the use of each of the various competing predictors, which depends on differences between pairs of variances and their sampling errors. And of course the probability of getting the same _sequence_ of splits is the product of the probability of getting the first (one minus the probability of getting any of the others) times the probability of making the same second split, etc., a product which diminishes in value rapidly. Of course it is possible to end up with the same breakdown of a sample with splits in different orders, i.e., one can split first on age and then on education, or the reverse, and end with the same final groups.

The examination of the predictors for competing alternative splits (two or more predictors where splits reduce the error variance about the same amount) for each subgroup provides a clear picture of the amount of intercorrelation among the predictors. If there are two competing predictors, and a split is made on one of them and the other retains no explanatory power over either of the resulting two groups, one sees clearly that they are alternative explanations and probably highly correlated with one another. Instead of a simultaneous process of "dividing up the credit" among correlated predictors, the sequential process used here selects one and reports that having once taken account of it (even in a single binary split) the other doesn't matter any more for that group. Once again, as with the extreme case problem, the results are transparent and face the researcher with his problems, rather than burying them in complex statistics.

Researchers accustomed to using numerical variables are often concerned with the use of binary divisions and with the possible loss of explanatory power when numerical predictors are converted into a few subclasses. The example below shows the potential loss of power from grouping a numerical predictor variable. It should be noted that when a predictor is used in several splits, dividing the sample into three or four subclasses on that dimension, the results are even better than in Table 1 because one has used only the best of the information. But even if one used all the classes available, as in dummy variable regression, it is clear that the losses in power are minimal, if the relation is actually linear. If the relationship is not linear, then one often does better by grouping a numerical variable in terms of explanatory power, to say nothing of the fact that one learns more about the real world.

Table 1

Percent of the Total Possible Explanatory Power Achieved By Using Equal Sized Subgroups Instead of Regression[1]

| | With: | | | |
|---|---|---|---|---|
| Number of Subgroups | Uniform Distribution (□) | Triangular Distribution (△) | Right Triangle (◿) | Bimodal Right Triangle (◺◿) |
| 2 | .75% | .67 | .67 | .89 |
| 3 | .89 | - | - | - |
| 4 | .93 | .89 | .91 | .96 |
| 5 | .96 | - | - | - |
| 6 | .97 | .95 | .96 | .98 |
| 7 | .98 | - | - | - |
| 8 | .98 | .97 | .98 | .99 |
| 10 | .99 | .98 | .99 | .99 |

Source: Graham Kalton, A Technique for Choosing the Number of Alternative Response Categories to Provide in Order to Coerce An Individual's Position on a Continuum (Memos of Nov. 7, 1960, Feb. 10, 1967 and March 10, 1967), Sampling Section, Institute for Social Research, Ann Arbor, Michigan.

[1] If a regression gives $r^2$ and the squared correlation ratio using K subgroups of that predictor is $\eta^2$, the table gives $\frac{\eta^2}{r^2}$. A useful approximation is is $\frac{\eta^2}{r^2} = 1 - \frac{1}{K^2}$ .

In fact, though, the comparison is not fair since the search process uses up many more degrees of freedom. A branching diagram with a dozen final groups usually accounts for as much of the variance of the dependent variable as a dummy-variable regression with sixty or more dummy variables (70 or more sub-classes of predictors).

Perhaps the most striking possible result from the program is the firm conclusion that some particular predictor may not matter. With an additive model, one is never sure about the possibility that a factor might matter for some subgroup of the population. But if that factor cannot account for any substantial fraction of the variance of the dependent variable over the whole sample or over any of the various different but homogeneous subgroups created by the program, then one can confidently dismiss it.

Results are independent of the order in which predictors are introduced, in spite of the sequential nature of the decisions made, but they are of course dependent on which predictors are used. Since there are often predictors which can affect other predictors but cannot be affected by them, the program allows for conducting the analysis in stages. One can introduce a set of basic background factors, remove their influence by calculating for each individual his deviation from the average of the final group to which he belongs, reassemble the full data set and analyze these residuals using another set of predictors. Since this process assumes no interaction between stages, one may want to reintroduce some of the initial predictors at the second stage. For instance, age, race, or education may be used in the first stage as background but can as well be used in the second stage. For example, the influence of moving to the city may depend on education.

It must be kept in mind that analysis of residuals, which is also done with ordinary regression, is not usually the best way to estimate the marginal or added power of certain predictors. The influence of the other things has been removed only from the dependent variable. True partial correlation requires removing their influence also from the predictor in question. (There also exists a concept called "part correlation" where the influence of other predictors is removed from the predictor in question, but not from the dependent variable.) It would be possible to derive two sets of residuals, using the present program, and correlate them as the tightest fair test of a nonspurious correlation.

The present program also provides evidence for marginal contributions of a predictor in the sense of their added effect on groups already freed from most of the effects of predictors already used to create them and free from the assumption of partial correlation coefficients that the marginal contribution is pervasive and universal, rather than restricted to a sub-part of the population. Or it provides such evidence in the form of the second-stage analysis of residuals from the first stage.

*If the dependent variable is a dichotomy*, the results are given in the form of proportions which are on one side of the dichotomy. Even with numerical variables it is sometimes useful in analysis to form two alternative dichotomies to find out whether the factors which push many people toward one end of a scale are actually the mirror image of those which push them toward the other end. This is not a non-linearity issue but a substantive theoretical issue. For instance, factors associated with increases in savings accounts are not the reverse of those associated with decreases.[1]

The capacity to produce residuals from one analysis (and expected values--the final group mean attached to each individual in the group), enables the program to identify and study extreme cases in more detail (cases deviant from their own group), or to develop an expected value of a variable that could be used as a predictor in a second analysis in the tradition of two-stage least squares or instrumental variables, reducing the errors-in-variables problem.

The remainder of this chapter is an examination of the features and options that are offered to the user.

## 2.2 Configurations

A rather special option unrelated to the main algorithm is a provision for finding the explanatory power of the subcell means of all possible combinations of a set of predictors. Instead of operating sequentially it subdivides the sample factorially, even though some combinations have few or no cases, and does a one-way analysis of variance components indicating what fraction of the total variance (around the mean) is accounted for by the subgroup means, i.e., by $\Sigma N_i \bar{Y}_i^2 - N\bar{Y}^2$.

It requires an extra prior step, sorting the data on all the predictors, in ascending order, so that all cases with any given combination of predictor values are together.

Meehl (1950) coined the term configuration and discussed what appeared to be a paradox in which dichotomous items taken singly had no correlation with a criterion, but their cross-product correlated. The use of configuration terms in scaling was then discussed by Stouffer, Borgatta, Hays and Henry (1953).

[1] See Eva Mueller and Jane Lean, "The Savings Account as a Source for Financing Large Expenditures," *Journal of Finance* 22 (September 1967), 375-393.

Horst (1954) showed that Meehl's configurational techniques were a special case of multiple regression with all variables having values of 1 or 0 and including all possible cross-product terms in the equation. Lubin and Osburn (1957) developed the rationale even further, relating it to what they termed "pattern analysis" by presenting a method for analyzing the relationships between a set of dichotomous items and a quantitative criterion. Their general polynomial equation for the optimal prediction of a criterion in its configural form was shown to have maximum "validity" in the least squares sense. They defined a dichotomous configural scale as follows:

1. Given a test of "t" dichotomous items, there are $2^t$ possible answer patterns (configurations) and a mean criterion score associated with each.
2. Assign this mean as the predicted criterion value for all individuals in an answer pattern.

They showed that the zero-order correlation of the configuration scale with the criterion was equal to or greater than the correlation of the criterion with any other set of scores based on the answers to the "t" dichotomous items. This follows immediately from the formula for the mean, since, by definition, it produces the smallest sum of squared deviations. Consequently, the pattern means must explain more variation than any other set of means. Sonquist (1970) discusses this further. The extension to polytomous predictors is straightforward.

Computation of the configuration score provides the analyst with some indication of what his predictors are worth in explanatory power when all the "stops are pulled." It is suggested that if the variation explained by the configuration is undesirably small, the analyst had best spend his time obtaining hints as to what other variables he might undertake to include in a subsequent investigation.

Basically this option calculates a single one-way analysis of variance asking what fraction of the total variance is accounted for by the subgroup means if one defines a subgroup for each combination of predictor-classes.[1] The fraction is of the sample, not of the population, as is true of all the fractions of variance "explained" as used in this program. Extrapolations to the population are difficult and depend on the variation in subcell sizes.

There is a limit on the number of subgroups (possible combinations) that can

[1] For one predictor at a time the eta squared in the optional predictor summary table is equivalent to a one-way analysis of variance, again for the sample, not extrapolated to the population.

be handled, making it advisable to recode predictors into trichotomies in order to use more of them. Five trichotomies produce 243 subcells. In any case, one should not stretch the limits because with enough detail one could always make the unexplained variance approach zero. The dependent variable should also have its extreme cases truncated, to avoid erratic results.

The result is usefully compared with dummy-variable regression to see the extent of the explanatory power lost with regression by assuming additivity. It can also be compared with the usual output to see what the tree gains by not assuming total detail.

## 2.3 Analysis Types[1]

Perhaps the most promising new feature or set of features was developed to deal with the problem of one dominant explanatory variable. Frequently in economic studies, income or education so dominates the dependent variable that the data are split on little else. One may then want to remove that effect to see what else matters. One could assume a particular relationship such as linear through the origin and simply divide the dependent variable into groups by that predictor. This often has the added advantage of improving the homogeneity of variance where the variance of the dependent variable is related to its level.[2]

Moreover, with non-orthogonal survey data, one may want to search out subgroups in which there are different relationships between the dependent variable and the "control." For instance, in much analysis of cross-section survey data, the economist is often interested in the effect of income on some behavioral variable, and on whether that effect (as represented by its slope) varies with other circumstances. The answer to this question will tell him whether it is necessary to disaggregate the data in the models used for forecasting, and the optimal way to do it.

Sociologists, psychologists and market analysts often face similar problems in which the purpose of the investigation requires isolating the effect of a particular variable under a wide variety of combinations of circumstances. For instance, intelligence, alienation and authoritarianism have all been the subject of repeated investigations in which the object has been to relate the particular

[1] Parts of this section are adapted from Sonquist, Baker and Morgan (1969).

[2] See L. R. Klein and J. N. Morgan, "Results of Alternative Statistical Treatments of Sample Survey Data," Journal of the American Statistical Association 46 (December 1951), 442-460.

factor to specific consequences in such a way as to specify the form of the relationship under various conditions and for particular types of people.

Another illustration is in the analysis of changes taking place over time. The initial value of a phenomenon under study clearly affects its value measured at a subsequent time. This is why the residuals from the regression of its $t_2$ value on its initial $t_1$ value are often used as a measure of change, instead of the raw increments.[1] However, this "initial value" effect may not be the same for all subgroups in the population. If, then, a single equation is fitted, a downward bias would be exerted on the correlations between change and those factors thought to be responsible for it. Thus, when residualizing a variable for study, a search should be made to determine if this effect is homogeneous throughout the population. Where "regression" toward normalcy over time is powerful, a two-stage analysis allows using the first analysis to estimate deviations from expected first-year levels (from final group averages) and using a recoded set of class intervals on them as the covariate or a predictor in a second-stage analysis of change.

To deal with these covariate problems, the AID algorithm has been expanded from the original means analysis to include a regression analysis, where the sum of squares is explained by differences in the two subgroup regression lines instead of the subgroup means.

In addition, one may ignore differences in the intercept and consider only differences in the slopes. This slopes criterion differs from the other two sufficiently to warrant a more detailed description.

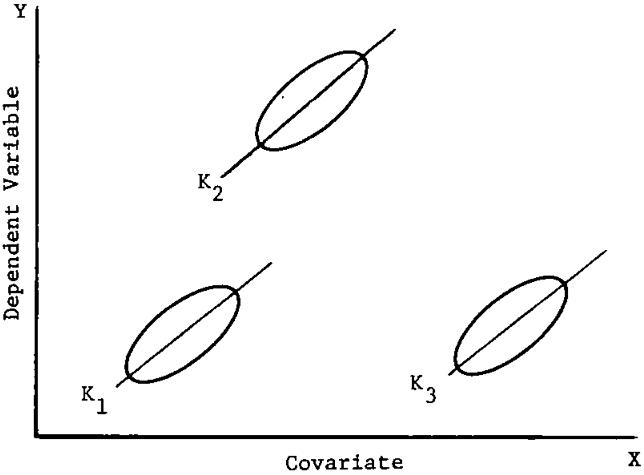
The difficulty with subgroup regressions (simple correlations) is that their explanatory power is dominated by differences in the <u>levels</u> of the regression lines rather than their slopes. And we may not even care to isolate groups with a high level on Y, being interested rather in groups with differences in the slope of the XY relationship (income elasticity, etc.). Hence a third option, the covariance search, calls not for a criterion of explanatory power for two separate regressions, but the power of two different regression <u>slopes</u> using the parent group level.

Since there may be several subclasses on each side of a split, the criterion is the power of the weighted average slope, not a pooled slope, on each side since different subclass means on X and Y can distort the pooled slopes.

The search for differences in <u>regression slopes only</u> turns out to be more complex than one might think. Suppose one wants to separate the K subclasses of

[1] For a thorough discussion of this problem see Lord (1950).

a characteristic into two groups, the first consisting of groups 1 through $K_1$ and the second consisting of groups $K_1$+1 through K. The first set may all have very steep slopes, the second very flat slopes, but if either their means on X or their means on Y vary, the pooled slopes of either of the two sets may have little resemblance to the subgroup slopes. A diagram may make this clearer. Suppose there are three subgroups in a set, forming separate clusters. While each of the three groups has a regression slope of approximately 1.0, the regression slope pooling $K_1$ and $K_2$ would be approximately 2.0 and that pooling $K_1$ and $K_3$ would be approximately 0.0, and that pooling all three groups would be negative.



Hence, whether or not the subclasses are rearranged in order of their regression slopes, a criterion which uses the two pooled slopes from the two subgroups ($1 \rightarrow K_i$, $K_i+1 \rightarrow K$) would hide a great deal, whenever the subclass means differ on X or Y or both. So, we use the weighted average slope for each of the two children, both as a criterion for deciding which split to make and as the criterion for calculating residuals where they are to be used in a subsequent analysis.

As the formulas show, one can think of the remaining error variance around predictions using such a weighted average slope, instead of a pooled slope as originally proposed. Indeed, the terms subtracted from total sum of squares separate into one table attributable to subgroup means and one attributable to the weighted average slope. There are two such "explained sum of squares" terms, one for each of the two children. The criterion for selecting the best split is the maximization of that term (not of the total explained sum of squares, which includes terms for the subclass means). (See Table 2).

This is somewhat intuitive. For instance, there is no proof that where the

Table 2

Analysis of Covariance

| Source of Variation | Sums of Squares | Degrees of Freedom |
|---|---|---|
| k individual slopes $b_i$ about their average slope $\bar{b}$. | $S_1 = \sum_i^k \frac{\left[\sum_v^{n_i}(x_{iv}-\bar{x}_{i.})(y_{iv}-\bar{y}_{i.})\right]^2}{\sum_v^{n_i}(x_{iv}-\bar{x}_{i.})^2} - \frac{\left[\sum_i^k\sum_v^{n_i}(x_{iv}-\bar{x}_{i.})(y_{iv}-\bar{y}_{i.})\right]^2}{\sum_i^k\sum_v^{n_i}(x_{iv}-\bar{x}_{i.})^2}$ | k-1 |
| Deviations for each of k groups about their individual slope $b_i$. | $S_2 = \sum_i^k\sum_v^{n_i}(y_{iv}-\bar{y}_{i.})^2 - \sum_i^k\left\{\frac{\left[\sum_v^{n_i}(x_{iv}-\bar{x}_{i.})(y_{iv}-\bar{y}_{i.})\right]^2}{\sum_v^{n_i}(x_{iv}-\bar{x}_{i.})^2}\right\}$ | N-2k |
| Deviations of the k group means about the regression line with slope b based on group means. | $S_3 = \sum_i^k n_i(\bar{y}_{i.}-\bar{y}_{..})^2 - \frac{\left[\sum_i^k n_i(\bar{x}_{i.}-\bar{x}_{..})(\bar{y}_{i.}-\bar{y}_{..})\right]^2}{\sum_i^k n_i(\bar{x}_{i.}-\bar{x}_{..})^2}$ | k-2 |
| Difference between $\bar{b}$ and $\hat{b}$. | $S_4 = \frac{\left[\sum_i^k\sum_v^{n_i}(x_{iv}-\bar{x}_{i.})(y_{iv}-\bar{y}_{i.})\right]^2}{\sum_i^k\sum_v^{n_i}(x_{iv}-\bar{x}_{i.})^2} + \frac{\left[\sum_i^k n_i(\bar{x}_{i.}-\bar{x}_{..})(\bar{y}_{i.}-\bar{y}_{..})\right]^2}{\sum_i^k n_i(\bar{x}_{i.}-\bar{x}_{..})^2} - \frac{\left[\sum_i^k\sum_v^{n_i}(y_{iv}-\bar{y}_{..})(x_{iv}-\bar{x}_{..})\right]^2}{\sum_i^k\sum_v^{n_i}(x_{iv}-\bar{x}_{..})^2}$ | 1 |
| Deviations about the average slope $\bar{b}$. | $S_w = S_1 + S_2$ | N-k-1 |
| Differences between Y-means, adjusted for $x=\bar{x}$, about $\bar{b}$. | $S_b = S_3 + S_4$ | k-1 |
| Deviations about the overall line (Error sum of squares) | $S_E = S_w + S_b = \sum_i^k\sum_v^{n_i}(y_{iv}-\bar{y}_{..})^2 - \frac{\left[\sum_i^k\sum_v^{n_i}(y_{iv}-\bar{y}_{..})(x_{iv}-\bar{x}_{..})\right]^2}{\sum_i^k\sum_v^{n_i}(x_{iv}-\bar{x}_{..})^2}$ | N-2 |
| Due to the overall regression line. | $S_R = \frac{\left[\sum_i^k\sum_v^{n_i}(y_{iv}-\bar{y}_{..})(x_{iv}-\bar{x}_{..})\right]^2}{\sum_i^k\sum_v^{n_i}(x_{iv}-\bar{x}_{..})^2}$ | 1 |
| Total sum of squares | $S_T = \sum_i^k\sum_v^{n_i}(y_{iv}-\bar{y}_{..})^2$ | N-1 |

subclasses are rearranged according to subclass slopes that one of the K-1 splits using that ordering will be the best.[1]

Not only must we use an average slope, rather than an overall slope, for the parent and for each child at any split, but we must calculate the net gain from using two different average slopes, one for each child, over using a single average slope for the combined parent, because the latter is not constant for changing predictors!

In the case of means, the explained sum of squares from knowing only the parent group mean is $N\bar{Y}^2$ regardless of the predictor, but with slopes, it is $\bar{b}\ N\sigma_x{}^2$ which will vary with the predictor classification since each one will produce a different $\bar{b}$.

Both with <u>regressions</u> and with <u>slopes only</u>, if the overall regression on the full sample accounts for much of the variance, the subgroup differences have less variance to account for, yet the criterion is a fraction of the original variance around the mean, hence the split reducibility criterion must be set lower. Slopes are also extremely sensitive to extreme cases, in X or Y or XY, and tend to become unstable very rapidly as subgroup sizes diminish. The different slopes may account for very little of the variance but may still provide some important information. And the procedure can be used to trick the computer into solving some other problems: One can use a dichotomous 0-1 covariate, like sex or race. The program then looks for groups with the largest differences in the racial or sexual differences, say in earnings. Or one can merge two separate surveys from different times or places, use a 0-1 covariate representing which time or place, and search for the groups with the largest differences between times or places.

The last possibility opens up vistas of powerful use of separate cross-section samples to search for social trends and their differences. The predictors, of course, should be things that do not change for individuals: race, sex, education, farm background, age (increases by one each year). The results should be far superior to the present system of finding groups that differ in one data set and looking for differentials among them in the differences with another data set. There is no reason to expect a correlation between the two, and we really need to search explicitly for the subgroups that differ on the differences.

Given the problems with weighted average slopes, however, it is advisable to reduce the number of categories of each of the predictors to five if the order is to

[1] See Appendix VI giving Ericson's proof for means which may or may not extend to slopes, reprinted from John A. Sonquist and James N. Morgan, <u>The Detection of Interaction Effects</u>, the Institute for Social Research, The University of Michigan, Ann Arbor, Michigan, 1964, pp. 149-157.

be maintained and three if it is not.[1] And it is particularly important to eliminate extreme cases in the dependent variable since they can have such a disturbing effect on estimated regression slopes. There may also be a problem if one of the two covariate groups is relatively small and is affected by something on which the majority group does not split evenly, so that not enough cases appear on one side of the split.

### 2.3.1 Means Analyses

If the total sum of squares for the parent group is

$$SS = \sum_{1}^{N} y^2 - N\bar{Y}^2 \quad ,$$

and the corresponding sums of squares for the two children are

$$SS_1 = \sum_{1}^{N_1} y^2 - N_1\bar{Y}_1^2 \quad \text{and} \quad SS_2 = \sum_{1}^{N_2} y^2 - N_2\bar{Y}_2^2 \quad ,$$

where $N=N_1+N_2$, then splitting the parent group such that the observations within each of the children are homogeneous is equivalent to minimizing the quantity $SS_1+SS_2$. Thus the reduction in the total sum of squares,

$$SS-(SS_1+SS_2) = \sum_{1}^{N} y^2 - \left(\sum^{N_1} y^2 + \sum^{N_2} y^2\right) - N\bar{Y}^2 + \left(N_1\bar{Y}_1^2 + N_2\bar{Y}_2^2\right) = N_1\bar{Y}_1^2 + N_2\bar{Y}_2^2 - N\bar{Y}^2 \qquad (1)$$

is maximized. But this is simply the between sum of squares term in a one-way analysis of variance (Table 3).

### 2.3.2 Regression Analysis

The corresponding between-sum-of-squares term for a regression analysis (means <u>and</u> slopes) can be formed in a similar manner. Table 4 is an analysis of variance representation of the regression of Y on X. (The derivation of this table may be found in Brownlee, pp. 338-341.) The error or residual sum of squares from estimating the regression line in the parent group is

[1] See Kalton, Table 1.

$$SS_R = \sum_1^N y^2 - N\bar{Y}^2 - \frac{\left[\sum_1^N (y-\bar{Y})(x-\bar{X})\right]^2}{\sum^N (x-\bar{X})^2} \qquad (2)$$

By splitting the parent group so as to minimize the residual sums of squares for the two children, the reduction in using two regression lines instead of the original regression line for the group is maximized, i.e.

$$SS_R-(SS_{R_1}+SS_{R_2}) = \sum_1^N y^2-NY^2-\frac{\left[\sum^N (y-\bar{Y})(x-\bar{X})\right]^2}{\sum^N (x-\bar{X})^2}$$

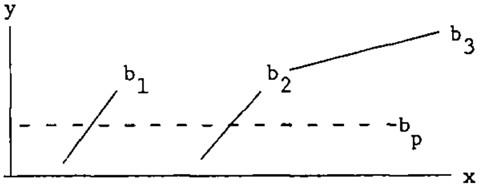
$$-\left\{\sum_1^{N_1} y^2-N_1\bar{Y}_1^{\,2}-\frac{\left[\sum^{N_1} (y-\bar{Y}_1)(x-\bar{X}_1)\right]^2}{\sum^{N_1} (x-\bar{X}_1)^2}+\sum_1^{N_2} y^2-N_2\bar{Y}_2^2-\frac{\left[\sum^{N_2} (y-\bar{Y}_2)(x-\bar{X}_2)\right]^2}{\sum^{N_2} (x-\bar{X}_2)^2}\right\}$$

$$= N_1\bar{Y}_1^2+N_2\bar{Y}_2^2-N\bar{Y}^2+\frac{\left[\sum^{N_1} (y-\bar{Y}_1)(x-\bar{X}_1)\right]^2}{\sum^{N_1} (x-\bar{X}_1)^2}+\frac{\left[\sum^{N_2} (y-\bar{Y}_2)(x-\bar{X}_2)\right]^2}{\sum^{N_2} (x-\bar{X}_2)^2}$$

$$-\frac{\left[\sum^N (y-\bar{Y})(x-\bar{X})\right]^2}{\sum^N (x-\bar{X})^2} \qquad (3)$$

### 2.3.3 Slopes Analysis

In a slopes-only analysis the analyst is concerned only in maximizing differences in slopes without regard to means. Thus, for a given predictor, the parent group should be split such that class slopes within a given child are homogeneous. For example, if the parent group has three classes, the first and second with identical slopes $b_1=b_2$ but different means. The overall or "pooled" regression line for a child with classes 1 and 2 will have a totally diverse slope $b_p$, and the group will be split between classes 1 and 2 rather than between classes 3 and 2.



In the slopes-only analysis then, it is necessary to disregard differences in class means in estimating a group slope.

If for a given predictor there are k classes in the parent group with $n_i$ observations in each class, then the average slope over the k classes can be shown to be: (see Brownlee, Chapter 11)

$$\bar{b} = \frac{\sum_{i=1}^{k} \sum_{\alpha=1}^{n_i} (y_{i\alpha}-\bar{y}_i)(x_{i\alpha}-\bar{x}_i)}{\sum_{i=1}^{k} \sum_{\alpha=1}^{n_i} (x_{i\alpha}-\bar{x}_i)^2} ,$$

and the resultant residual sum of squares using $\bar{b}$ is:

$$R = \sum_{i=1}^{k} \sum_{\alpha=1}^{n_i} y^2{}_{i\alpha} - \sum_{i=1}^{k} n_i\bar{y}_i^2 - \frac{\left[\sum_{i=1}^{k} \sum_{\alpha=1}^{n_i} (y_{i\alpha}-\bar{y}_i)(x_{i\alpha}-\bar{x}_i)\right]^2}{\sum_{i=1}^{k} \sum_{\alpha=1}^{n_i} (x_{i\alpha}-\bar{x}_i)^2} \qquad (4)$$

Note that the essential difference between equations (2) and (4) is in the means. The regression analysis takes deviations from an overall group mean while the slopes-only analysis takes deviations from class means. This results in a different total sum of squares term for each predictor.

Splitting the parent group on a given predictor such that the reduction in the parent group residual sum of squares is maximized is equivalent to maximizing

$$R - (R_1+R_2) = \frac{\left[\sum_{i=1}^{k_1}\sum_{\alpha=1}^{n_i}(y-\bar{y}_i)(x-\bar{x}_i)\right]^2}{\sum_{i=1}^{k_1}\sum_{\alpha=1}^{n_i}(x-\bar{x}_i)^2}$$

$$+ \frac{\left[\sum_{i=1}^{k_2}\sum_{\alpha=1}^{n_i}(y-\bar{y}_i)(x-\bar{x}_i)\right]^2}{\sum_{i=1}^{k_2}\sum_{\alpha=1}^{n_i}(x-\bar{x}_i)^2} - \frac{\left[\sum_{i=1}^{k}\sum_{\alpha=1}^{n_i}(y-\bar{y}_i)(x-\bar{x}_i)\right]^2}{\sum_{i=1}^{k}\sum_{\alpha=1}^{n_i}(x-\bar{x}_i)^2} \qquad (4a)$$

where $k = k_1+k_2$, since the $\sum y^2$ and $\sum n_i\bar{y}_i^2$ terms cancel.

Clearly, selecting which of several subgroups are to go on one side of a split according to subgroup regression slopes and also allowing re-ordering of the subclasses of a predictor, is a doubly dangerous procedure, quite likely to produce idiosyncratic splits and results difficult to explain. Regression slopes are less stable than means, affected by extreme cases in either the covariate or the dependent variable but particularly by any cases extreme on both at the same time.

On the other hand, if one is searching for differences in regression slopes, not in levels of the dependent variable, the regression option is unsatisfactory, since different regression line levels account for so much more of the variance than differences in their slopes. Differences in level would dominate the selection of predictors in covariance search processes using regression.

The best policy if one is searching for slope differences, would seem to be to (a) be doubly careful to eliminate or truncate any extreme cases on the dependent variable, or on the covariate, and (b) recode the predictors to collapse each to three or four classes for those to be left "free" and four or five classes for those whose rank order is to be maintained.

## 2.4 Pre-Set Divisions

The program has a simple procedure for specifying a sequence of splits, after which the usual search procedure can attempt further splits. The purpose of allowing the analyst to force the first few splits is mainly to allow a prior division of the sample into some obviously different groups that are not expected

Table 3

Analysis of Variance for Differences in Means

| Source of Variation | Degrees of Freedom | Sum of Squares | Mean Square |
|---|---|---|---|
| Observations around grand mean<br>$(Y - \overline{Y})^2$ | N-1 | $(N - 1)s_y^2 = SST$ | $s_y^2$ |
| Between group means<br>$(\overline{Y}_j - \overline{Y})^2$ | k-1 | $\sum N_j(\overline{Y}_j - \overline{Y})^2 = SSB$ | MSB |
| Within groups<br>$(Y - \overline{Y}_j)^2$ | N-k | $\sum_{i=1}^{N_j}(Y_{ij}-\overline{Y}_j)^2 = SSW$ | MSW |

Table 4

Analysis of Variance for Regression

| Source of Variation | Degrees of Freedom | Sum of Squares | Mean Square |
|---|---|---|---|
| Regression estimates around $\overline{Y}$<br>$(\hat{Y} - \overline{Y})^2$ | 1 | $(N-1)r^2 s_y^2$ | $(N-1)r^2 s_y^2$ |
| Observations around regression estimates<br>$(Y - \hat{Y})^2$ | N-2 | $(N-1)(1-r^2)s_y^2$ | $\frac{N-1}{N-2}(1-r^2)s_y^2$ |
| Observations around $\overline{Y}$<br>$(Y - \overline{Y})^2$ | N-1 | $(N-1)s_y^2$ | $s_y^2$ |

Where $r^2 = \dfrac{[\Sigma(X-\overline{X})(Y-\overline{Y})]^2}{\Sigma(X-\overline{X})^2\,\Sigma(Y-\overline{Y})^2}$

$$\hat{Y} = \overline{Y} + b_{yx}(X-\overline{X})$$

$$s_y^2 = \frac{\Sigma(Y-\overline{Y})^2}{N-1}$$

$$b_{yx} = \frac{\Sigma(X-\overline{X})(Y-\overline{Y})}{\Sigma(X-\overline{X})^2}$$

to show the same patterns. But the procedure has other uses. One may want to take output derived from another set of data, or half the sample, and force it on a set of data to see how well it explains that data set (percent of variance explained). One cannot compare two different trees very well--they differ in too many ways. But the differences in the subgroup means and in the variance explained, when the same tree is imposed on a second set of data, provide important insights into the stability of the first findings. It would technically be possible to make estimates of the probability of arriving at the same tree with a different set of data, but it depends on the sampling errors of differences between variances--the sum of squares that are compared at each split in deciding which split to make. If there are other possible splits which would explain nearly as much variance, then the probability that in another sample they would actually explain more, is large. And the probability of arriving at the same whole tree is clearly the product of the probabilities of making each of the successive splits the same way--a number which falls rapidly as the tree grows.

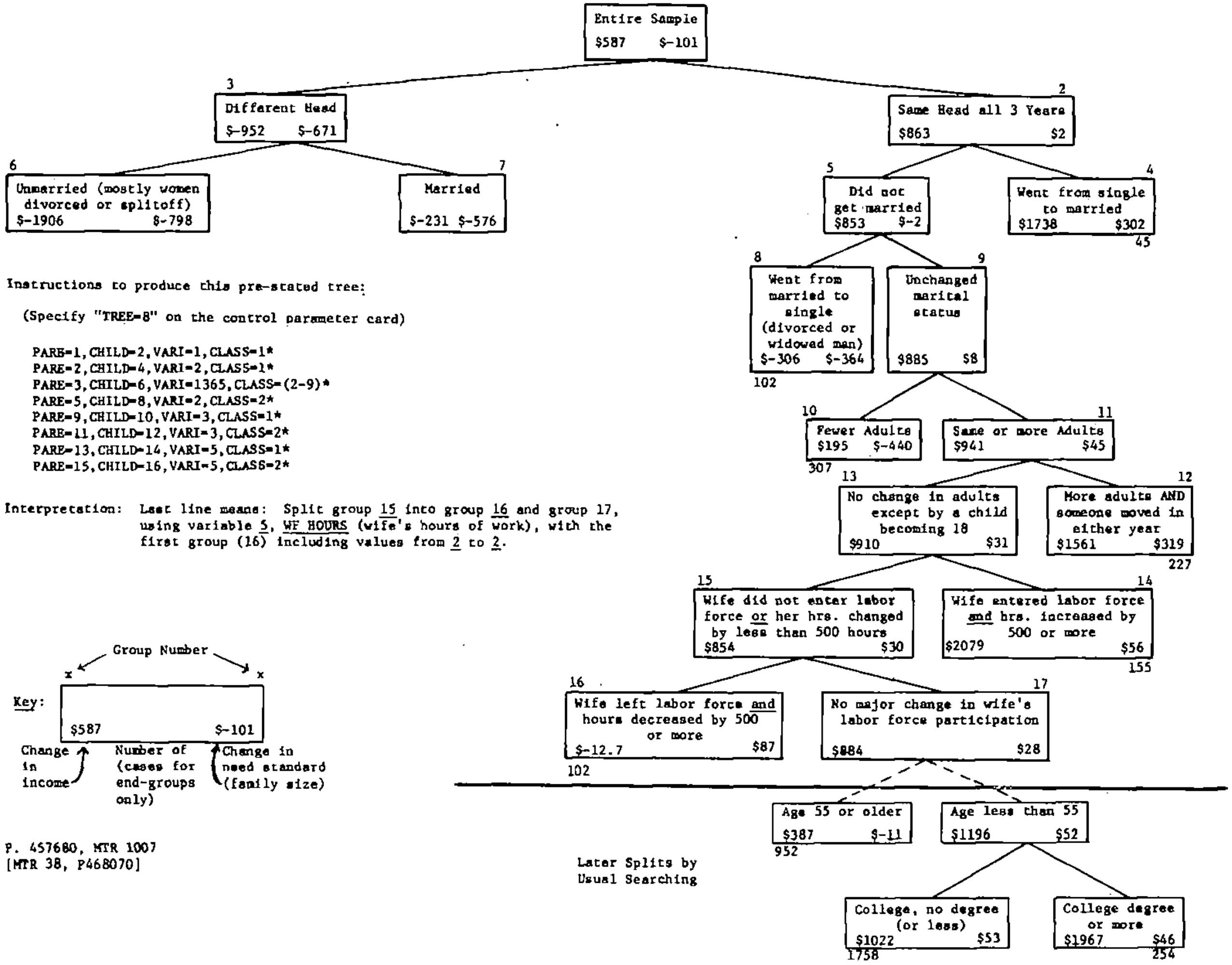
Or one may want to look at a set of related dependent variables, for groups which differ a lot on one of them. Another use of the pre-set tree is to disaggregate a sample sequentially according to some logical order. Poverty, for instance, may be explained by some clearly exogeneous and irremovable forces: old age, physical disability, a single adult who has children or disabled people to care for, lack of education or job skills. Similarly, in looking at change in family income, one may want to remove sequentially those with a changed family head, a shift in marital status, changed number of adults, etc. Figure 3 gives an example of this, where the covariance option was used in order to provide two means for each group, income change and change in heads, by pretending that the latter was the covariate.

It is not always a good idea to force splits merely because there is some deviant group with an obvious explanation--such a group may often be found by the regular process of splitting.

A major purpose of the original program was to avoid the arbitrary selection of subgroups for separate analysis, and to allow the data to suggest the appropriate subgroups, divide the sample into them, and proceed. The criterion of power in reducing error variance used with the program assures that the groups so split off will be both different enough and large enough to deserve separate treatment.

There are, however, situations where some group is important out of all proportion to its frequency in the sample, for theoretical or ethical or public policy reasons. In this case one may want to force a subdivision at the start. But

Figure 3. Average Annual Change in Income and In Needs, by Groups with Changes in Family Status or Wife's Work Status (For 1967, 1968 and 1969 Incomes)


Entire Sample
$587 $-101
3
Different Head
$-952 $-671
2
Same Head all 3 Years
$863 $2
6
Unmarried (mostly women divorced or splitoff)
$-1906 $-798
7
Married
$-231 $-576
5
Did not get married
$853 $-2
4
Went from single to married
$1738 $302
45
8
Went from married to single (divorced or widowed man)
$-306 $-364
102
9
Unchanged marital status
$885 $8
10
Fewer Adults
$195 $-440
307
11
Same or more Adults
$941 $45
13
No change in adults except by a child becoming 18
$910 $31
12
More adults AND someone moved in either year
$1561 $319
227
15
Wife did not enter labor force or her hrs. changed by less than 500 hours
$854 $30
14
Wife entered labor force and hrs. increased by 500 or more
$2079 $56
155
16
Wife left labor force and hours decreased by 500 or more
$-12.7 $87
102
17
No major change in wife's labor force participation
$884 $28
Age 55 or older
$387 $-11
952
Age less than 55
$1196 $52
College, no degree (or less)
$1022 $53
1758
College degree or more
$1967 $46
254
Later Splits by Usual Searching
Group Number
x
x
Key:
$587 $-101
Change in income
Number of (cases for end-groups only)
Change in need standard (family size)

Instructions to produce this pre-stated tree:

(Specify "TREE=8" on the control parameter card)

```
PARE=1,CHILD=2,VARI=1,CLASS=1*
PARE=2,CHILD=4,VARI=2,CLASS=1*
PARE=3,CHILD=6,VARI=1365,CLASS=(2-9)*
PARE=5,CHILD=8,VARI=2,CLASS=2*
PARE=9,CHILD=10,VARI=3,CLASS=1*
PARE=11,CHILD=12,VARI=3,CLASS=2*
PARE=13,CHILD=14,VARI=5,CLASS=1*
PARE=15,CHILD=16,VARI=5,CLASS=2*
```

Interpretation: Last line means: Split group 15 into group 16 and group 17, using variable 5, WF HOURS (wife's hours of work), with the first group (16) including values from 2 to 2.

P. 457680, MTR 1007
[MTR 38, P468070]

one may also want simply to look at that subgroup within each of the groups generated by the analysis to see whether it differs from the majority anywhere in the population. The difficulty with the forced procedure is that the group will be so small that the criterion for allowing further splits may not allow any.

An alternative for examining such minority groups is to assign them weights appropriate to one's values and proceed with a weighted analysis. If they have been oversampled, and weights have been used to reduce the influence of that group to its proper proportion in the population, one can simply run an unweighted analysis to allow the oversampled group more influence. The recoding flexibility allows any complex generation of a new weight variable for an analysis without rewriting a new data tape.

## 2.5 Lookahead

Experimentation with the original AID algorithm to determine its behavior under known conditions has been carried out using contrived data.[1] While the original procedure was found to be capable of dealing adequately with many two-way interactions, others were identified as being difficult for it to deal with. These were seen to consist of interaction relations characterized by consistency, i.e., by balance or symmetry. One such example is the "exclusive-or" model shown below. No main effects appear in these cases, so no split is made that would reveal the mutually offsetting interaction effect inside.

| | B | Not B |
|---|---|---|
| A | High | Low |
| Not A | Low | High |

"EXCLUSIVE-OR"

The obvious test is to take each predictor's best split on the group in question and make one or two additional splits (the best possible) on one or both of the resulting subgroups. One then asks which set of two (or three) splits provides the largest total sum of squares explained, makes the first split, and proceeds. It is possible that a weak first split would allow subsequent splits that were sufficiently powerful to offset that fact. Certainly even a two-split sequence would uncover the offsetting interaction effect just described.

[1] An extensive discussion is presented by Sonquist (1970).

Although the frequency with which variants of this model actually occur in real data is not known, it is notable that at least one realization has received considerable attention in the recent sociological literature, the concept of status inconsistency.[1] Analysis techniques for dealing with this class of models are of importance for economic, educational and psychological research as well as for sociology and marketing.

It can be seen that the earlier sequential partitioning algorithm which examines only the zero-order effects of A and B separately could not discover the consistency effect in a number of these models. In some cases there are really two A effects and they cancel each other out in the total group. Of course, the additive assumptions required in regression or Multiple Classification Analysis would also tend to conceal the real state of the world.

However, the extended AID algorithm incorporated into this version of the program can be instructed to partition the sample tentatively, first on one explanatory variable and then on the other (as well as making tentative partitions on other variables). This makes it possible first to reveal the consistency effect to the analyst by means of profiles of means and differential changes in explanatory power, then to make an appropriate partition and, finally, to continue with the rest of the sequential search procedure.

In general, such a two-split scanning algorithm appears to provide complete and positive identification of all two-way interactions existent in the data. It will even provide leads or clues to the existence of three-way interactions. This is seen to be a simple extension of the way in which the present algorithm provides clues to the existence of two-way interactions. An algorithm which examines the cross-classification of p predictors simultaneously can identify completely terms composed of p raw variables regardless of the symmetry of the term. However, such an algorithm also appears capable of revealing a term involving p + 1 raw variables if the term is asymmetric.

For instance, if we have the three variable negative "and" model:

"If A and B and C, then Y = 0, otherwise Y = 4"

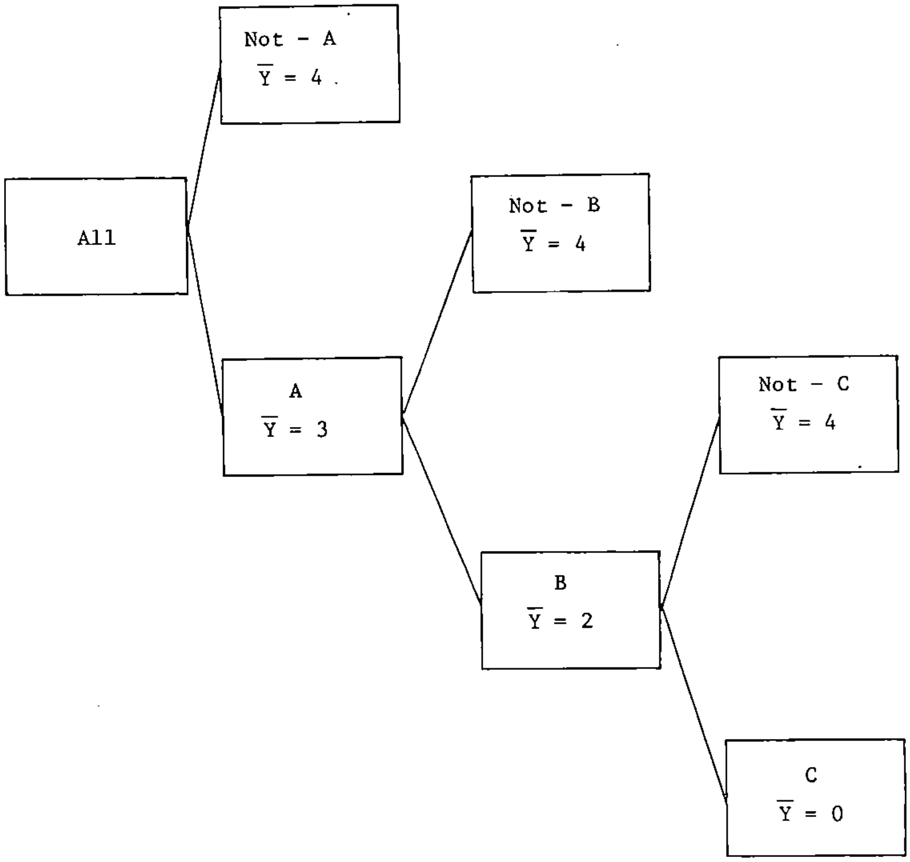
the algorithm using a two-split strategy would produce the sequence of partitions illustrated in Figure 4 and reveal the basic structure.

Of course the amount of computing required to search out combinations of three or more variables increases as an exponential function of the number of variables considered simultaneously. Hence constraints have to be put on the process to permit the elimination of unpromising leads and thus the examination

[1] For an example, see Blalock (1966).

Figure 4

A Three Variable Negative "and" Model



ABC Implies Y = 0, Else Y = 4

of the subsequent partitions.

### 2.5.1 Lookahead Algorithm

There are two parameters governing the lookahead process:

(1) the number of lookahead steps, i.e., partitions made after the tentative partition of the parent group; and

(2) the number of partitions for which all variables are to be permuted, i.e., where the algorithm "forces" the group to be tentatively split on each predictor.

Thus, in a 1-step lookahead, two successive partitions are made and the permute parameter must be 1; and in a 2-step lookahead, three successive partitions made and the permute parameter may be 1 or 2.

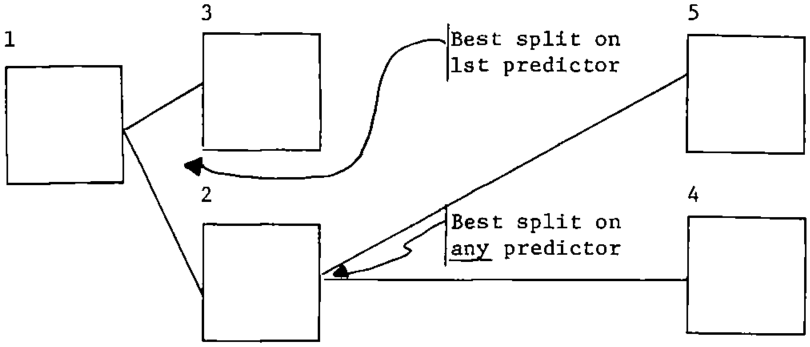
A 1-step lookahead with 1 forced split is executed in the following manner:

(A) Every predictor is examined as a possible basis for partitioning the candidate group. The "best" split definition is saved for each predictor.

(B) A tentative partition is made on the first predictor based on its best split from (A).

(C) The resulting new group with the largest unexplained sum of squares is the new candidate group, and the best split for each predictor is determined for this group.

(D) The predictor found to be the most powerful is selected as the partitioning variable for this second group, and the total amount of variation explained by the two partitions (three groups) is saved.



(E) Steps (B), (C), and (D) are repeated with each predictor in turn used at step (B) to create the first tentative partition.

Thus, if there are k predictors, the entire process is repeated k times;

each time the explained variation is recorded. At the end of the k repetitions, the k tentative levels of explained variation are compared. Then, that partition rule for group 1 which had been associated with the configuration providing the largest obtained level of explained variation is now actually used to split group 1 into two parts. At that point, the algorithm starts over again from the beginning.

### 2.5.2 Stopping the Lookahead Partitioning

Permitting the algorithm to wind itself so closely around the data as is done in, say, a three-step lookahead, runs the risk of spurious splits.[1] Hence this version of AID has had incorporated into it a series of controls that permit the user to reduce the probability of obtaining incorrect or unstable, unreproducible results. Provision has been made to require that if the lookahead option is exercised partitions based on it must explain proportionately more variation than a partition based only on permutations of the predictors in a single group.

When a lookahead of k splits is used, a parallel number of split reducibility parameters are submitted, one for each of the k tentative partitions. A split reducibility parameter is that percent of the total sum of squares which must be explained by the proposed partition in order for it actually to take place. These parameters, each consisting of a percentage (e.g., .6%, 1.8%, etc.) can be set by the user in such a way as to require that in order to be used, a partition based on a lookahead of length two would have to explain more variation than a partition of the same group based on a lookahead of length one, and that a partition based on a lookahead of length three would have to explain more than a partition based only on a lookahead of two, etc., etc. Further, the analyst is given control over how much more powerful a split based on a lookahead of length k + 1 must be in comparison to one of length k.

The analyst may require that any partition of length k explain $P_k$ of the original total variation in the dependent variable. This is accomplished by setting $P_k$ to a percentage between .01% and 100%; for instance, setting $P_2$ to 1.8 would mean that the three groups tentatively generated by a lookahead of two splits would have to explain at least 1.8 percent of the original total sum of squares for the partition associated with these three particular groups actually to be used. Each of the $P_k$ are set by the user independently of each other. This means that the lookahead can be set to exhibit a preference for partitions

[1] See Sonquist (1970)

based on two or three tentative splits. These preferences are expressed as percentages of the total sum of squares.

It is recommended that in the absence of other guidelines, the first lookahead be required to explain at least twice (and that the second lookahead be required to explain at least three times) the variation required for a split to take place at all. That is, lookahead steps should be required to explain at least an additive function of the initial requirements.

The lookahead algorithm always works from the "top" down; that is, it attempts first to make a partition using the longest lookahead that has been permitted by the user. If it cannot make a partition using a lookahead of this length (i.e., the reducibility criterion for that length lookahead cannot be met or all of the resulting partitions would cause some of the new groups to be below the minimum size) it then tries to partition the group using a lookahead one step shorter. It repeats this process if, at any given point, none of the resulting partitions meet the explained variation and minimum size requirements. Finally, if it cannot make a partition of the group under the requirements for no lookahead, it marks the group as a final, unsplittable one.

We now give a more formal statement of the operation of the lookahead under the reducibility criteria.

Let $TSS_o$ be the total sum of squares for the dependent variable. Let S be the lookahead length specified by the user. Let $B_i$ (S) be the explained variation resulting from a proposed split on variable i with lookahead length S.

Then $B_i(o)$ is this maximization function for the original AID algorithm. $B_{max}(S)$ is the largest $B_i$ of all those computed at lookahead length S--that is

$$B_{max}(S) \geq B_i(S)$$

where i ranges over all candidates resulting from the lookahead.

Then if

$$B_{max}(S) \geq P_S \times TSS_o; \text{ S specified,}$$

the parent group is split on the original predictor i which yields $B_{max}(S)$. If no $B_i$(S) satisfies this inequality, then the maximum $B_{max}(S-1)$ at the previous step (S-1) in the lookahead is compared with the required minimum explanatory power $P_{S-1}$.

Finally, if, upon examination, $B_{max}(0) < P_o \times TSS_o$, the parent group is termed final and no further attempts to split it are made. The lookahead will not continue beyond any step where a resulting group is smaller than the minimum group size specification. Also, the process is stopped when the maximum number of splits has been reached.

## 2.6 Predictor Constraints

The original AID algorithm provided for constraining the ordering of the classes of each predictor. The present variation of the program allows this constraint of ordering, and, in addition, allows the user to require that certain predictors be ranked so that some must be used before or after others in the partitioning process. Further, some variables may be inserted as predictors, but ranked so that statistics are obtained for them, but they are never actually used to partition the input sample.

### 2.6.1 Monotonic vs. Free

Any input variable may be used as a predictor, provided it is either read in as an integer in the range 0 to 31 or is recoded so that it falls in this range. Predictors are also classed either as Free or Monotonic. Monotonic predictors will have the order of their coded values (0,1,2,...,29,30,31) maintained during the partition scan. In this case, the classes of the predictor will not be rearranged by sorting them into ascending sequence using the within-class mean value of Y (means option), or the within-class slope of the regression of Y on X (slopes or regression option). Thus, this option is intended for ordinary use with predictors which are ordinary scales or which consist of class-interval codes established for a continuous variable.

The classes of a "free" predictor are rearranged to find that partition which maximizes the sum of squares between the two resulting groups. For a predictor with k classes, the partition of these classes into two sets with m classes in one and k-m classes in the other (m=1,2,...,k-1) that maximizes the between sum of squares, $n_1\bar{y}_1^2 + n_2\bar{y}_2^2 - n\bar{y}^2$ , is that one where the m class means in the first group are less than or equal to the k-m class means in the second, i.e., $\bar{y}_1, \ldots, \bar{y}_m \leq \bar{y}_{m+1}, \ldots, \bar{y}_k$.[1] Thus, one need only examine k-1 partitions, after arranging the k groups in ascending order according to their means on the dependent variable.

No proof exists that the correspondence between sum of squares in a covariate analysis is maximized by sorting on class slopes, however, it seems a reasonable assumption especially since it cuts the number of possible combinations to be examined considerably.

The usual use for the free predictor designation is for variables that are nominal scales, or for other cases in which it is desired not to constrain the

[1] See Appendix VI for a proof of this by W. Ericson.

classes which are to be placed together in the resulting two new groups.

The free option should be used sparingly, since it vastly increases the number of things looked at and the possibility of idiosyncratic splits. If a set of categories does not form a natural ordering, it is quite possible that it represents more than one dimension, e.g., occupation which contains elements of skill, white versus blue collar, managerial responsibilities, entrepreneurial activities, self-employment, etc. In such cases, it is better to convert the classification into a series of dichotomies, or even to maintain the order, not allowing splits with odd combinations on each side. It is better to recode, to put such codes as "inapplicable" and "missing information" in a reasonable place, than to leave a whole predictor free because of them.

Even non-monotonic relations can be handled unless they are symmetrical, and the output of a run that maintains order will reveal what reordering might be substituted.

### 2.6.2 Ranking

To facilitate user control over the order in which predictors are used in the partitioning process, two predictor ranking options have now been incorporated into AID3. These are termed "simple ranking" and "range ranking." In both cases, each predictor is assigned a rank. Ranks may range from 0 to 9.

Rank zero has a special significance; all variables assigned to rank zero will have statistics computed for them in every parent group that is selected for a partition attempt. However, rank zero variables are prevented from entering into the actual partitioning process and they are not examined in the look-ahead. This permits the user to insert one or more variables as predictors, and examine their effect profiles in various parts of his sample without permitting them to enter into the partitioning process. Rank 0 can be assigned to any number of variables whose effects it is desired to observe and it may be used with either of the ranking options. It is particularly useful for variables which may either affect *or* be affected by the dependent variable.

The user may elect to assign as few as two of the available ranks or he may use all ten. He may assign the same rank to several of his variables. He may or may not wish to use zero as a rank.

#### 2.6.2.1 Simple Ranking

The objective of simple ranking is to permit the user to govern the order in which various sets of variables are permitted to enter the partitioning process.

The algorithm then uses the ranking assignments as follows. The total sample is partitioned using only those variables in the highest non-zero rank (smallest non-zero integer). Statistics are computed for all ranks, however, including such variables as may have been given rank zero. If none of these variables are capable of producing a partition rule that meets the split reducibility and minimum group size requirements supplied, then that rank is chosen which is next highest (i.e., which has the next assigned number). All of the variables which had been assigned this next highest rank are now eligible to be used as the basis for a partition and may now be used in the lookahead, and the first (highest) rank is abandoned. If, again, none of these variables can be used, the program abandons this rank and proceeds to the next one. This continues either until a rank is reached at which a successful partition can be made or until there are no more predictors.

When a successful partition is made, the program is said to be "at" this rank. All higher level ranks are abandoned. Variables in the abandoned higher level ranks are no longer eligible for being used as the basis for a partition, although the program will continue to compute statistics for them in the parent group. They will no longer continue to be considered in the lookahead computations, however. Lower level ranks have not yet become eligible.

Further partitions of the new groups that have been created as a result of a partition will be made "at" that rank provided they meet the reducibility and minimum group size requirements. Whenever the requirements are not met by any variables having the rank where the program is "at," it moves downward to the next highest rank (i.e., which has the next larger assigned rank number).

Note that various branches of the tree may work downward through the ranks at different speeds. Where the program is "at" in any given branch depends only on where it was "at" in the preceding node and on the ability of those and the next ranked predictors to produce a satisfactory partition.

This can be illustrated as follows: consider the following ten variables ranked as indicated in Table 5. Variable ten is ranked 0, variable one is ranked 1, variables two and three are ranked 2, etc. When the partitioning starts, variable one is the only predictor eligible for use, since it is the only predictor in the first rank. However, statistics are produced for variables in rank 0 and for the other ineligible predictors. The lookahead could not be used since several of the ranks have only one predictor and there must be at least k+1 predictors in every rank used if a lookahead of length k is used.

If variable one did not meet the reducibility and minimum group size criteria, the algorithm would move to rank 2, abandoning rank 1. Variables two and

Table 5

Ten Variables and Six Ranks

| Variable | Rank |
|---|---|
| 10 | 0 |
| 1 | 1 |
| 2<br>3 | 2 |
| 6<br>7 | 3 |
| 4 | 4 |
| 5<br>8<br>9 | 5 |

three would then both be eligible for consideration, but variable one would not be. However, again statistics are produced for all input predictors. If a partition were made, say, on variable three, then the algorithm would start its next partition attempt again using the rank 2 variables, variable numbers two and three.

### 2.6.2.2 Range Ranking

The second ranking option, range ranking, provides somewhat more flexibility in the way in which variables are used in the partitioning process. In addition to the ranks themselves, two ranges of ranks and a preference are supplied.

The preference can be set "UP" so that the algorithm chooses its partitioning variables starting with the highest rank (provided they meet the reducibility and minimum group size criteria). Alternatively, it can be set to "DOWN" reversing the preference order, or to "AT" providing a third alternative. The preference is effective within the range of ranks specified by the user. Variables outside the range (either because of abandonment or because progression "downward" into the ranks that far has not occurred yet) are excluded from eligibility as in simple ranking, although, as above, they always have statistics reported for them.

As in simple ranking, the algorithm is initially "at" rank 1; subsequently, the "at" rank is determined by the last partition in that branch of the tree.

The eligibility range is defined as a certain number of ranks "up" and a certain number of ranks "down" from the rank where the algorithm is "at." Any variable in that range of ranks is eligible for use in a given partition.

The preference option can be set to cause the algorithm to start at one end or another of the eligibility range. Alternatively, the preference option can be set to cause the selection of variables to start where the algorithm is "at." In each case, the algorithm first determines whether or not according to the reducibility criteria a split could be made on the basis of one of the variables in the preferred rank. If there is at least one that meets these eligibility and minimum group size requirements, the actual selection of a variable to use as the basis of the partition is made from the variables in the preferred rank. If there are none that meet the criteria, the algorithm attempts to make its selection from the variables in the next preferred rank. If, after failing the entire length of the range (no variable has been found which works) the algorithm moves "down" one rank, bringing the next lower rank within the range.

If the preference is for higher ranked variables, the algorithm will start at the highest (1,2,3, etc.) end of the ranking range. If the preference is set for lower ranked variables, the algorithm starts at the lowest (8,9) end of the ranking range moving back upward if not successful. When the preference is set for lower ranked variables, and the unsuccessful attempts to find a split variable have caused a progression back upward all the way to the end of the eligibility range, the algorithm again moves downward so a new rank is brought in at the "downward" end of the eligibility range and an attempt is made to use these newly eligible variables in the partition.

If the preference is set for variables at the rank where the algorithm is "at" and the search is unsuccessful, then both adjacent ranks are made available for scanning and, if possible, a variable is chosen from a higher rank, i.e., towards 1. If the variable eventually chosen for use is in one of the adjacent ranks, the algorithm progresses upward or downward accordingly, and is then "at" a new rank. A summary of the various possible options is given in Figure 5.

A separate range is provided to govern the lookahead operation, i.e., tentative splits of other than the parent group. The operation of the lookahead range is the same as that of the parent group except that statistics for variables at rank zero or outside the current range are not reported. The lookahead range must be at least equal in width to the range used for the partition, and it may be wider. More specifically, the number of ranks "up" must be equal to or greater than the number of ranks "up" specified for the parent group range. Similarly, the number of ranks "down" must be a number greater than or equal to the corresponding "down" range specified for the selection of variables for the parent group.

### 2.6.2.3 Using the Ranking Options

The purpose in providing these types of ranking is to permit the analyst to impose whatever theoretical considerations he may have on the order of the splitting process.

The simplest use is to look at the effects of a variable but not split on it (assigning it rank 0). This allows looking at its relationships without allowing it to make divisions, an especially useful procedure if a variable is not clearly either cause or effect of the dependent variable. This is particularly true of attitudes, where one may want to know their relation to the dependent variable without assuming that they cause it, rather than result from it. In covariance analysis (see above), one may want to use classes of the covariate in order to see whether there are non-linearities which the covariance analysis assumes away (assigning the bracket rank 0). The print-out will provide means of the dependent variable within groups according to the covariate at the same time it is computing the regression slopes. It is always possible that a relationship that is linear overall is not linear within some subgroups, and this procedure

Figure 5

Results of Alternative Rank Specifications

| RANK | Range UP (L)<br>"AT-L" | Range DOWN (K)<br>"AT+K" | Eligible Rank Numbers, in Order of Preference<br>(Low number = high rank) | |
|---|---|---|---|---|
| SIMPLE RANKING | | | | |
| ALL | 0 | 0 | 1, 2, 3, ... M (if M ranks specified) | |
| | L | 0 | Not allowed | |
| | 0 | K | | |
| | L | K | | |
| RANGE RANKING (see NOTE below) | | | | |
| UP | 0 | 0 | Equivalent to "ALL" | |
| | 0 | K | A, A+1, A+2,...A+K (Preference for A) | |
| | L | 0 | A-L, A-L+1,...A (Preference for A-L) | |
| | L | K | A-L, ..A, A+1, ... A+K (Preference for A-L) i.e., always tries lower rank numbers and works up (down the the ranks) to A, then on to A+K. | |
| AT | 0 | 0 | Equivalent to "ALL" | |
| | L | 0 | A, A-1, A-2, ... A-L | Preference for A |
| | 0 | K | A, A+1, A+2,...A+K | |
| | L | K | A, A±1, A±2, ...A+K,A-L | |
| DOWN | 0 | 0 | Equivalent to "ALL" | |
| | L | 0 | A, A+1, A-2,...A-L (Preference for A) | |
| | 0 | K | A+K, A+K-1,...A+1, A (Preference for A+K) | |
| | L | K | A+K,...A,... A-L (Preference for A+K) | |

NOTE: If "A" denotes the AT rank, "L" the number of ranks UP, and "K" the number of ranks DOWN, then the eligible range is [A-L, A+D].

On the first split, A=1. A-L is bounded by 1, and A+K is bounded by 9, e.g., if A=4, L=5, K=6, then the eligibility range is still only 1-9.

"UP", "AT", and "DOWN" refer to rank numbers, so "UP" the numbers means down the ranks, the usual procedure.

"UP, L, K" tries the full set of ranks at each split, starting with A-L which can be set equal to 1, but in practice the results are likely to be the same as "ALL".

will reveal the problem.

But perhaps the most important use of ranking is in the situation where the explanatory variables are not all at the same stage in the causal process--some being clearly logically prior to others.[1] Then one may want to give Rank 1 to the exogeneous, background, and constraint variables, and only then allow variables that represent the current situation, motives, opportunities, and recent changes, to come into play at Rank 2. Such a procedure is an alternative to a two-stage analysis using pooled residuals from one stage as the dependent variable in a second analysis. If one feels that there are interaction effects between variables at the two levels, that is, that the groups according to background variables will respond differently to current situational variables, then it would be better not to pool. But pooling does have advantages in providing larger groups, more stability, and more degrees of freedom.[2] Finally, one may want to put at a last and highest rank, variables which may be either cause or effect, but whose relationship to the dependent variable is of interest.

A minor option with ranking is a choice whether one allows only the predictors at that rank (UP, 0, 0), or allows all those already tried and exhausted to come back in if they can (UP, L, 0). It is minor because the chance that they will do so is small.

For range ranking, a preference must be stated for UP, AT, or DOWN. The best eligible predictor in the preferred rank will be chosen over predictors in other ranks regardless of explanatory power. Simultaneous inclusion of <u>all</u> previous ranks can be achieved by redefining the predictor ranks.[3] Using the previous example of Table 5:

| STEP | rank 1 variables | rank 0 variables |
|---|---|---|
| 1 | 1 | 2-10 |
| 2 | 1-3 | 4-10 |
| 3 | 1-3,6,7 | 4-5,8-10 |
| 4 | 1-4,6-7 | 5,8-10 |
| 5 | 1-9 | 10 |

[1] For example, see Sonquist and Morgan (1964), pp. 105-109.

[2] For extensive use of pooled residuals in two- and three-stage analysis, see Morgan, Sirageldin and Baerwaldt, <u>Productive Americans</u>, 1966.

[3] See section 2.9 and Appendix III (an analysis step).

## 2.7 Premium for Symmetry

Particularly when there are several predictors in close competition for splitting on the dependent variable, it is tempting to suggest that one would prefer a symmetric tree if forcing that symmetry did not result in loss of too much explanatory power. The advantage of a symmetric tree or section of a tree is that it is simpler. Indeed, a totally symmetric tree implies an n-dimensioned table where all the subcells are important and different. What is meant by symmetry? It means that if one of a pair of groups has already been split on particular classes of a predictor, the other of the pair is split in the same way. A looser definition would be that the second is split on the same predictor, but not necessarily with the same sets of subclasses on each side of the split. The advantages in simplicity of this partial symmetry seemed minor, so we use the complete symmetry.

One can, of course, set the premium at 100 percent, forcing symmetry, but still leaving open the decision about the first split made on each of a pair. This will still not force total symmetry; since once one has two pairs of groups, the first splits on each of the pairs can be different!

The option allows selection of a loss-function specifying how much one is willing to lose at any splitting decision, relative to the best split, in order to achieve symmetry. In other words, one might end up with <u>some</u> symmetric splits in an otherwise nonsymmetric branching diagram. A ten percent premium for symmetry (penalty for assymetry) means that the symmetric split will be made if its power is at least 90 percent that of the best split. For example, the symmetric split is made if its BSS satisfies:

$$\text{BSS for the symmetric split} \geq (1 - \frac{\text{Premium for symmetry}}{100}) \times (\text{BSS for best split}).$$

When the symmetry option is specified, the algorithm selects the symmetric branch of the tree to split on next (as opposed to that group with the largest unexplained sum of squares) whenever possible. Symmetry takes precedence over ranking, provided the symmetric split is an eligible one.

If one really wants to know the total loss from total symmetry, then the configurational option gives the explanatory power of a total tree using all the details of a set of predictors and can be compared with dummy-variable regression using the same predictors, which is total symmetry.

We have not found very many places where this feature was useful, and it borders on another problem--the dominant variable. If each of several groups splits on the same predictor because it is dominant, a better solution may be to go to the covariance approach described above in Section 2.3.

Even perfect symmetry does not imply additivity. Just because each of two groups can be split in the same way on the same predictor does not mean that the differences between the resulting pairs will be the same, absolutely or relatively.

Additivity of effects (absence of interaction effects) is a sufficient but not necessary condition for symmetry of a branching diagram. It is possible for the same predictor to be the most powerful for subsequent splits of each of a pair of groups, but not to have a uniform effect on the two groups. The branching diagram below is a real case, hence makes the point only weakly (Figure 6a). The data produced a symmetric tree but an interaction effect clearly exists.

If we take the weighted means for whites and nonwhites, and for young and old, to estimate the interior of the table assuming additive effects, we have Figure 6b, where the first entry is the actual proportion, the second the expected proportion, and the third the difference.[1] It is clear that the older nonwhites are more likely and the younger less likely, to approve of mothers' working, than an additive model would suggest.

Actually, one could also take the unweighted means for whites and nonwhites, and for young and old, to estimate the interior of an additive table, though such data are unavailable unless one has the detailed table in the first place. In this case we get Figure 6c, which says that older nonwhites and younger whites are more likely to approve, and the other two groups less likely.

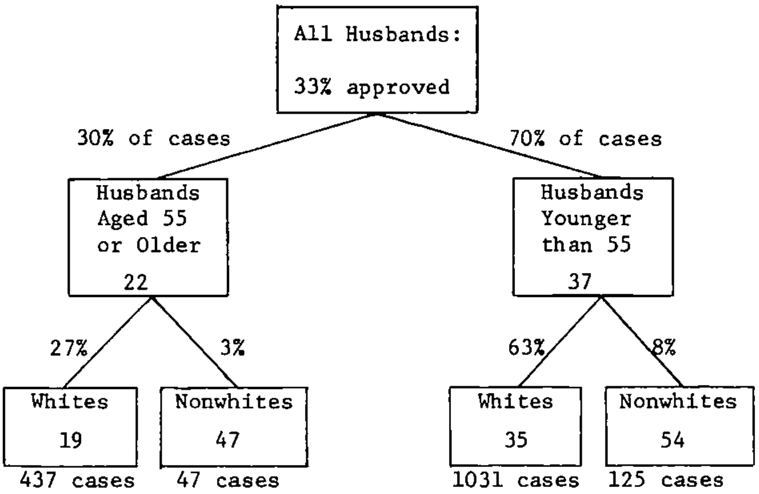
## 2.8 Elimination of Extreme Cases

The least squares criterion almost universally used in statistics, and in this program as well, is very sensitive to extreme cases. In much real data, moreover, the extreme cases are likely to involve either errors of measurement or conceptual problems. In any case, we may not want our findings to be dominated by a few cases, or to face the likelihood that another sample would produce widely different results because of them. One can use the recode capability either to truncate extreme cases or to give them a zero value on the filter variable so they will be excluded. However, this requires defining extreme cases uniformly regardless of their situations. In the population as a whole, a house value greater than $75,000 may not be extreme, but among lower income families where the head is less than 65 years old, such a value might well be extreme.

[1] One estimates the expected values in the subcells as follows: Take the deviations of the means by age or race from the grand mean as estimates of age or race effects. For any subcell, add the appropriate age and race effect to the overall mean to estimate that subcell mean.

Figure 6a

Proportion of Husbands Approving of Mothers' Working*
(For all 1640 married heads of families)



*The question was: "Suppose a family has children but they are all in school--would you say it is a good thing for the wife to take a job or a bad thing or what? Source: Morgan *et al*. *Productive Americans*, p. 330.

Figure 6b
Deviations from Weighted Expected Values

| | Nonwhite | White | |
|---|---|---|---|
| Youth | 54-56=-2 | 35-34=+1 | 37 |
| Old | 47-41=+6 | 19-19=0 | 22 |
| | 52 | 30 | 33 |

Figure 6c
Deviations from Unweighted Expected Values

| Nonwhite | White | |
|---|---|---|
| 54-56=-2 | 35-33=+2 | 45 |
| 47-44=+3 | 19-21=-2 | 33 |
| 50 | 27 | 39 |

The user defines "extreme" in terms of the number of standard deviations from the mean in either direction, where the mean and standard deviation are those for the group in question. If the identification numbers of the extreme cases are needed in order to look them up, that variable must be read in with the other data, and its "variable number" specified on the set-up forms.

Extreme cases are dealt with in two stages. Initially, the entire data set is read and recoded (only cases filtered with the global filter are excluded). The mean and standard deviation are calculated on the dependent variable for the sample, and any cases lying outside of $\bar{y} \pm n\sigma$, where n is specified by the user, is considered an outlier. These cases may be removed from the data, or simply cause a warning to be printed. At this stage, one would hope to find the gross errors in the data.

Subsequently, extreme cases are defined by the mean and standard deviation of each group as it is being split. These outliers may be removed before making the split. After each group, including the first, has been searched and the best split decided, the extreme cases are located and removed so that they do not appear in the two resulting subgroups. Note that the data given for each group are before the extreme cases in that group are removed. This allows extreme cases to affect one split before they are taken out, and to affect the total sum of squares and hence the actual value of the split reducibility criterion. This is unlikely to be a problem, except at the beginning, and the extreme cases for the full sample could be taken out by the filter anyway.

Experimentation with this feature on a regression analysis of house value on income, throwing out cases more than five standard deviations from each group mean, showed that twelve cases were thrown out, in four different places, and the resulting tree was different--at the third split on one branch and the fourth split on the other. The results were, however, very much the same as when we truncated house value at the beginning.

Since the purpose is to eliminate the rare extreme cases, no more than twenty-five cases can be eliminated from any group, and if more are eligible, only the first twenty-five encountered will be eliminated. However, the others would almost surely be eliminated from the two subsequent subgroups before they were split (but after the split had been selected).[1]

This feature could be used for cleaning data of errors, in an effective but biased way (increasing the apparent fraction of the variance explained). Or one could isolate small groups of deviates for special analysis.

[1] This does not apply to the initial outliers treatment.

## 2.9 Multi-stage Processing

AID3 operates in three modes: input, compute, and output. The input mode is executed automatically at the beginning and again after an output mode. However, the user specifies compute and output commands. By not specifying the output command, once the partitioning process has stopped the user may re-define certain parameters (e.g., maximum number of splits, symmetry) as well as defining forced splits and continue the partitioning under the new definitions. This type of user control would be most useful in an interactive system, but it may be useful when the analyst has some concept as to the structure of the partitioning process. For example, it might be used in conjunction with ranking to allow greater control over the algorithm.[1]

[1] See section 2.6.2.3.

# III
# Input Features

This chapter details the requirements and options for the input data file. The input setup, i.e., parameter cards, is described in Appendix III. The reader is referred to other publications for more extensive treatment of research uses.[1]

## 3.1 Data Structure

The program assumes a rectangular data structure: each logical record is one data observation containing all variables to be transmitted to the program, plus any other variables which may also be contained on the file.

This file may or may not be pre-sorted on one or more variables used as keys.[2] Pre-sorting requires a separate job-step using a sort-merge program, but is only necessary if the configuration option (section 2.2) is used.

The data file is read in integer fields. Scale factors are provided for the dependent variable, covariate, residual, and predicted value. Predictors must be integers in the range $0 \leq p \leq 31$.

Since the recoding routines operate in integer mode, decimal places are truncated, and variables must therefore be appropriately scaled before dividing to obtain the desired accuracy (e.g., by multiplying by 1000).

Decimal points within data fields are not allowed, but may be bypassed in the following manner: a 4-character field AB.C may be read as two fields, AB

[1] See Morgan and Sonquist (1963), Sonquist and Morgan (1964), Andrews, Morgan and Sonquist (1967), Sonquist, Baker and Morgan (1969), Sonquist (1969), Sonquist (1970), Sonquist (1970b).

[2] A parameter setting permits computation of "configuration" statistics for sorted files. See section 2.2.

and C and converted back by 10xAB+C.

### 3.1.1 Weighting the Data

In some sampling designs for collecting data, multi-stage probability designs are used that require "weighting" the resulting observations. AID3 has been designed to accommodate those types of designs where the resulting "weight" attached to each resulting observation is a positive integer. If the user designates one of his input variables as a "weight," the program treats the data as though it were receiving multiple copies of each observation, the number of such copies being determined by the associated weight. Non-integral values of the weight variable are not permitted. The range that may be taken on by a variable designated as a weight is $1 \leq V_w \leq 999$. However, users with weights larger than 99 and sample sizes of 1000 or more are cautioned that floating-point arithmetic rounding errors may occur. Such users are encouraged to seek local individual advice on rounding errors in sums of squares. Users with one- or two-digit weights, dependent variables in the range 0 to 999,999 and samples not greater than about 3000 should experience no difficulty with rounding problems.

### 3.1.2 Multiple Response Variables

AID3 will only accept single-valued variables. So-called "multiple-response" variables must be read as separate variables and the recoding routine used to create as many single-valued variables as desired from them. For instance, if, in response to the question "Which magazines do you read regularly?" a respondent says "Time, Life, and Progressive," the analyst might wish to reserve five or six 2-column fields in his file for up to that many answers, recording as successive positive integers those mentioned (e.g., 01, 08, and 27 for those mentioned above). This might be coded 0108270000 if five fields were reserved as a 2-digit "multiple response" variable. If this configuration is to be used in an analysis using AID3, then the analyst would have to create one or more variables each of which has a single value (e.g., 1 if person reads any magazines of a certain type, 0 if not).

### 3.1.3 Scale Factors

Although all variables must be supplied to the program in integer form using implied decimal places, it may be desirable to shift decimal points on the dependent variable and the covariate for readability and compatibility with other program output. This is true also if residuals are computed for output. Consequently, input scale factors have been provided for the dependent variable and the covariate. The user simply indicates in "powers of ten" notation where he wishes the program to place a decimal point for that variable. By using a positive scale factor, the decimal point is moved to the right by the number of places indicated. A negative scale factor causes it to be moved to the left. For example, if the integer field supplied as the dependent variable has three implied decimal places, the user may supply an associated scale factor of -3, causing the variable to be multiplied by $10^{-3}$, or 1/1000 and be properly scaled and rounded.

The use of an appropriate output scale factor causes the desired number of significant digits to be retained during the computation of residuals. Generally two more decimal places than used on input are sufficient for accuracy in generating residuals. If the dependent variable is dichotomous (one or zero), the user may wish to round these residuals back to either 1, 0 or -1, however.

## 3.2 OSIRIS vs Formatted Data Files

Input data files may be described with an OSIRIS dictionary or with a Fortran IV format statement.

For data sets without a dictionary, up to three cards of format information must be supplied.[1] The format statement describes an input (or output) data case. All variables transmitted to the program from the data file, whether used in the analysis or not, must have a format code or field descriptor (e.g., I10). The transmitted or input variables should be in integer mode (non-integer variables may be transmitted if they are only to be carried along, i.e., not used in the analysis, and later outputted in a formatted residual file). The variables are read in the order listed on the input variable list card.[2] In addition, field descriptors for each variable created during execution, i.e., with the internal recode or the residual options, must also be supplied if a formatted output file is requested. These "output" variable fields will naturally follow the "input" fields.

The OSIRIS multivariate recode, bad data option, and global filter cannot be used with formatted data files.

A detailed account of OSIRIS dictionary and data files may be found in

[1] See the IBM System/360 and System/370 FORTRAN IV Language Manual for information on formats.

[2] See Appendix III and the OSIRIS/40 User's Manual for a description of variable list cards.

Appendix D of the OSIRIS/40 CSF User's Manual.

## 3.3 Recoding

It is recognized that users of complex statistical programs often need a built-in capability for transforming their variables at the time a statistical run is made. This has been found to be particularly true in the case of research tasks to which AID is frequently addressed. Predictor variables typically involve nominal scales (classifications) which reflect several dimensions and should be converted into separate dichotomies, as well as continuous variables which must be converted into sets of ordered classifications to be used as AID predictor variables. Also, AID users typically wish to generate interaction terms for later use in a Multiple Classification Analysis.[1] In addition, if distributions are skewed, dependent variables may need transformations using square roots or logarithms. Users may wish to locate and re-assign missing values, the assignments sometimes being made on a probability basis. Users may wish to exclude certain observations from their analysis. The reasons for exclusion may include the presence of missing values of the dependent variable or covariate, the analyst's desire to analyze only a subset of his data, or the presence of some observations which have so much missing information that they must be excluded from the analysis. Or extreme values of the dependent variable may be reduced to some limited value.

AID3 provides capabilities for accomplishing these tasks of recoding and selection of subsets of observations. A powerful, newly-developed recoding language is appended to the input section of the program, permitting the user to generate almost any type of new variable he chooses.[2]

The recoding control language is actually a kind of special purpose programming language; that is, it operates sequentially. The user submits a series of instructions which are executed in order. Execution is initiated once for each observation in the user's input data file. Each instruction consists of a logical clause, an operational clause, or both.

A logical clause is a simple proposition about the arithmetic relationship (equality, inequality, larger than, etc.) between two input variables or between a variable and one or more constants. When values of the variables are supplied

[1] See Sonquist (1970a).

[2] A complete description of the AID3 internal recode including examples is given in Appendix II.

from an observation that has been read in and altered as desired by any preceding computation this proposition acquires a defined "value." It is either "true" or "false" for the values given. This result is then recorded as the value of a "truth switch." Any number of logical clauses may be concatenated to form a compound logical clause.

Similarly, any number of operational clauses can be concatenated to form a block of ordered computational instructions. These instructions are used to establish values for new (or old) variables, and to perform arithmetic operations. In addition, one instruction, GO TO, enables the user to control the sequence of computation any way he wishes. In a block of contiguous operational clauses, individual instructions are always executed in the order submitted, except where the sequence is altered by execution of a GO TO command. A print instruction PRNT allows the user to look at 1 or 2 variables.

Logical control over actual execution of the recoding instructions is accomplished by use of the "truth switch." When the recoding routine starts it executes operational clauses until it encounters a logical clause. When it encounters the beginning of a logical clause, it determines whether the proposition represented by the clause is true or false and records the results in the truth switch. If the following clause is also a logical clause, it is concatenated with the results of the previous one using the Boolean operator, and a new value for the truth switch is computed. Other logical clauses immediately following are treated similarly. On the other hand, if, after encountering the beginning of a logical clause, the next instruction is found to be an operational command, the truth switch is interrogated to determine whether the operational clause should, in fact, be executed.

If the truth switch is "true," then execution of the operational clause is initiated, and all successive operational clauses are also executed. Then when the beginning of the next logical clause is encountered, the program returns to logical mode and starts computing a new value for the truth switch. However, if the truth switch is "false" when an operational clause is encountered, the operational clause is not executed nor are any other operational clauses that may be concatenated to it; the program does nothing until it encounters the next logical clause. What it does then depends on the type of logical clause it finds.

Logical clauses are of two types--"initial" and "subsequent." A subsequent clause cannot act as the "beginning" of a logical clause, either single or compound. Upon encountering a subsequent clause not concatenated to an initial clause, the program simply returns to logical mode, but it does not evaluate this clause and hence a new value for the truth switch is not computed. The program is now in "limbo" since a return to logical mode also means that no further operational clauses can be executed until a new value for the truth switch has been established. Yet this cannot happen until the program encounters an "initial" clause and has evaluated it, whereupon the whole logical process is re-initiated from scratch. As a result, encountering a "subsequent" clause after computation simply turns off all activity until a subsequent "initial" clause starts it again.

This organization permits the user to write down a set of conditions (compound logical clause) under which a specified block of computation is to be done. He then simply writes the block of computations. If the conditions are not fulfilled the computation is not performed. He may then write an alternative set of conditions and specify a second alternative computational block. If the first set of conditions is "true" for a given observation and the second set is "false," the first block of computations is performed and the second suppressed. (If both conditions are true then both blocks of computations are executed. If neither, then there is no computation.) Thus, alternatives can be set up to accomplish the various kinds of assignments that are to be made during the recoding process.

Provision is made for the user to supply a residual, or "alternative" condition which corresponds to a "none of the above" condition (i.e., true if and only if previous conditions were all false), completing the logical capability.

The sixteen Boolean operators that are permitted for concatenating logical clauses include "and," "or," and "exclusive or," "not," and "implication" as well as all of the less well known operators. The arithmetic operations that can be performed include establishing a value for a variable, all four of the elementary arithmetic operations, as well as square root, logarithmic, modulo and arcsine functions, and the generation of random numbers. The relational operators include less than, greater than, equality or inequality and membership in a closed interval. For error reduction and simplicity, all operations are integer, implied decimal points being assumed in the input. Four-digit statement labels and the GO TO operation provide complete user control over computational sequences.

To facilitate sophisticated use, a simple form of indirect referencing of input variables is provided, enabling user-written subroutines which can be inserted and applied to several variables as desired, as well as facilitating repetitive operations.

Instead of eliminating cases with extreme values on the dependent variable, one can truncate them, converting all values larger than some amount to be equal to that amount, or combine several items (sum, ratio, etc.) into a new variable and then truncate it. Since this is done case-by-case as the data are read in, one cannot calculate within this program the distribution of a new variable and then decide how to truncate it.[1]

Given the earlier warning about maintaining the order of predictors in order to reduce the possibility of idiosyncratic splits, it may be necessary either to alter the scale value of the 0 or the missing information code, especially if they are at the wrong end of a scale; or a set of categories which do not form a scale can be converted into a set of dichotomies. Unless one is willing to assume that a variable such as religious preference or occupation forms an ordinal scale, it is advisable to convert it to a set of dichotomies, one per class.

Another warning: Complex recoding is easily done incorrectly, and a substantial analysis can be done before one discovers the error. The usual safeguards against incorrect specifications do not operate in this program. The best hedge against expensive mistakes is to have a small data-tape like the main one but with only 25 or 100 cases with a listing available of all the data on those cases. One can then do the analysis with these few cases, which takes very little computer time, and make sure all the newly created variables and filtering have been done correctly.

The same recoding capacity is available during each input stage of a multistage run, and there are several recoding possibilities that exist before doing a second or later-stage analysis. One can take the residuals from a first analysis (after taking account of background factors) and transform or truncate them to make a better dependent variable for use in the next analysis. Or they can be converted to a set of categories and used as a predictor in a second analysis as in studies of regression to normalcy; or one can take the expected values (means of final subgroups), convert them to a set of categories, and use this as one of the predictors in a second analysis. This is an application of the principle of two stage least squares, where the expected values are considered freer of measurement error and provide a less biased estimate of the effect sought. Or one can take the actual identification number of a group (only some of which remain) and transmute that into a set of categories which produce a new predictor, namely the final groups identified in the first run. Of course the second-stage analysis would be very likely to split on that predictor most often (though not necessarily), and one could assign it a low or zero rank to suppress it altogether (see Section 2.6.2) in the second analysis. This makes it available for observation but allows other predictors to make the splits. Or one could use the group numbers to generate a filter variable to select only certain

[1] An alternative method is to allow the program to throw out extreme cases in each parent group; see Section 2.8.

groups to use in a second analysis.[1]

### 3.4 Culling the Data

Several methods for eliminating or subsetting the data are available to the user. If a residual file is to be generated, any data cases eliminated from the analysis step will be outputted with missing data values generated for the residual, predicted value, and group number. The exception to this is, no observation will be outputted for data eliminated with a global filter or the bad data options.

### 3.4.1 Bad Data Treatment

With OSIRIS dictionary defined data sets, the standard bad data option is available: eliminate the case; substitute a missing data value; or terminate the run. With formatted data sets, records which cause a reading error are ignored. Cases discarded as bad data are not outputted on the residual file.

### 3.4.2 Missing Data on the Dependent Variable

Cases which have missing data codes for the dependent variable may be eliminated from the analysis (see Appendix C of the OSIRIS/40 User's Manual). This option may be used with all data files, however, missing data are not defined for formatted input variables.

### 3.4.3 Illegal Predictor Values

The user specifies a maximum allowable code for each predictor. Any predictor value greater than the specified maximum will be eliminated from the analysis.

### 3.4.4 Filters

The OSIRIS global and local filters are described in Appendix H of the OSIRIS/40 User's Manual. The global filter may only be used with OSIRIS data files.

The program also provides for the optional interrogation of a subset selector variable. Since the decision to include or exclude a given observation is made after the recoding routine operates on that observation, the analyst can

[1] But one would probably want to see the first analysis results before deciding which groups to use in the second analysis to save some elaborate recoding.

simply use his first few recoding instructions to generate a new variable according to whatever specifications he wishes, indicating what classes of observations to include or exclude. He then designates this as the subset selector variable. If the value of this selector variable is zero when interrogated after the recoding, the entire observation is simply excluded from input.

### 3.4.5 Outliers

The mean and standard deviation of the dependent variable are calculated on the sample after global filtering and elimination of bad data. Any cases lying outside n standard deviations from the mean (n specified) will cause a warning message to be printed. These cases may also be excluded from the analysis.[1]

[1] See section 2.8.

# IV Interpretation of Output

## 4.1 Basic Output and Notation

Output from AID3 is generated during each of the three control modes. General sample statistics are printed during the input mode and include the optional 1-way analysis of variance on the predictor configuration. Also, data cases excluded from the analysis will be outputted onto the residual file.

During execution of the compute mode, a record of the partitioning process including statistics on the parent group may be printed (i.e. , the trace). Groups which cannot be split will be outputted onto the residual file.

During the output mode, a 1-way analysis of variance is computed on the final groups. Groups which have not been split are considered final and outputted onto the residual file. Summary tables are then generated.

Data is input to the program after global filtering or exclusion of cases using the bad data option. These input cases constitute the sample. For each analysis packet, the data are culled using local filters, missing data options, etc. All formulas pertain to this culled sample[1] and conform to the following basic notation:

$w$ = weight value

$Y'$ = unweighted value of the dependent variable

$Y = wY'$ = weighted Y value

$Y^2 = wY'^2$ = weighted Y-squared value

$X = wX'$ = weighted value of the covariate

$X^2 = wX'^2$ = weighted X-squared value

$YX = wY'X'$ = weighted cross product (also denoted Z)

[1] The exception to this is the mean and standard deviation of the dependent variable calculated on the full sample and used to calculate the initial boundaries defining an outlier. See section 3.4.5.

## 4.2 Initial Statistics

The program lists the total number of cases read, how many cases were excluded, and the number of cases used in the analysis. These remaining cases make up the total sample for the analysis packet and are therefore the first group to be split. Totals for the sample are:

$N$ = Total number of observations in the sample

$$W = \sum_{\alpha=1}^{N} w_\alpha = \text{sum of weights}$$ [1]

$$MW = W - \frac{W}{N}$$

$$\Sigma Y = \sum_{\alpha=1}^{N} w_\alpha Y'_\alpha = \text{sum of } Y$$

$$\Sigma Y^2 = \sum_{\alpha=1}^{N} w_\alpha Y'^{2}_\alpha = \text{sum of } Y\text{-squared}$$

$$\overline{Y} = \frac{\Sigma Y}{W} = \text{mean of the dependent variable}$$

$$TSS = SS = \Sigma(Y - \overline{Y})^2$$

$$\sigma^2_Y = \frac{SS}{MW} = \text{variance of } Y$$ [1]

$$\Sigma X = \sum_{\alpha=1}^{N} w_\alpha X'_\alpha = \text{sum of } X$$

$$\Sigma X'^{2} = \sum_{\alpha=1}^{N} w_\alpha X'^{2}_\alpha = \text{sum of } X\text{-squared}$$

$$\overline{X} = \frac{\Sigma X}{W} = \text{mean of the covariate}$$

$$\sigma^2_X = \frac{\sum(X - \overline{X})^2}{MW} = \text{variance of } X$$

$$b_{y.x} = \frac{\Sigma(Y - \overline{Y})\ (X - \overline{X})}{\Sigma(X - \overline{X})^2} = \text{slope of } Y \text{ on } X.$$

[1] If W is small, say W < 50, and the run is unweighted, then it may be advisable to correct for small sample sizes and $\alpha_{adj} = \sqrt{N/(N-1)}$ where N is the number of observations over which summation has taken place.

## 4.3 Trace

Trace statistics include parent group as well as resultant group statistics for each possible partition. The parent group totals

$$N,\ W,\ \Sigma Y,\ \Sigma Y^2,\ \Sigma X,\ \Sigma X^2,\ \Sigma Z,\ SS$$

correspond to the quantities given in the preceding section for the sample, but are summed only over the parent group.

For each predictor, the non-empty classes are listed in the order in which the tentative partitions are made. If classes 5,1,4,2,6 appear, then

BETWEEN 4
AND 2

denote the partition results in one group with classes (5,1,4) and the other with classes (2,6). The statistics

$$N,\ W,\ \overline{Y},\ \sigma_Y^2,\ \overline{X},\ \sigma_X^2,\ b\ \text{(slope)}$$

are given for both of the two resulting groups as well as the between sum of squares (BSS) corresponding to the split. If L1 and L2 are the group numbers for the children, then formulas for the slope and BSS terms are[1]:

(1) Means Analysis

b undefined

$$BSS = W_{L1}\overline{Y}_{L1}^2 + W_{L2}\overline{Y}_{L2}^2 - W\overline{Y}^2$$

(2) Regression Analysis

$$b = \frac{\sum_{\alpha=1}^{N_i} (y_\alpha - \overline{Y}_i)(x_\alpha - \overline{X}_i)}{\sum_{\alpha=1}^{N_i} (x_\alpha - \overline{X}_i)^2} = \text{pooled slope, } i=L1,\ L2$$

[1] See section 2.3 for the derivations.

$$\text{BSS} = W_{L1}\,\bar{Y}_{L1}^{2} + W_{L2}\,\bar{Y}_{L2}^{2} - W\,\bar{Y}^{2} + \frac{\left[\sum^{N_{L1}} (y - \bar{Y}_{L1})\,(x - \bar{X}_{L1})\right]^{2}}{\sum^{N_{L1}} (x - \bar{X}_{L1})^{2}}$$

$$+ \frac{\left[\sum^{N_{L2}} (y - \bar{Y}_{L2})\,(x - \bar{X}_{L2})\right]^{2}}{\sum^{N_{L2}} (x - \bar{X}_{L2})^{2}} - \frac{\left[\sum^{N} (y - \bar{Y})\,(x - \bar{X})\right]^{2}}{\sum^{N} (x - \bar{X})^{2}}$$

(3) Slopes Analysis

$$b = \frac{\sum_{k=1}^{M_j} \sum_{\alpha=1}^{n_k} (y_{k\alpha} - \bar{Y}_k)\,(x_{k\alpha} - \bar{X}_k)}{\sum_{k} \sum_{\alpha} (x_{k\alpha} - \bar{X}_k)^{2}}$$

= average slope of group i for classes $k=1,\ldots,M_j$ of predictor j, $i=L1,\ L2$

NOTE: There is a different average slope of group i for each predictor.

$$\text{BSS} = \frac{\left[\sum_{k=1}^{M_{i_j}} \sum_{\alpha=1}^{n_k} (y_{k\alpha} - \bar{Y}_k)\,(x_{k\alpha} - \bar{X}_k)\right]^{2}}{\sum_{k=1}^{M_{i_j}} \sum_{\alpha=1}^{n_k} (x_{k\alpha} - \bar{X}_k)^{2}} + \frac{\left[\sum_{k=1}^{M_{2_j}} \sum_{\alpha=1}^{n_k} (y_{k\alpha} - \bar{Y}_k)\,(x_{k\cdot} - \bar{X}_k)\right]^{2}}{\sum_{k=1}^{M_{2_j}} \sum_{\alpha=1}^{n_k} (x_{k\alpha} - \bar{X}_k)^{2}}$$

$$- \frac{\left[\sum_{k=1}^{M_j} \sum_{\alpha=1}^{n_k} (y_{k\alpha} - \bar{Y}_k)\,(x_{k\alpha} - \bar{X}_k)\right]^{2}}{\sum_{k=1}^{M_j} \sum_{\alpha=1}^{n_k} (x_{k\alpha} - \bar{X}_k)^{2}}$$

Also note that the focus is on the explanation of the sample variance, not that of the population. The searching operations make the specification of degrees of freedom necessary for extensions to the population difficult.[1]

[1] See Lansing and Morgan (1971), pp. 304-306, or Snedecor and Cochran (1956), p. 274, or Anderson and Bancroft (1952), p. 327.

The user may suppress any or all of the trace. The options are:

(1) Suppress the entire trace;

(2) Print only parent group statistics and the best eligible split on each predictor;

(3) Print parent group statistics and all eligible splits on each predictor;

(4) Same as (3) but include partitions which do not explain sufficient variation, i.e. , the BSS does not meet the reducibility criterion;

(5) Same as (3) but include partitions whose resulting groups do not meet the minimum group size criterion;

(6) Print the entire trace.

Option #5 is recommended if some minimum group size is used because splits with BSS large enough to qualify but splitting off very few cases will then be visible even if the split is not made, warning the user of extreme case problems.

With use of the lookahead option, option #4 or #6 should be used here since an actual split may be made even though its BSS does not qualify in cases where it is optimal when combined with one or two subsequent splits.

Option #6 is often useful, however, in allowing one to force the tree one split further beyond any final group, or to see the actual effect (not just the power) of some predictor on each of the final groups.

## 4.4 Final Tables

One of the design objectives in this version of AID was improvement of the form in which information was presented to the analyst. In previous versions of the program much of the useful information was scattered throughout the trace of the partitioning process. In the present version this detail has been collected and placed in several tables specifically geared to the decisions the analyst must make about the explanatory power of his predictors and their effect profiles in various parts of the sample. The analyst uses information about explanatory power and its changes throughout the partitioning process to make judgements about the presence of interaction effects. He uses effect profiles in a similar fashion.[1]

The final table printed as part of the basic output is an analysis of variance table over the final groups generated by AID. The within- between- and total sums of squares are presented, together with the proportion of variation

[1] For a discussion of display techniques see Sonquist (1969) and Sonquist (1970a), (1970b).

"explained" by the entire branching process.[1] (Tables 2 and 3 give the terms for covariate and means analyses respectively.) The total proportion of variation explained is

$$\frac{\sum_i BSS_i}{TSS},$$

where $BSS_i$ is the between sum of squares term for the appropriate analysis type (defined in section 4.3) resulting from the partition of group i, i.e., the reduction in unexplained variation from splitting group i.

In addition, there are four types of summary tables which may be printed.

1. GROUP SUMMARY TABLE: Number of groups and number of final groups, followed by a recapitulation of the actual splits made and the fraction of the original total variance explained by each split. Branching diagrams can be made expeditiously using just this table.

2.(a) 100*BSS/TSS TABLE FOR N-STEP LOOKAHEAD: For each predictor used for the first split for each group, the explanatory power of that plus one or two more splits. Tagged with < if less than N+1 splits were made (because of other criteria). Replaced with **** if a resultant group too small. If a minimum group size rule excludes a particular division by a predictor on a group, but another division on the same predictor and group is allowable, the "second best" split's BSS/TSS will appear.

(b) 100*BSS/TSS TABLE FOR 0-STEP LOOKAHEAD: Gives the results of a single best eligible split on each predictor for each group. Only eligible splits are reported as with previous table even if "second best," and replaced by **** if a resultant group did not contain enough cases.

(c) 100*BSS/TSS TABLE FOR 0-STEP LOOKAHEAD, MAXIMUM BSS REGARDLESS OF ELIGIBILITY: Gives the power of the best split for each predictor with each group, except the final groups, even if group size or criterion makes that an ineligible split. Provides a warning of extreme cases even if minimum group size prohibits their isolation.

3. PROFILE OF CLASS MEANS AND SLOPES: Gives means of Y (and of X, and slopes if covariance), and N for each predictor subclass for each group. Useful for getting detailed profiles, since the trace tables give only the two pooled groups for each split. Arranged by predictors and for each predictor gives the ETA for its full subclass detail over the whole sample, that is, the explanatory power of that predictor over the whole sample using all its subclasses.

4. PREDICTOR SUMMARY TABLE: Gives more detail on each subclass of each predictor over each subgroup, including ETA, ETA (NSTEP), and ETA(0).

[1] This may be compared with the configuration analysis of variance, since the latter represents the total variation that could be explained by all main effects and interactions for the given set of predictors and as such is the upper bound on explained variation (see Lubin and Osburn, 1957).

The first uses all the detail, the second is the power of the binary splits, and the third of the first binary split.

The least useful is the Predictor Summary Table, most of which is elsewhere in the output. If no lookahead is used, and a small minimum group size, only one of the BSS/TSS tables is needed. The group summary table is essential and the profile of class means and slopes very useful.

### 4.4.1 Group Summary Table

The Group Summary Table provides a record of the actual partitions that took place during the analysis and summarizes the statistical effects of the covariate. The following quantities are printed for each group:

1. Group number and children into which the current group is split.
2. Predictor number, name and class identifiers forming the basis for the split.
3. Percentage of the total variation explained by the split and percentage of the total variation remaining in this group: BSS/TSS and SS/TSS.
4. Number of observations in this group; sum of weights in group.
5. Statistics for the dependent variable; $\overline{Y}$ and $\sigma^2_Y$ .
6. Statistics for the covariate, where applicable;

   $\overline{X}$, $\sigma^2_x$, $r_{xy}$, $b_{y\cdot x}$, a, a(norm)

   (where $b_{y\cdot x}$ is the pooled or average slope depending on whether a Regression or Slopes-only analysis was used--see section 4.3)

$$r_{xy} = \frac{\sum (y - \overline{Y}_L)\ (x - \overline{X}_L)}{\sqrt{\sum (y - \overline{Y}_L)^2 \sum (x - \overline{X}_L)^2}}$$

= the correlation coefficient between x and y

$$a \quad = \overline{Y}_L - b_L\overline{X}_L \ ,$$

a(norm) = $\overline{Y}_L - b_L\ (\overline{X}_L - \overline{X}_1)$ , where L denotes the current group and $\overline{X}_1$ is the original sample mean. Hence a(norm) is the expected value of Y for a member of the group whose X-value is equal to the overall sample mean; it is a measure of the level of the subgroup regression line.

### 4.4.2 BSS/TSS Tables

Three BSS tables are available and may be printed when the BSS options are activated and a lookahead is used. The lookahead BSS table shows the explanatory power of the groups resulting from the partitioning of the parent group and the successive lookahead results. For instance, if a two-step lookahead is used, creating a total of four groups, the BSS shown in the table for the i-th predictor for this parent group is the explanatory power of all four of the groups resulting from the lookahead.

The entries in the lookahead BSS/TSS table are

$$\frac{BSS_{ijm}}{TSS} \times 100 = xx.x$$

for the best partition on each predictor in each group. In the formula above the references are to the i-th predictor for the j-th parent group using a lookahead of length m. The partition may use other variables out in the lookahead splits after the maximizing action taken by the lookahead is attached to that variable forming the initial split in the parent group. Each variable has the maximum BSS associated with its initial use attached to it.

In each column of the table (one column per parent group) a flag appears if the partition was eventually based on less than the maximum lookahead permitted. Final groups are flagged also. Each entry is replaced with asterisks if that predictor was constant in that group, and it is replaced with a zero if the corresponding partition had been too small to meet the minimum group size requirement. Thus, the nonzero elements of the table are all valid, though possibly very small indications of explanatory power. Each variable has the maximum eligible BSS associated with its initial use attached to it. If the STARTING TREE option is specified, the entries in the BSS table may not correspond to the actual BSS's from pre-specified splits, since the former are governed by the FREE/MONOTONIC predictor parameter and are based on a 0-step lookahead. Should the ordering of classes differ from that in the trace, then the entry in the table is based on that of the trace.

The BSS/TSS table with no lookahead corresponds to the lookahead table with m=0, the explanatory power of each predictor in the parent group.

The BSS/TSS table regardless of eligibility contains the maximum BSS/TSS regardless whether that BSS was obtained from an eligible split.

### 4.4.3 Profile of Class Means and Slopes

The means/slopes profile is broken down into separate tables, one for each predictor. The mean of the dependent variable (Y) is shown for every class of every predictor in every group, together with the number of observations upon which that mean is based.

In the case of a Regression/Slopes analysis, the mean of the covariate (X) is also given for each class in addition to the slope for that class. For Group 1 only (whole sample) ETA is also printed, showing the explanatory power of the full detail of each predictor.

$$\text{ETA} = \frac{\sum_{k=1}^{M_j} W_k \bar{Y}_k^2 - W\bar{Y}^2}{\text{TSS}} \text{, for classes } k = 1, \ldots, M_j \text{ of predictor } j.$$

### 4.4.4 Predictor Summary Table

Data for each predictor's behavior during the partitioning process are collected and organized in this table. One set of statistics for the predictor as it behaves in each group is printed. Data are presented for each class of the predictor.

For each class the following statistics are printed:

1. Number of observations and sum of weights for that class.
2. The sum of weights expressed as a percent of that group and a percent of the total sample.
3. The sum of Y (ΣY); and this sum expressed as a proportion of the total ΣY for the sample and of the total (ΣY) for the parent group.
4. The mean and variance of Y ($\bar{Y}$ and $\sigma_y^2$).
5. The sum of X (ΣX), and this sum expressed as a proportion of the total ΣX for the sample and of the total X for the parent group.
6. The mean and variance of X ($\bar{X}$ and $\sigma_x^2$).

In addition, three summary statistics are printed, showing the explanatory power of the predictor in the group in question.

1. ETA--the proportion of variation that can be explained by this predictor in this group, using all the classes of this predictor. This is the variation in this group that would be explained if a one-way analysis of variance were run on the observations in this parent group, using as grouping all of the classes of the predictor in question.

$$\text{ETA} = \text{BSS}_i/\text{SS}_i$$

for the i-th parent group, where

$$SS_i = \sum^{N_i} (y - \overline{Y}_i)^2$$

and

$$BSS_i = \sum_{k=1}^{M_j} W_k \overline{Y}_k^2 - W_i \overline{Y}_i^2$$

where the summarization is over the $M_j$ classes of predictor j in the i-th group. It is the square of what is called the correlation ratio or $\eta$.

2. ETA (LOOKAHEAD)--The proportion of variation that can be explained by the specified lookahead groups based on a partition of the parent group using this predictor. The computation is the same as ETA, above, but the summation is over the N-step lookahead groups created from binary splits instead of the k classes of one predictor, i.e., summed over N+1 groups.

$$\text{ETA } (n) = BSS_{ijn}/SS_i$$

for a lookahead of length n using parent group i and variable j as the initial partitioning variable for the parent group.

3. ETA (NO LOOKAHEAD)--The proportion of variation that can be explained by a partition of the parent group using this predictor summed over the two resulting groups. This is the proportion explainable by the single split, itself.

In addition, components of sums of squares of Y about the fitted regression lines are printed.[1] These components are described in Table 2. They provide an analysis of covariance over the subclasses of each predictor separately, for each group created.

## 4.5 Residual Files

This version of AID can compute residuals, and predicted values, and can tag each observation with the group number of the final group into which it was placed. In addition, all input and generated variables can be transferred to the file containing the residuals, predicted values and group tags. All of these quantities are incorporated into the input data vector for each observation initially received by the program and are transferred along with this vector into a new data file. Residuals and predicted values may be appropriately scaled by separate output scale factors, and their field widths must be specified for the output file. The final group number is always put out as a 3-column field, hence

[1] See Walker and Lev (1953), p. 396, and Brownlee (1965), p. 378.

no field width specification is necessary.

The output file may be generated as a standard OSIRIS data file with a dictionary or as a format described file. In the latter case, output fields for all generated variables must have been included (following input fields) in the input FORMAT cards: Generated variables are outputted in the order they appeared in the parameter setup (Appendix III). A complete variable list is printed before the analysis begins and the user would do well to try a test run to insure his format fields correspond to the correct variables.

The scratch data file (ISR01) used by the program is updated with all generated variables and used as the data file in subsequent analysis packets. This allows the user to specify residuals without generating an output file.

Formulas used in calculating the residuals and predicted values are:

Means analysis: $\tilde{Y}_\alpha = \overline{Y}_i$

$$R_\alpha = \tilde{Y}_\alpha - Y_\alpha$$

for individual $\alpha$ as a member of final group $i$.

Slope or Regression analysis:

$$\tilde{Y}_\alpha = a_i + b_i\,(X_{i\alpha} - \overline{X}_i)$$

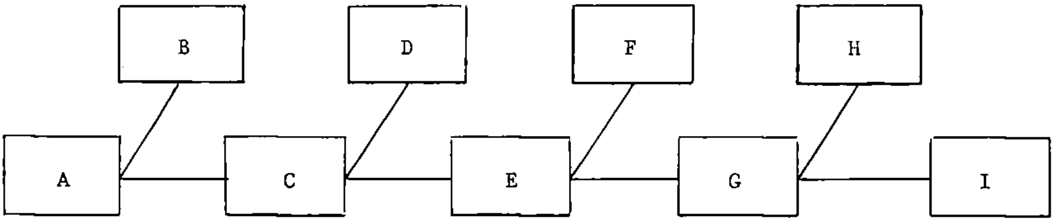
$R_\alpha = \tilde{Y}_\alpha - Y_\alpha$ for individual $\alpha$ in final group $i$.

## 4.6 Structure of the Trees[1]

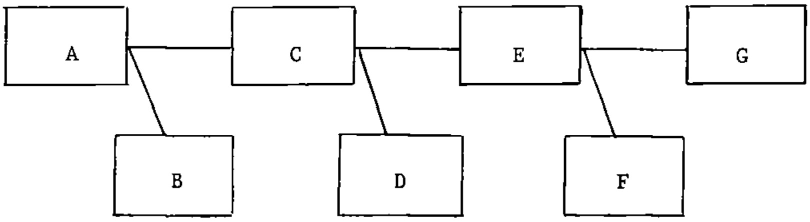
The results of the program can show a series of different characteristic tree patterns. Each tree has sections that can be described as a combination of two configurations, based on the useful convention of showing the group with the highest mean as the uppermost branch. One may be termed a trunk-twig structure, the other a trunk-branch structure.

The trunk-twig structure is a main branch from which small groups are split off and are not themselves split again. This may take three forms: top-termination, bottom-termination, and alternating-termination. The top-termination structure may be termed an "alternative advantage" model. Group B consists of those observations possessing the "advantage" represented by that characteristic which split group A into groups B and C. Once group B has been

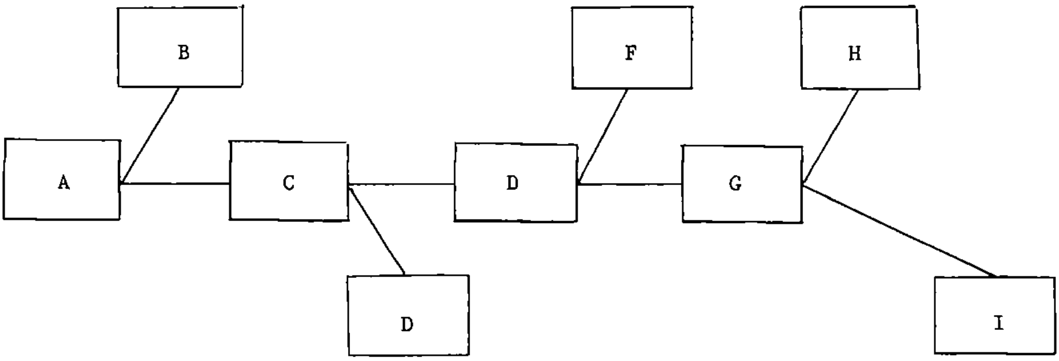
[1] The following sections were extracted from the original monograph, The Detection of Interaction Effects.



TOP TERMINATION



BOTTOM TERMINATION



ALTERNATING TERMINATION

established, it cannot be split further by the program.

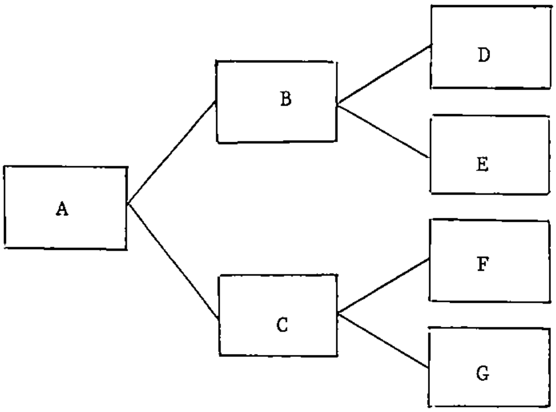
The bottom-termination structure may be termed an "alternative disadvantage" model, and is analogous. The possession of any one of a number of characteristics is enough to prevent an observation from achieving a high value on the dependent variable.

The interpretation of the alternating termination configuration is similar. In all three types, the interpretation to be made depends on the characteristics of the final groups themselves, especially on the number of observations in the group, its variance, and whether or not there existed predictor variables which "almost worked" in the attempt made by the program to split it.

Another property of the tree is its symmetry or nonsymmetry in terms of the extent to which the same variables are used in the splits on the various trunks. Nonsymmetry implies interaction, i.e., effects of combinations of factors. If a variable is used on one of the trunks, and if it shows no actual or potential utility in reducing predictive error in another trunk, then there is clear evidence of an interaction effect between that variable and those used in the preceding splits. The utility of a predictor in reducing predictive error is evaluated by statistic $(BSS_{mpr}/TSS)_i$ for each predictor at each branch in the tree. This output is produced by the program and represents the proportion of the variation in the group to which the predictor is being applied that would be explained if it were used in a binary split of that group.

Trees may, of course, be symmetrical with respect to the way in which top-termination, bottom-termination and alternating-termination configurations appear in the main trunks.

The trunk-branch structure is usually typical of the first few splits of any tree. In this case, each group produced by a split is further subdivided.



TRUNK-BRANCH STRUCTURE

Some of the early groups may remain unsplit. If this is so, then the most important aspect of the interpretation of this structure has to do with the fact that there remains within-group variation which can be explained. At each step, the analytic question that should be asked is, "What are the reasons why there is as much variation in each of the groups as there is?" This question will be discussed below in more detail.

A further property of each tree is the number of final groups that result from the analysis. This is, of course, a function of the input sample size, the statistical properties of the algorithm, and the relationships between the characteristics of the predictor variables and the dependent variable.

Based on the present characteristics of the algorithm, we can distinguish three types of final groups: small groups, explained groups, and unexplainable groups. A small group is one containing too few observations to warrant an attempt to split it. An explained group is over this minimum size, but has too little variation in it (less than, say, 2 per cent of the original variation) to warrant an attempted split. An unexplainable group is sufficiently large and spread out, but no variable in the analysis is useful in reducing the unexplained variation contained within it. Each tree will generally have some of each of the three types. But the total number of final groups is heavily dependent on the rules used to stop the splitting process.

## 4.7 The Behavior of the Variables in the Trees

The analysis of the behavior of the predictors and their relationship to the dependent variable during the partitioning process can be approached through a series of questions, asked with reference to each partition.

### 4.7.1 Chance Factors

The first question is, "Given the minimum group size rule and split eligibility rule used, what is the likelihood that this split occurred by chance?" This problem may still occur even if the above-suggested rules have been used for minimizing the probability of its happening. If a variable actually used in the split is the only one which shows up as important, according to the criteria used, then the probability of its predictive power being based largely on sampling variability is relatively slight, unless it is an unconstrained variable with a large number of classes. When several variables are almost equally good as predictors, in any given split, then the likelihood is greater that sampling variability has had a hand in selecting one, rather than another, as that variable to be actually used in the split. The $(BSS/TSS)_i$ tabulation provides a guard against basing an interpretation only on those variables actually based in the partition process, since the explanatory power of the unused predictors is presented in all its detail.

The overall structure of the tree provides a clue as to the probability that sampling variability is operating together with a skewed distribution.

In the case where the dependent variable is badly skewed and has a tail extending toward the right (positive skewness), a top-terminating trunk-twig structure is likely to appear in several main branches of the tree. These terminal groups will have large, positive means, and will contain few (1-5) observations. Typically, they will result from splits on several different variables. Sooner or later the program will find some predictor which enables it to split out these extreme cases from the group in which they happen to be.

A careful re-reading of interviews may turn up a variable, certain values of which most of these extreme cases will have in common. This variable may then be inserted into a subsequent analysis. One may be reasonably confident that these observations will then be placed together in one group via a split on this variable. Good strategy would, therefore, dictate a preliminary investigation of the skewness of the dependent variable before the main analysis starts.

One might construct a dummy variable which has the value one if an observation is out in the skew tail and zero if it is not. A preliminary AID analysis, using this as the dependent variable, together with the predictors to be used in the main analysis will provide information as to which classes of the sample are out in the tail, rather than being in the main body of the distribution. It may be that one set of variables will be found optimal to explain being out on the tail of a distribution. Another set might prove best for explaining overall variation or variation in the main body of the destruction. This possibility would, of course, be of considerable theoretical importance.

Of course this technique need not be confined to observations out in a skew tail of the dependent variable distribution. For some analytic purposes it may be desirable to use this technique to determine what combination of variables are associated with an observation's being, say, in the second quartile of the distribution, or less than some specified value.

It should be noted that a variable which is not skewed in the total sample, may become skewed during the partitioning process. This cannot be caught in advance. Hence even when a preliminary investigation of skewness has been made, the analyst should be on his guard for the appearance of this particular trunk-twig structure. A bottom-terminating trunk-twig structure with small terminal groups would provide a signal for negative skewness. The provision for identifying and tagging and/or eliminating outliers should be of help here.

### 4.7.2 Conceptualization Problems

A second question that should be asked is, "Does this split reflect conceptualization problems in applying the framework of predictor variables to the sample, or sections of it?" A number of interpretation problems in the trees may stem from measurement or coding errors, or from the use of variables that were designed for other statistical purposes. This technique is at its best when the predictors have a clear, uni-dimensional reference. We have found an example of a conceptual problem that looked, initially, like a somewhat contradictory finding, until coding decisions were uncovered which appeared to misclassify uneducated people living on the fringes of cities of 50,000 and over, with respect to the rural or urban nature of their surroundings. Indices having several components also tend to behave in a somewhat peculiar fashion. Presumably, this is because the items in these indices, though related both theoretically and statistically, may affect the dependent variable in different ways, particularly if some of them interact with other variables in the tree and others do not. Splits involving such variables may or may not "make sense." See Coombs (1964) for a thorough discussion of scaling problems.

Perhaps the most important point to be made here is that problems like these are often revealed only by large standard errors that may accompany a multiple regression analysis. They tend to stand out quite clearly in the tree display of the AID results.

### 4.7.3 Manipulation of Variables

A further question which should be asked with reference to any given split is, "Are there competing predictors correlated with the one actually used in the split? If so, does their explanatory power increase, decrease, or stay the same in subsequent splits?" The logic to be employed here is developed extensively by Hyman in his discussion of spuriousness, and in his presentation of M- and P-type elaboration. He presents a formalization of the logic of examining the relationship between two variables when a third factor is introduced. The two factors under examination are referred to as $x$ and $y$, and the third is called $t$. In our notation, $x$ is the variable used to split group $i$ into groups $j$ and $k$; $y$ is the dependent variable, and $t$ is multiple and consists of each of the other predictors in the analysis. We are interested in the relationship between variable $t$ and variable $y$, as represented by the statistics $(BSS/TSS)_i$, $(BSS/TSS)_j$ and $(BSS/TSS)_k$ for each predictor $t$. If, in addition, we consider whether or not there is a logical, theoretical justification for a correlation between $x$ and $t$, and if so, whether $x$ can be conceptualized as antecedent to $t$ in a causal chain, we have a systematic application of the analysis strategies of:

1. Interpretation ($t$ is an intervening variable)
2. Explanation ($t$ is antecendent to $x$ and is logically related to it)
3. Control for spuriousness ($t$ is antecendent to $x$ and cannot be related logically to it)
4. Specification ($t$ is neither antecedent to $x$ nor subsequent to it, but is logically related. Here $x$ is a circumstance that affects the extent to which $t$ is related to $y$.)

The reader is referred to Hyman and to Blalock (1961) for the details of the logic.

We note that we have reverted to a form of the analysis question, "Other things being equal, how does $x$ affect $y$?" but in a somewhat different form. We now have the question, "When we extract variation associated with predictor $x$, how do the relationships between $t_1$, $t_2$, ..., $t_p$ and $y$ change?"

In providing an answer to this question that is meaningful, the question of the substitutability of variables in the analysis must be taken into consideration. This is the problem of intercorrelations between the predictors.

Another consideration related to the above question is, how does one decide whether to rank predictors, allowing some clearly exogenous or background variables to work first, and then asking whether certain intervening or attitudinal or environmental variables add anything, or to pool the residuals after allowing the background variables to work, and running them against a new set of predictors? The first alternative allows the second set of variables to operate differently in different subgroups, i.e., allows for interaction effects between the two sets of predictors. The second allows none, or allows only those built into the analysis by including first-stage predictors also in the second stage. The second option is also more economical of degrees of freedom, of course, and may be necessary if the sample is only of middling size (1000 or so).

For example, one might, in explaining earnings, want to put education, age, race, sex, farm background in a first analysis, and then go on, with or without pooling, to introduce mobility, occupational choice, current environment, etc. But even if one pooled residuals, one might want to reintroduce education and race in the next stage, in case the effects of mobility are dependent on one's education or race.

It is impossible here to consider all the problems associated with the relationship between a variable and the concept(s) it purports to represent, but a few points should be emphasized.

Some intercorrelations are built into the data by the coding process. Other high correlations may result because two predictors may themselves be the results of a third factor which may or may not be represented in the analysis by a variable. Still others are there because things go together in the real world. But it is on exactly this structure of relations that we are trying to get a grip. What is required is a strategy for minimizing the interpretation problems.

One way to deal with this is to put in the most clearly exogenous, most orthogonal and uni-dimensioned variables into a first-stage analysis or ranked highest, together with a relatively high reducibility criterion and fairly large minimum group size, and then use the richer matrix of predictors for an analysis of the residuals. Where a tight test is desired as to whether a variable which is of considerable theoretical importance has effects, this variable may be held out of the first-stage analysis and entered in the second stage to see whether it enables the explanation of residual variance. If a low eligibility criterion is used, the present algorithm will make a final sweep over all the final groups before dropping them from consideration, thus providing information on how all of the predictors are distributed within each group. These distributions can be used to provide information as to whether the group occupies its present place because of its actual pedigree or because of some other factor(s) correlated with the ones used to form it.

Moreover, it would certainly be desirable to obtain information on the zero-order correlations among the predictors in the sample. Since they are classifications, this is not easy. A complete set of bivariate frequency distributions provides a general impression. Further improvements in the algorithm itself should provide for a satisfactory method of computing the intercorrelation matrix of predictors at each branch of the tree.

If there are some variables which, because of high intercorrelations or low logical priorities, must be put into a second-stage analysis, one will not know (and has decided not to ask) what their influence would have been in the formation of the first-stage groups. The second stage, however, will show whether or not their influence on the dependent variable has already been accounted for. Re-introducing the first-stage variables into the second stage will also provide an answer to the question of whether there is a small, but universal, effect across all groups which will appear when they are pooled for the residual analysis.

In some cases, the first-stage analysis will identify groups which are clearly constrained in some special way, and explained so clearly that they really should be eliminated from the subsequent analysis.

Concentrating on explaining the level of the dependent variable may tend to obscure other information contained in the tree which may be extremely important. The homogeneity of the final groups, especially if some of them appear after only a few splits, and are large in size, may be more interesting and important than their average on the dependent variable. Since the program produces the standard deviation as well as the mean of each group, one can examine the variance, or relative variance of each final group. If any group has a larger variance than the others, it raises the question of whether there is some other factor which affects this group, or varies more over it, but which was not included in the analysis.

The use of the tree strategy calls one's attention to the possibility that one or two variables may be sufficient for explaining the variation associated with some of the observations, whereas, additional theoretical sophistication may be required for an adequate explanation of the remainder of the sample.

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# Appendix II
# Recoding

This section contains the detailed instructions for using the recoding routines that have been incorporated into AID. The recoding instructions consist of a series of clauses. One or more clauses constitute an instruction. The instructions are executed once for each observation read by the program during the input. They are executed in sequence, except as altered by GOTO instructions and those that are skipped because of the operation of the logical clauses and the truth switch. This is explained below.

## A. Instruction Format

Each instruction consists of a statement label (optional) and one or two clauses. There may be a relational clause and/or an operational clause. An optional comments field is included with the statement but is merely printed on the user's output, having no effect on the computations.

All four parts of a single statement are punched on a single IBM card.

### (1) Statement Labels

A positive integer may be attached to a statement. This is ordinarily optional and merely serves to identify the statement. It is required if the statement is to be referenced by a GOTO instruction. Not every statement need have a statement label and statement labels may be used in any order. A given positive integer may not be used as the label for more than one statement.

### (2) Relational Clauses

A relational clause consists of a logical tag and, depending on the tag, it may also have a test variable reference, a relational operator, and A or (A and B) operands. If the logical tag used does not require the use of the remaining fields, their contents will be ignored. If there is no logical tag, no test variable number, no relational operator and no A and B operands, the instruction is deemed to have no relational clause as part of it. If at least one of these fields exists, blank tags, test variable references and relational operators are assumed to be the same as those in the immediately preceding logical clause. (Note: A and B operands must always be supplied when needed.) This permits the user to avoid excessive repetition in writing commands. If there is no logical clause, the instruction is deemed to consist only of computation. Logical clauses may be written without subsequent computational clauses, and thereby concatenated into a compound logical expression. This compound expression is of the form:

(((A)op B)op C)op D) etc. etc.

The user may make use of this nesting in adapting the commands to construct an expression reflecting the logical structure of his problem. A compound logical expression must begin with an Initial Clause. This is discussed in detail below.

The testing part of the relational expression uses four fields, a test variable reference, a relational operator and either one or two operands. The B operand is used only for the IN or OUT relational operators. It is ignored otherwise. When the logical clause is executed, the test is performed according to the requirements of the tag and the relational operator. The current values of the referenced variables are used. The result is a new value for the truth switch (see below), which is dependent on the old truth switch value as well as the test. The rules for referring to test variables and A and B operands are discussed below.

The test variable field may contain a test variable reference. This may be direct or indirect, but it must be a reference to a variable, not a constant.

An A or B operand may refer either to a variable, directly or indirectly, or it may contain a constant. Rules for forming A and B operands are discussed below.

Any relational operator may be combined with any legitimate test variable reference and any legitimate A (and, when appropriate, B) operand to form a test. The test should be formed by the user only if the tag requires a rest--and then it must be present.

### (3) Operational Clauses

An operational clause may or may not be present in a given statement. When present its form depends on the type of operation to be performed. If the operator used does not require certain of the available fields, they will simply be ignored (see below). As with the logical clauses, the complete absence of all values in any of the fields indicates the absence of the clause.

If at least one is present, the clause is deemed to exist. Missing values of the Resultant Variable and the Control Operator are then supplied automatically from the previous operational clause. Values for operands must always be supplied if the operand is to be used. The Result Variable, where required, must be a direct or indirect variable reference. The C and D operands may be either direct or indirect variable references or integer constants.

Figure A2.1 illustrates the format required.

## B. The Interpreter and Executor

The instructions supplied by the user are interpreted and stored by "the interpreter." The executor is a two-state machine. It operates either in logical mode or in computational mode. It initiates execution of the instruction sequence supplied to it once for each data-unit read by the computer. It starts in computation mode, executing operations in the order submitted by the user, except where the sequence of computation is altered by the execution of a GOTO instruction. Computation is its ordinary mode of operation.

When any logical clause of any kind is encountered, the machine exits immediately from computational mode and no further computational operations can be executed until a return from logical mode has occurred.

When this exit from computational mode takes place the value of the machine's "truth switch" first becomes undefined. Then the machine reenters logical mode and the only operation that can be executed is the evaluation of logical expressions and the establishment of a new value for the truth switch. When in logical mode and with an undefined truth switch the only type of operation that can be executed is one which can define a new value, either true or false, for the truth switch. Then, after the truth switch has been defined an exit to computational mode is possible. This can occur when the truth switch has the value "true" and a computational operation follows. Upon detecting this condition, the machine reverts to computational mode and the computation is executed. If subsequent instructions contain only computational clauses, the machine remains in computational mode.

The truth switch can become defined only through assignment or through evaluation of an Initial Clause. There are two logical constants (T, true, and F,

Figure A2.1

MULTIVARIATE RECODING FORM

Coder: ____________________ Job: ____________________

Date: ____________________ Data: ____________________

| STATEMENT NO. | LOG. TAG | TEST VAR. NO. | REL. OPER. | A OPERAND | B OPERAND | RESULT VAR. NO. | CONTROL OPER. | C OPERAND | D OPERAND | COMMENT |
|---|---|---|---|---|---|---|---|---|---|---|
| 5<br>□xxxx | 10<br>□xxxx | 15<br>xxxxx | 20<br>□xxxx | V 30<br>Pxxxxxxxxx | V 40<br>Pxxxxxxxxx | V 45<br>Pxxxx | 50<br>□xxxx | V 60<br>Pxxxxxxxxx | V 70<br>Pxxxxxxxxx | 80<br>□□xxxxxxxx |
| 1 | T | | | | | | | | | TRUNCATE |
| | IF | V0005 | GT | 100000 | | V0005 | = | 100000 | | |
| | | | | | | | | | | |
| 2 | IF | V0010 | LE | -25 | | V0010 | = | 1 | | BKT V10 |
| | ØR | | IN | -24 | -5 | | | 2 | | |
| | | | | -4 | 4 | | | 3 | | |
| | | | | 5 | 24 | | | 4 | | |
| | | | | 25 | 99 | | | 5 | | |
| | | | GE | 100 | | | | 6 | | |
| | | | | | | | | | | |
| 3 | IF | V0020 | IN | 0 | 2 | | | | | |
| 30 | AND | V0021 | | 0 | 2 | V0100 | = | 1 | | |
| 31 | ALT | | | | | | = | 0 | | |
| | | | | | | | | | | |
| 4 | IF | V0030 | EQ | 1 | | | | | | |
| | ØR | V0031 | IN | 2 | 9 | V0101 | = | 1 | | |
| | | | | | | | | GOTO | 100 | |
| | IF | V0040 | EQ | 1 | | | | | | |
| | AND | V0041 | IN | 2 | 9 | V0101 | = | 2 | | |
| | ALT | | | | | | | 3 | | |
| 100 | T | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |

false) which can be used to assign a value to the truth switch.

An Initial Clause is the first (and may be the only) part of a relational proposition. In other words, a relational proposition must begin with an Initial Clause or Assignment and may (or may not) be followed by one or more Subsequent Clauses. An Initial Clause is always headed by the tag IF (if) or NIF (if not). It is never headed by another type of tag. Subsequent Clauses are always headed by one of the other types of tags (e.g., OR, NOR, EXOR, etc.), and are never headed by IF, NIF, T, or F. The truth switch acquires a defined value as soon as an initial clause has been evaluated or it is assigned T or F. Its new value is dependent only on the current operation, not upon past values. The new value is established accordingly on the basis of the truth tables associated with IF, NIF, T, or F (see Table A2.1).

Since the truth switch has a new value defined after the evaluation of the Initial Clause, computational statements may be placed between the Initial and Subsequent Clauses or between Subsequent Clauses. If such statements are encountered in logical mode when the value of the truth switch is T, the machine will exit to computational mode and any number of logically consecutive statements can be executed. However, if such computational statements are encountered when the machine is in a logical mode, when the value of the truth switch is still F, then the machine will remain in logical mode. The computational statement(s) will then be passed over and the machine will proceed to evaluate the next Subsequent Clause. If, while in logical mode and seeking a Subsequent Clause, the machine encounters another Initial Clause, the truth switch will simply be reset according to the tag associated with this new Initial Clause. Then the machine will proceed in logical mode as above, again seeking a following Subsequent Clause or exiting to computational mode if the truth switch happens to be T when a computational statement is encountered.

When in computation mode the only instructions the machine can execute are another computation operation or the evaluation of an Assignment or Initial Clause, the latter two causing a change to logical mode and immediate redefinition of the truth switch. Encountering a Subsequent Clause when in computation mode also causes an immediate exit to logical mode, but with the value of the truth switch undefined.

When in logical mode the machine can do nothing but evaluate an Initial Clause or an Assignment, until its truth switch acquires a defined value. After that, it can evaluate Subsequent Clauses and can exit to computational mode if a computational instruction is encountered at the time the truth switch is T. If the truth switch is defined but equals F, and the machine is in logical mode, it

Table A2.1

Truth Tables for Boolean Operators

Test

| switch | 1 | 0 |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 0 | 0 |

F

| | 1 | 0 |
|---|---|---|
| | 1 | 0 |
| | 0 | 0 |

AND

| | 1 | 0 |
|---|---|---|
| | 0 | 1 |
| | 0 | 0 |

NIMP

| | 1 | 0 |
|---|---|---|
| | 1 | 1 |
| | 0 | 0 |

ALSO

| | | |
|---|---|---|
| i | 0 | 0 |
| 0 | 1 | 0 |

BIF

| | |
|---|---|
| 1 | 0 |
| 1 | 0 |

IF

| | |
|---|---|
| 0 | 1 |
| 1 | 0 |

EXOR

| | |
|---|---|
| 1 | 1 |
| 1 | 0 |

OR

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 0 | 1 |

NOR

| | |
|---|---|
| 1 | 0 |
| 0 | 1 |

CONS

| | |
|---|---|
| 0 | 1 |
| 0 | 1 |

NIF

| | |
|---|---|
| 1 | 1 |
| 0 | 1 |

NBIF

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 1 |

ALT

| | |
|---|---|
| 1 | 0 |
| 1 | 1 |

IMP

| | |
|---|---|
| 0 | 1 |
| 1 | 1 |

NAND

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

T

NOTE: The Test values appear across the top as indicated and the current value of the truth switch appears at the left side. The resulting value of the truth switch appears in the body of the table.

can only evaluate relational clauses, either Initial or Subsequent (all computational instructions being passed over and ignored). Note that Subsequent Clauses which follow computation and which are not preceded by an Initial Clause can _never_ be evaluated and that computation which follows them can _never_ be executed.

### (1) The Truth Switch

Computation is started with the truth switch equal to T. Then when the first exit to logical mode occurs, the value of the truth switch becomes undefined. Later, when an Initial Clause is evaluated or an Assignment takes place, the truth switch acquires a defined value of T or F.

Each logical tag has associated with it a truth table for relating the current value of the switch and the results of the current test to a new value for the switch. Thus, when the machine is in logical mode and an expression is evaluated (as we have seen expressions are not always evaluated), a new value for the truth switch is defined. The truth tables for all of the logical tags were presented in Table A2.1.

### (2) Logical Tags

The truth switch and the truth value of the current test are used together to establish a new value for the truth switch. The way in which these two quantities are used together to establish this new value is determined by the logical tag associated with the relational expression.

Each logical tag has associated with it a truth table (see Table A2.1). The truth switch has a defined value either T(true) sometimes represented by a "1" or F (false), sometimes represented by a "0." Similarly, the relational expression has a value of true or false, depending on the values of the variables that are supplied to it when the test is made. The new value of the truth switch is simply obtained from the corresponding row and column of the appropriate truth table.

The tags correspond to the sixteen unique elementary Boolean operators.[1] They are:

1. T -- The Boolean constant, "true."
2. F -- The Boolean constant, "false."
3. IF -- A univariate operator dependent only on the value of the test.
4. ALSO-- A univariate operator dependent only on the value of the truth switch.
5. AND -- Corresponds to set intersection. True only if both conditions are true.

[1] Hohn, 1966.

6. NAND-- The negative of AND. False only if both conditions are true.
7. OR -- Corresponds to set union. True if either or both conditions are true.
8. NOR -- The negation of OR. True only if both conditions are false.
9. NIF -- "If not true that"--the negation of IF, a univariate operator, dependent only on the value of the test. True only if the test is false.
10. ALT -- "Alternatively," a univariate operator dependent only on the value of the truth switch. It is true only if the truth switch is false; thus, it reverses the value of the truth switch.
11. IMP -- "Implication" operator, false only if the first condition is true, but not the second.
12. NIMP-- "Does not imply," the negation of implication. True only if the first is true, but not the second.
13. BIF -- "But if." True only if the first condition is false and the second one true.
14. NBIF-- The negation of "but if."
15. CONS-- The consistency, or biconditional operator, requiring both true or both false for the result to be true.
16. EXOR-- Exclusive OR. True if one or the other is true, but not both.

Not all tags will be found to be equally useful, especially since many are simply negations of the others. However, providing all of them makes it possible for the user to express the logical aspects of his problem in a manner most convenient for the way in which he thinks of it.

The tags likely to be found to be of most use are IF, AND, OR, ALT and T, and the user is directed toward an understanding of these first.

Well-formed logical expressions begin with an Initial Clause and end with a Subsequent Clause, usually headed by an ALT. This insures that the truth switch is "true" when subsequent operations are attempted. It is good practice to head a block of computation with a T to avoid possible earlier paths that arrived at this point with the truth switch undefined.

## C. Operands

All operands are assumed to be integer in mode. There are three kinds of operands permitted: variables, constants and pointers.

### (1) Variables

An understanding of the notation used is facilitated by considering the vector form of the data that is supplied to the program. All of the variables for a single unit of analysis (person, interview, transaction, automobile) are stored as a vector of integers. The file consists of a sequence of these vectors.

The program reads one vector at a time during its input phases. After this vector has been read, execution of the variable generation instructions is initiated, starting with the first instruction. The path taken through the instructions depends on the values of the input variables for that particular vector and the relational tests and computations that are performed upon them.

The variables are simply numbered using integers up to 9999, and there are up to 300 permitted in the current implementation of the program. Fewer than that may actually have been read in, but the user is free to use any of the others. They will all have been set to zero before the first analysis vector is read. Only those actually read in each time will have new values for each vector. The others are under the subsequent control of the user.

A variable is ordinarily referred to by placing it in the operand position of an instruction. This reference is of the form

Vnnnn

where V is a capital letter which is placed in the indicated column in the operand field, and nnnn is a positive integer. The positive integer must be right-adjusted in the 4-column space reserved for it. Some examples are:

```
V0005  (variable five)
V0019  (variable nineteen)
```

(2) Constants

If the V is not present, and the column usually containing it is blank, the operand will be interpreted as an integer constant which may be as much as eight digits in width, or seven digits and a minus sign. The constant must be right-adjusted in the 8-column space provided in the operand field for it. Some examples are:

```
 5
05
+5
-5
```

Minus signs must be punched and are to be placed just to the left of the most significant digit. Plus signs may be punched but are not ordinarily used. Use of leading zeroes is optional. Constants must be positive or negative integers, not alphabetic characters or real numbers containing a decimal point.

Several simple examples are given on Figure A2.1. The two lines starting with statement 1 first turn the truth switch to true. They are not really necessary since the next statement starts with IF, but a reminder. The next line truncates variable 5 so that all values over 100,000 are changed to 100,000 (to keep extreme cases from dominating things, without omitting them altogether).

The six lines starting with statement 2 change variable 10 from a numerical field to a bracket or interval code. Note that the ranges go from the smallest real number up, e.g., from -24 to -5. Note also the economy in the implied ditto signs when OR, IN, etc. are repeated automatically.

The three lines starting with statement 3 illustrate the logical (AND) which means "both A and B," i.e., the new variable is coded 1 only if both things are true. ALT reverses the truth switch so that the remainder (only A, or only B, or neither) can be coded 0.[1]

The seven lines starting with statement 4 illustrate the GOTO, used to skip one group past the ALT statement so they will not be recoded to 3 after having been recoded to 1. The implied code created is:

| Code | Meaning |
|---|---|
| 1 | V30 = 1 and/or V31 = 2-9 (either or both) |
| 2 | V40 = 1 <u>and also</u> V41 = 2-9 |
| 3 | Remainder (neither of the above are true) |

Statement 100 turns the switch to True just in case the next operations are not to start with IF.

## (3) Indirect References and Pointer Variables

A variable may also be referred to in an operand by means of a pointer. A pointer is established and given a value by setting any variable so that it contains a positive integer. This is accomplished using an arithmetic operation with the variable to be used as the pointer as a result, e.g., V101 = 26.

Subsequently, the pointer variable is referred to in an indirect reference and identified as a pointer by the reference. If this occurs, the operand is interpreted as a reference to the variable <u>whose number is contained in the pointer</u>.

Any variable can be used as a pointer. A positive integer can be placed in it using arithmetic operators. The contents of any pointer can be modified at any

[1] NOTE: The operational clause in statement 31 <u>must</u> have a non-blank field for either the result variable or the operator since the C and D operands are <u>NOT</u> non-zero or blank, e.g., without the = the clause is ignored and V100 is never set to 0.

time using arithmetic operations and referring to it by ordinary variable references to it. It can be subjected to relational tests. In fact, the only difference between a pointer and a regular variable is what happens when the pointer variable is used as an indirect reference in an operand. When this occurs, the pointer is used simply to find out which other variable is to be used as the operand.

The difference between direct and indirect references to variables using pointers is best elucidated by giving an example.

```
01      V0051 = 111
02      V0016 ADD P0051     V0015
```

In instruction 01 the constant 111 is placed in variable 0051. A direct reference is made to V0051 to put it there. In statement 02, an indirect reference is made to variable 0051. It is used as a pointer, as indicated by the reference P0051. The pointer points to variable 111. Thus, instruction 02 really reads "add the contents of that variable whose number is contained in V0051 to the contents of V0015 and put the result into V0016." But V0051 already contains the number 111 because of instruction 01. Thus instruction 02 is interpreted as though it actually read

```
02      V0016 ADD V0111   V0015
```

Thus, variable 111 is added to variable V0015 and the results placed in variable V0016.

If we had the set of instructions shown in Figure A2.2, we could obtain the sum of those variables in the sequence V0011, V0012, V0013,..., V0030, V0031 which did not have the value 9. The results would be placed in V0044. For instance, one might want to build an index adding the code values for a number of different variables except where they are 9 (for not ascertained).

Instructions 1, 2 and 3 establish the initial conditions desired, i.e., the new variable to be generated (V0044) is set to zero and the pointer is set to the first of the sequence of variables to be added, V0011. Then control is passed to instruction 6. This instruction adds the variable "pointed to" to the partial sum being developed in V0044 and puts the results back in V0044. However, the operation is performed only if the variable "pointed to" does not have 9 as its value. (Let 9 be the "missing-data-code" for each of this series of variables V0011-V0031.)

If the value of the variable currently being pointed to is not equal to 9, its contents are added to V0044. But if its value is equal to 9, the

Figure A2.2

MULTIVARIATE RECODING FORM

Sum Variables 11 to 33

Coder: ____________________ Job: ____________________

Date: ____________________ Data: ____________________

| STATEMENT NO. | LOG. TAG | TEST VAR. NO. | REL. OPER. | A OPERAND | B OPERAND | RESULT VAR. NO. | CONTROL OPER. | C OPERAND | D OPERAND | COMMENT |
|---|---|---|---|---|---|---|---|---|---|---|
| 5<br>□ xxxx | 10<br>□ xxxx | 15<br>xxxxx | 20<br>□xxxx | V 40<br>Pxxxxxxxxx | V 30<br>Pxxxxxxxxx | V 45<br>Pxxxx | 50<br>□ xxxx | V 60<br>Pxxxxxxxxx | V 70<br>Pxxxxxxxxx | 80<br>□xxxxxxxx |
| 1 | | | | | | V0044 | = | 0 | | Set |
| 2 | | | | | | V0101 | = | 11 | | initial |
| 3 | | | | | | | GOTO | 6 | | values |
| 4 | | | | | | V0101 | ADD | V0101 | 1 | Increment |
| 5 | IF | V0101 | GT | 31 | | | GØTØ | 9 | | and test |
| 6 | IF | P0101 | NE | 9 | | V0044 | ADD | V0044 | P0101 | |
| 7 | ALT | | | | | | NØØP | | | Perform |
| 8 | T | | | | | | GOTO | 4 | | task |
| 9 | | next instruction | | | | | | | | |
| | | | | | | | | | | |
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addition does not take place and the alternative (instruction 7) is executed instead. This results in nothing being added to V0044 from the current variable being pointed to. Then, after one of these two paths has been taken, instruction 8 sends control back to instruction 4.

The purpose of instructions 4 and 5 is to increment the integer in the pointer by 1 and then to check it to determine if the value of the integer in the pointer has now exceeded the subscript of the last variable we wished to add. Then either an exit takes place to the remainder of the work to be done (statement 9) or the next variable pointed to is checked and, if appropriate, is added and the loop repeats once more. In instruction 4 the constant 1 is added to the number in the pointer. (This started at 11 and thus eventually will take on as its value in turn all the integers between 11 and 32 inclusively.) Its value at the time statement 9 is executed is 32. Note that when the number in the pointer is larger than 31 we have completed our task. At this point, the relational expression in instruction 5 will be true and the operational clause will be executed, taking control to statement 9 in computational mode.

On the other hand, if the integer in the pointer is equal to or less than 31 this expression will be false; consequently, the operational clause will not be executed and control will pass in logical mode to statement 6. Statement 6 starts with an Initial Clause so logical mode is re-entered and the truth switch acquires a value based only on statement 6. If it is true, the operational part of statement 6 will be executed and computational mode will be assumed. This will prevent the ALT tag in statement 7 from being executed, since ALT heads a Subsequent Clause. Since the program is in logical mode, the operational part of statement 7 also is not executed. However, statement 8 contains an Assignment; hence the truth switch is set to T and the operational part of the statement is executed, sending control to statement 4 via the GOTO.

In the case where the test for a value of 9 is met, computational mode will be entered at statement 6 and the operational part of statement 6 executed. The occurrence of a logical clause in statement 7 causes a return to logical mode, but the tag is a Subsequent Clause and is ignored as above. The truth switch remains undefined until statement 8.

Eventually the value of the pointer variable reaches 32; but we do not wish to add V0032 to V0044. However, the relational expression in statement 5 would be true when the pointer reached 32. This would cause control to be transferred out of the loop upon execution of the operational part of instruction 5. On the other hand, if we had not yet finished our task, this relational expression would still be false and the operational part of instruction 5 would not be executed. Consequently, control would pass to instruction 6, which is what we desire since this would process the next variable in its turn.

This procedure would examine each variable in turn, adding it to V0044 if its value were not 9. Then, when variables V0011 through V0031 inclusively had been inspected (and added where appropriate) the pointer, V0101, would have the value 32 and control would pass to statement 9.

It can be seen from the above example that the benefits to be derived from using pointers are associated with repetitive operations that would take a great many instructions if written out once for each case. These benefits increase with the size of the block of "work" instructions that are to be applied repetitively.

A second example illustrates another kind of use of the pointer variable. (See Figure A2.3.) Here, a complex bracket code is applied both to variable 0061 and to Variable 0062, replacing each with the appropriate bracketed value. The reader is encouraged to proceed through the example in some detail, following out alternative paths that follow from different values of V0016 and V0062.

## D. Operators

All "a" and "b" operands are assumed to be in integer mode, as well as the test variable, Vn.

### (1) Relational Operators: IN, OUT, NE, EQ, LE, GE, LT, GT

| | | | | |
|---|---|---|---|---|
| Vn | IN | a | b | True if Vn lies inside the closed interval where "a" is the lower bound and "b" is the upper bound. The lower bound must always be placed as the "a" operand. |
| Vn | OUT | a | b | True if Vn lies outside the closed interval, where "a" is the lower bound and "b" is the upper bound. The lower bound must always be placed as the "a" operand. |
| Vn | NE | a | | True if Vn is not equal to "a" algebraically. |
| Vn | EQ | a | | True if Vn is equal to "a" algebraically. (Note that EQ Is used here; = for arithmetic operation) |
| Vn | LE | a | | True if Vn is less than or equal to "a" algebraically (e.g., -6 is less than -5). |
| Vn | GE | a | | True if Vn is greater than or equal to "a" algebraically (e.g., -1 is greater than -2). |
| Vn | LT | a | | True if Vn is less than "a". |
| Vn | GT | a | | True if Vn is greater than "a". |

Figure A2.3

MULTIVARIATE RECODING FORM

Applying a Bracket Code to Two Variables Using Pointers

Coder: ______________ Job: ______________

Date: ______________ Data: ______________

| STATEMENT NO. | LOG. TAG | TEST VAR. NO. | REL. OPER. | A OPERAND | B OPERAND | RESULT VAR. NO. | CONTROL OPER. | C OPERAND | D OPERAND | COMMENT |
|---|---|---|---|---|---|---|---|---|---|---|
| 5<br>▯ xxxx | 10<br>▯ xxxx | 15<br>xxxxx | 20<br>▯xxxx | 30<br>V Pxxxxxxxxx | 40<br>V Pxxxxxxxxx | 45<br>V Pxxxx | 50<br>▯ xxxx | 60<br>V Pxxxxxxxxx | 70<br>V Pxxxxxxxxx | 80<br>▯▯ xxxxxxxx |
| 1 | | | | | | V0201 | = | 61 | | |
| 2 | | | | | | V0202 | = | 901 | | |
| 3 | | | | | | | GOTO | 9001 | | |
| 901 | | | | | | V0201 | = | 62 | | |
| 4 | | | | | | V0202 | = | 902 | | |
| 5 | | | | | | | GOTO | 9001 | | |
| 902 | | | | | next instruction | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | procedure to be applied | | | | | |
| 9001 | IF | P0201 | LE | 2499 | | P0201 | = | 1 | | |
| | OR | | IN | 2500 | 4999 | | | 2 | | |
| | | | | 5000 | 7499 | | | 3 | | |
| | | | | 7500 | 9999 | | | 4 | | |
| | | | | 10000 | 14999 | | | 5 | | |
| | | | | 15000 | 999998 | | | 6 | | |
| 9002 | ALT | | | | | | | 9 | | |
| 9003 | T | | | | | | GOTO | V0202 | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |

### (2) Arithmetic Operations: =, SET, ZAD, ZSB, ABS, ADD, SUB, MPY, DIV, MOD

All operands and results are assumed to be in integer mode. Note that the relational operator for equals is "EQ" but the arithmetical operator is "=". The first is a logical test, the second is an instruction that a variable assume a certain value.

Vn = c<br>
Vn SET c<br>
Vn ZAD c

The contents of "c" are placed in Vn. Note "=" is used, not "EQ".

Vn ZSB c

The contents of "c" are placed in Vn and the sign is is reversed (plus becomes minus, and vice versa).

Vn ABS c

The contents of "c" are placed in Vn and the sign of Vn is set to +.

Vn ADD c d

The contents of "c" and "d" are added together algebraically and the result is placed in Vn. The largest result is $2^{31}$. Overflows will not be detected.

Vn MOD c d

This is a modulo operator. The contents of "c" are divided by the contents of "d" using truncated integer division (without rounding). The remainder is placed in Vn. The quotient is lost. The sign of the number placed in Vn is the same as the sign of the c/d quotient.

Vn SUB c d

The contents of d is subtracted from c algebraically and the result stored in Vn.

Vn MPY c d

The contents of c are multipled by the contents of d algebraically and the results are placed in Vn. Ordinary rules apply as far as the sign of the result. Overflows will not be detected.

Vn DIV c d

The contents of c are divided by the contents of d using integer division. The integer part of the result is placed in Vn. The remainder is lost. There is no rounding; rather, truncation takes place. The sign of the resultant is according to regular algebraic conventions. If the result is between zero and 1 in absolute value, then Vn will have the value zero.

The acceptable range of integers is $2^{31}$ to $-2^{31}$ and customary rules apply to all signs; e.g., -10 multipled by -2 = +20.

### (3) Functions: NRAN, FRAN, LOGT, LOGE, SQRT, ASIN

Arguments are assumed to be integer in mode. Results will appear as integers with the appropriate number of implied decimal places as indicated by the d operand.

Vn NRAN c d A normally distributed random number with mean 0 and variance 1 will be stored in Vn with d implied decimal places. If c is not equal to zero or blank, then Vn will always be in the range $-c \le r \le + c$. All generated numbers outside this range will be rejected and another number generated until one meets the requirements; this is then stored in Vn. On the other hand, when c = 0, then the first number generated will be used. Numbers will be rounded to d implied decimals.[1]

Vn FRAN c d A random number from a flat distribution in the range $0 \le r \le c$, and having d implied decimals will be stored in Vn. Numbers will be rounded to d implied decimals.[1]

Vn LOGT c d The logarithm to the base 10 of c is stored in Vn. The number located at c is assumed to have d decimals. The logarithm stored in Vn always has three implied decimal places. If $c \le 0$, Vn = 0.

Vn LOGE c d The natural logarithm of c is stored in Vn. The number in c is assumed to have d decimal places. The result stored in Vn always has three implied decimal places. When $c \le 0$, then Vn = 0.

Vn SQRT c d The square root of the number stored in c is stored in Vn. This number stored in c is assumed to have d implied decimal places. The result stored in Vn will have d implied decimal places. If $c \le 0$, Vn = 0.

Vn ASIN c d The arcsin of c is stored in Vn. The number in c is assumed to have d decimal places. The result stored in Vn always has three implied decimal places. The function applies to all values of c such that $-1 \le c \le 1$. For all other cases Vn = 0.

[1] The pseudo-random number generation used is due to Hastings (1955). The methods were programmed and tested by Messenger (1970). A flat random distribution is generated by a non-overflow modulo type of algorithm using a call to the clock to obtain a starter number. A transformation is applied to produce a normal distribution. The cycle length is greater than 10,000 (cycle length is the number of random numbers generated when the first exact duplication occurs). Other tests of randomness have been examined (Messenger, 1970) and show satisfactory results, including Chi-square tests of fit to uniform (0,1) at the .05 level based on 10,000 observations, no statistically significant serial correlations in tests varying lags from 1 to 16, and 95% confidence intervals of the sample mean and standard deviation allowing acceptance of the null hypothesis that the sample was drawn from a uniform (0,1). About 350 random numbers/sec are drawn on a 360/40.

(4) Control Operators: GOTO, EXIT, NOOP, PRNT

GOTO c — Control is transferred to the statement number whose integer identifier is referred to by the c operand. Statement numbers are positive integers in the range $1 \leq c \leq 9999$. The truth switch is not affected (its value is "on" at this time anyway). If there is no such statement number control will pass to the next sequential instruction.

EXIT — Control is returned to the calling program in which the recoding statements are imbedded. No further recode statements will be executed. An EXIT statement is generated automatically when an END statement is encountered.

NOOP — No operation. Used as a "landing field."

PRNT Vn Vm — Causes variables n and m to be printed. The latter, Vm, may be omitted.

(5) Pseudo Operators: END

END — This operator is used to mark the physical end of the recoding statements, indicating that no more follow, even on the same physical IBM card. Its appearance causes transfer of control to the calling program, since all recoding statements have been read and processed. During execution of the recoding, an END statement will be treated like an EXIT.

All of the operators, operands, tags, comments and statement labels are ilustrated and listed in Figure A2.4.

Figure A2.4

MULTIVARIATE RECODING FORM

Summary of Recoding Instructions

Coder: ______________ Job: ______________

Date: ______________ Data: ______________

| STATEMENT NO. | LOG. TAG | TEST VAR. NO. | REL. OPER. | A OPERAND | B OPERAND | RESULT VAR. NO. | CONTROL OPER. | C OPERAND | D OPERAND | COMMENT |
|---|---|---|---|---|---|---|---|---|---|---|
| 5<br>□xxxx | 10<br>□xxxx | 15<br>xxxxx | 20<br>□ xxxx | V 30<br>Pxxxxxxxxx | V 40<br>Pxxxxxxxxx | V 45<br>Pxxxx | 50<br>□ xxxx | V 60<br>Pxxxxxxxxx | V 70<br>Pxxxxxxxxx | 80<br>□xxxxxxxx |
| 0001 | T | Vxxxx | IN | Vxxxx | Vxxxx | Vxxxx | = | Vxxxx | Vxxxx | COMMENTS |
| 9999 | AND | Pxxxx | ØUT | Pxxxx | Pxxxx | Pxxxx | ZAD | Pxxxx | Pxxxx | |
| | NIMP | | NE | xxxxxxxx | xxxxxxxx | | SET | xxxxxxxx | xxxxxxxx | |
| | ALSØ | V0001 | EQ | -9999999 | -9999999 | V0001 | ZSB | -9999999 | -9999999 | |
| | BIF | V0300 | LE | 99999999 | 99999999 | V0300 | ABS | 99999999 | 99999999 | |
| | IF | | GE | | | | ADD | | | |
| | EXØR | P0001 | LT | V0001 | V0001 | V0001 | SUB | V0001 | V0001 | |
| | ØR | P0300 | GT | V0300 | V0300 | P0300 | MPY | V0300 | V0300 | |
| | NØR | | EXIT | P0001 | P0001 | | DIV | P0001 | P0001 | |
| | CØNS | | END | P0300 | P0300 | | MØD | P0300 | P0300 | |
| | NIF | | | | | | NRAN | | | |
| | NBIF | | | | | | FRAN | | | |
| | ALT | | | | | | LØGT | | | |
| | IMP | | | | | | LØGE | | | |
| | NAND | | | | | | SQRT | | | |
| | F | | | | | | ASOM | | | |
| | END | | | | | | GØTØ | | | |
| | | | | | | | NØØP | | | |
| | | | | | | | EXIT | | | |
| | | | | | | | PRNT | | | |
| | | | | | | | END | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |

# Appendix III
# Set-up Instructions

## A. General Description

AID3 is a generalized data analysis program which uses analysis of variance techniques to explain as much of the variance of a given dependent variable as possible.

The AID3 search algorithm makes successive dichotomous partitions on the sample, using independent variables to "predict" the dependent variable, in such a way as to maximize differences among the split groups.

The algorithm may be set

(1) to maximize differences in group means, slopes, or regression lines;

(2) to examine the explanatory power of 1, 2, or 3 successive splits before selecting the "best" split;

(3) to rank the predictors, weighting them as to preference in the partitioning;

(4) to sacrifice explanatory power for symmetry;

(5) to start after a specified partial tree structure has been generated;

(6) to run in successive stages, e.g. , redefining predictors, or creating and then pooling residuals in a 2nd stage analysis.

## B. Input

1. Data file
2. Dictionary file (optional)
3. Control cards

## C. Output

1. Initial printout
   (a) statistics for the sample

(b) analysis of variance on predictor configuration (optional)

2. Trace printout (split statistics)

   (a) all splits for each predictor

   (b) best split for each predictor

3. Final tables

   (a) analysis of variance on final groups

   (b) group summary (optional)

   (c) BSS/TSS with lookahead (optional)

   (d) BSS/TSS no lookahead (optional)

   (e) BSS/TSS regardless of eligibility (optional)

   (f) profile of class means/slopes (optional)

   (g) predictor summary (optional)

4. Output residual file (with optional dictionary)

## D. Restrictions

1. Maximum number of predictors: NP≤63
2. Largest valid class value for a predictor: 31
3. Sum of largest class values for all predictors: 400-NP
4. Maximum number of partitions: 89
5. Maximum number of variables: 300
6. R-type variables (see OSIRIS Recode) should not have the same variable number as V-type variables if an output dictionary is requested.

## E. Missing Data Treatment

1. Missing data on the dependent variable may be excluded.
2. Missing data defined in the input dictionary is passed on to the output dictionary.
3. Missing data is generated for the residual and predicted value as a field of 9's.
4. Missing data is generated for the group number as '000'.
5. Missing data may be excluded on the predictors by specifying maximum class values less than missing data values.

## F. Setup Summary

```
JOB cards

// EXEC ISRSYS

//FT07F001  DD  parameters describing input data file
                (omit if using an OSIRIS dictionary)

//FT08F001  DD  parameters describing output data file
                (omit if not generating a FORMATTED residual file)
```

```
//DICTx     DD  parameters describing input dictionary
                (omit if on cards or if a FORMATTED data file)
//DATAx     DD  parameters describing input data file
                (omit if on cards or if a FORMATTED data file)
//DICTy     DD  parameters describing output dictionary
                (omit if residual file not created or a FORMATTED output
                 file)
//DATAy     DD  parameters describing output data file
                (omit if residual file not created or a FORMATTED output
                 file)
//SETUP     DD  *
$RUN  AID3
$RECODE card          }
   Recode statements  }    optional (may be used only with OSIRIS
                                     data files)
$SETUP card
   1.  Global filter card (optional - only used with OSIRIS data files)
   2.  Label card
   3.  Global parameter card
   4.  Format cards (optional)
   5.  Input variable list card                              ┐
   6.  Local filter card (optional)                          │
   7.  Local label card                                      │
   8.  I/O parameter card                                    │ Analysis
   9.  AID3 internal recode cards (optional)                 } Packet
  10.  Predictor cards                            }          │
  11.  Control parameter card                     } Analysis │
  12.  Predefined split cards (optional)          } Step     ┘
$ DICT         }
  dictionary   }     If dictionary on cards
$ DATA         }
  data         }     If data on cards
/*
```

G. Description of Control Cards - defaults are underlined.

1. Global Filter (optional): See OSIRIS User's Manual
   For use with OSIRIS data files only

2. Global card: 1-80 columns used as a heading

3. Global parameter card:

   INFILE=IN/xxxx — For OSIRIS data files only. Up to 4 characters used as the input ddname suffix

   PRINT=DICT/NODICT — For OSIRIS data files only. Option to print or suppress printing of the input dictionary file

   BADDATA=TERM/MD1/MD2/SKIP — For OSIRIS data files only. When non-numeric characters (including embedded blanks and all blank fields) are found in numeric variables:

   TERM: Terminate the run

   MD1: Convert the value to the 1st missing data code and fields of &'s and -'s to nines + 1 or 2 respectively

   MD2: Convert the value to the 2nd missing data code and fields of &'s and -'s to nines + 1 or 2 respectively

   SKIP: Skip the case

   a message indicates the number of cases so treated

   FORMAT=0/n — For FORMATTED data files only. The number ($1 \leq n \leq 3$) of Format cards needed to describe the input data file FT07F001

4. FORMAT cards: If FORMAT=n, n>0, was specified on the global parameter card, n cards of format information. This should include all input fields, and if a formatted output file is requested the input descriptors should be followed by output fields for all generated variables (see section H for the order in which variables are generated), i.e., AID3 recoded, residual, predicted value, and group number variables. Each item constitutes one variable. Variables are read in the order listed on the variable list card, e.g., if the format is (I6,2I5,I4) and the variable list is V10, V6, V20, V1 * then the first format (I6) is for variable 10, the second (I5) is for variable 6 etc.

5. Input Variable List card: a list of all variables from the input data file plus OSIRIS recode variables (R-type) to be used in the run. (See OSIRIS User's Manual).

ANALYSIS PACKET

6. Local Filter (optional): See OSIRIS User's Manual

7. Local Label

8. I/O Parameter card:

YVAR=(n,m) — n=dependent variable number, no default m=scale factor. The y-values are multiplied by 10 to the scale factor, i.e., $10^m$ default: 0.

NOWEIGHT/WEIGHT=n — Weight variable number (optional).

ANALYSIS=MEANS/SLOPES/REGRESSION

Type of analysis to be used

XVAR=(n,m) — n=covariate number, no default if a SLOPES or REGRESSION analysis is specified m=scale factor, default: 0.

SUBSEL=n — Subset selector variable number (optional). If a subset selector variable is given, all zero values of this variable are filtered from the analysis.

MDOPTION=BOTH/MD1/MD2/NONE (optional)

Missing data option: (See OSIRIS User's Manual)

NONE: Ignore missing data.

MD1: Eliminate cases with missing data 1 values for the dependent variable (Y=MD1).

MD2: Eliminate cases with missing data 2 values for the dependent variable
$Y>MD2$ , $MD2>0$
$Y\leq MD2$ , $MD2<0$.

BOTH: Eliminate cases with MD1 or MD2 values for the dependent variable.

NORECODE/RECODE — Internal AID3 recoding option. If RECODE is specified, then internal recode cards must be supplied (See 9. below).

OUTLIERS=INCLUDE/EXCLUDE — Option to include or exclude outliers from the analysis. An outlier is defined by the OUTDIST parameter.

OUTDIST=5./n — Number of standard deviations from the parent group mean defining an outlier (punch decimals).

IDVAR=n — Variable number for the identification variable printed with each case flagged as an outlier.

NOCONF/CONF — Analysis of variance on predictor configuration option.

TABLES=(BASIC/NONE,NOBSS/BSS,NOEL/ELIG,NOPR/PRED,NOME/MEANS)

Final tables options

BASIC: Print Group Summary and BSS/TSS with lookahead tables.

BSS: Print BASIC tables plus BSS/TSS with no lookahead.

ELIG: Print BASIC tables plus BSS/TSS of maximum split regardless of eligibility.

PRED: Print BASIC tables plus predictor summary.

MEANS: Print BASIC tables plus class means/slopes profile.

RESID=(n,m,w)

Omit if residuals are not to be generated.
n=residual variable number, no default.
m=residual scale factor, no default.
w=residual filed width, needed for OSIRIS output files only, no default. This should include a space for a sign character (+,-).

The following parameters are ignored if the residuals option is not specified:

RESNAME='name'

1-24 character name for the residual. Default: 'AID3 RESIDUAL'.

CALC=(n,m,w)

Omit if the predicted value is not to be generated.
n=predicted value variable number
m=predicted value scale factor
w=predicted value field width, needed for OSIRIS output files only.

CALNAME='name'

1-24 character name for the predicted value. Default: 'PREDICTED VALUE'.

GROUP=n

n=variable number for the final group to which each case belongs. Omit if not to be generated.

GRONAME='name'

1-24 character name for the group numbers. Default: 'GROUP NUMBER'.

NOFILE/FILE

Option to generate an output file. The current data set which is stored on a scratch data file is updated within the program enabling the user to use residuals in a multi-stage analysis even if NOFILE is specified.

OUTFILE=OUT/ xxxx/FORM

For OSIRIS output data files, 1-4 characters used as the output ddname suffix (see section I).
For Formatted output files: specify 'FORM'. FORM=n, n>0 must have been specified on the global parameter card. The same format is used for the output file as for the input file (see 4. FORMAT cards and section H). The output file is generated onto unit FT08F001.

NOSTAN/STANDARD

Option to standardize residuals. If STANDARD is specified residuals will be divided by the standard deviation of the final group to which they belong.

9. AID3 RECODE cards: RECODE must be specified on the I/O parameter card (8). The RECODE option is turned off after the INPUT mode is completed. See Appendix II for RECODE specifications.

ANALYSIS STEP

10. Predictor cards:

PRED=($p_1$,$p_2$,...) List of predictor variable numbers (up to 63) carrying the characteristics specified below.

PRENAME='name' 1-24 character name used for all predictors listed. Default: input dictionary name.

M/F Monotonic or Free characteristic
M: do not sort on predictor class means/slopes before executing the algorithm
F: sort on class means/slopes

MAXCLASS=9/n Maximum class value allowed for the predictors. Values greater than the specified maximum are eliminated from the analysis.

RANK=1/n Predictor rank, $0\leq n\leq 9$.

END Must be specified on the last predictor card.

11. Control Parameter Card

LOOKAHEAD=(n,m) n=the number of lookahead steps (0-2)
=the number of splits - 1.
m=the number of permuted steps.
If n is non-zero, m must be greater than zero. Default (0,0).

REDUCIBILITY=(for parent group; for 1st lookahead step; for 2nd lookahead step)

Reducibility criteria for each step in the lookahead expressed as a percentage e.g., .8%=.008. Default (.8;na,na) (Punch decimals).

MIN=25/n Minimum allowable number of cases in a group. If a SLOPES or REGRESSION analysis is specified then n must be at least 3. Otherwise, for mean analyses $n\geq 1$.

MAX=25/n Maximum number of partitions. $1\leq n\leq 89$.

SYMMETRY=0/n Percentage premium for symmetry option. (Punch decimals e.g., 60.%=.60).

RANK=NORANK/ALL/UP/AT/DOWN

Predictor ranking option
NORANK: no ranking - predictor ranks are ignored
ALL: simple ranking

UP: range ranking with a preference for predictors with high ranks, i.e., towards 1.

AT: range ranking with a preference for predictors with a rank "at" the current rank.

DOWN: range ranking with a preference for low ranked predictors, i.e., towards 9.

RANGE=(nup;ndown;lup;ldown)

for range ranking only

| | |
|---|---|
| eligible range =<br>[a-NUP,a+NDOWN] | nup: number of ranks "UP" for actual splitting<br>ndown: number of ranks "DOWN" for actual splitting |
| lookahead range =<br>[a-LUP,a+LDOWN] | lup: number of ranks "UP" for the lookahead$\geq$nup<br>ldown: number of ranks "DOWN" for the lookahead$\geq$ndown |

where a is the current rank at which the algorithm is operating.

TREE=0/n — Number of pre-specified splits to be made (see predefined splits cards, 12. below).

TRACE=ELIG/NOTR/BEST/BSS/MIN/ALL

Print trace option

ELIG: print only eligible splits

NOTR: suppress all printing

BEST: print only the best split for each predictor

BSS: print "ELIG" + splits which do not meet the reducibility criterion, i.e., BSS too small

MIN: print "ELIG" + splits with a resultant group with too few cases

ALL: print the entire trace

COMPUTE/NOCOMP — Command to execute the COMPUTE mode: generate any predefined splits if TREE=n,n>0; execute the splitting algorithm under current parameters, and if RESID is specified, generating residuals for those groups which cannot be split.

OUTPUT/NOOUT — Command to execute the OUTPUT mode: generate residuals for any remaining final groups; write final Tables.

12. Predefined split cards: TREE=n,n>0, must be specified on the control parameter card. The TREE option is turned off after the splits are made. For each split one card:

PARENT=n — Number of the group to be split.

CHILD=n — Number of the first resultant group. (Even number, the second group will be numbered n+1.)

| | |
|---|---|
| VARIABLE=n | Variable number for the predictor used to make the split. |
| CLASS=($c_1$,$c_2$,...) | List of predictor classes to be put into the first resultant group. Remaining classes go to the second child. |

13. If the NOOUTPUT command appears on the control parameter card (11) the analysis step is repeated (predictor cards, etc.). Parameters default to the previously defined values with the exception of the COMPUTE and OUTPUT commands. Predictor cards need only be included for those predictors being redefined. New predictors may not be defined. Also, the maximum class value may not be altered.

    If the OUTPUT command was specified, then the next analysis packet is executed (local filter, etc.). Parameters take on the original defaults.

## H. Generated Variables on 'Formatted' Output Files

Generated variables are added to the input variable list in the following order.[1]

| | Variable | Relevant Keyword |
|---|---|---|
| 1. | Dependent variable | YVAR |
| 2. | Weight variable | WEIGHT |
| 3. | Covariate | XVAR |
| 4. | Subset selector variable | SUBSET |
| 5. | Identification variable | IDVAR |
| 6. | Residual | RESID |
| 7. | Predicted value | CALC |
| 8. | Group number variable | GROUP |
| 9. | Recode variables, in the order in which they appear in AID3 internal RECODE stream | (RECODE) |
| 10. | Predictors, in the order they appear | PRED |

Note: Only those variable numbers not listed on the input variable list are added, and they follow the input variables. For example if the variables are

| Variable | Number |
|---|---|
| Y | 20 |
| predictors | 2-5,12 |
| residual | 10 |
| other input | 1,6-9 |

[1] This information is necessary when generating a formatted output file so that the output field descriptors are in the correct order on the FORMAT cards.

and the variable list card is V1-V9* then the output variable list order is V1-V9, V20,V10,V12*.

## I. OSIRIS Residual Files[1]

If OUTFILE=OUT is specified on the I/O parameter card, then an OSIRIS type 3 dictionary is generated with the following specifications:[2]

| Variable | Field Width[3] | Number of Decimals | Missing Data | Name |
|---|---|---|---|---|
| Y,X | 7 | p | 9...9 | s |
| Predictors | 2 | 0 | - | p/s |
| Residual | p | p | 9...9 | p/'AID3 RESIDUAL' |
| Predicted Value | p | p | 9...9 | p/'PREDICTED VALUE' |
| Group Number | 3 | - | 000 | p/'GROUP NUMBER' |
| Other input variables | s | s | s | s |
| AID3 Internal Recode generated variables | 2 | 0 | - | AID VARIABLE NUMBER x |

OSIRIS recode variables (R-type) may not have the same variable number as V-type variables if an OSIRIS residual file is generated. R-type variables become V-type variables on the output data file (i.e., R-type variables are designated by negative numbers, e.g., R10=-10, the absolute value is taken as the variable number for the output dictionary).

[1] See the OSIRIS User's Manual for a complete description of dictionary files.

[2] s denotes a specification in the input dictionary, if any; p denotes a parameter specification.

[3] Field widths may be overridden by specifying a variable number v, $9100 \leq v \leq 9999$. The hundredths digit of v is the field width. For example, variable 9476 has a field width of 4.

Appendix IV

# Program Documentation

AID3 was written for an IBM 360/40 computer. AID includes 30 FORTRAN IV subprograms, OSIRIS subroutines written in FORTRAN IV and assembly language, and an overlay structure (see Diagram 1). The size of the program after linking with the overlay is 85,000 bytes, and the core requirement is 104K. It uses five scratch files all written without format control. Parameter and variable information are stored in labelled common blocks.

## A. Program Structure

The program flow is controlled by the main program (ZAID3). After calling an initialization program to read and check parameter cards, read the dictionary, and initialize the input and output data and scratch files, the main program determines in which mode the program should be operating (input, compute, or output) and calls the appropriate subroutines. The input mode is executed automatically after the initialization stage and again after completion of each output phase. Compute and output commands from the user control the other two modes.

1. Initialization: The input and output units are defined; the input dictionary or format is read; if specified, an output dictionary is generated; all parameter cards are read and checked for errors. Execution is terminated if any errors occur.

2. Input: The input and output parameters are defined; the data set is read, recoded, and stored; non-valid data is eliminated for the current analysis packet and outputted onto the residual file if requested; sample statistics are generated and printed, including the optional 1-way analysis of variance on the predictor configuration.

3. Compute: Computation parameters are defined; any splits defined for the starting tree option are made; the lookahead algorithm is initiated and operates under the given parameters--symmetry takes precedence over

ranking which in turn takes precedence over the lookahead; any group which cannot be split is deemed a final group and if specified is outputted onto the residual file. When the splitting process terminates, if the output mode is not specified, the program redefines computation parameters and continues the splitting process from where it stopped under the new parameters.

4. Output: Any remaining unsplit group becomes a final group and, if specified, is outputted onto the residual file; final tables are generated, including the analysis of variance on final groups.

Diagram 2 gives the overall structure of the program.

## B. Program Storage

AID3 uses five temporary scratch files. Three are direct access files (ISR01, ISR02, FT05F001), and two are sequential (FT03F001, FT04F001).

1. AUNIT (ISR01): a direct access file (written and read with the OSIRIS subroutine DIRECT) used to store the input data matrix, one observation per record. Two additional records are written, following the data matrix, containing the missing data codes. If NV denotes the total number of variables referenced by the program, then the record length is 4(NV+7) bytes $\leq$ 1228. Chart 1 shows the storage needed.

2. BUNIT (ISR02): a direct access file (written and read with DIRECT) used to store information for each group: a list of the record numbers in AUNIT containing the observations in the group; group totals; and class totals for each predictor. The record length is 896 bytes. Chart 2 shows the storage needed for each group.

3. CUNIT (05): a direct access file (FORTRAN IV DEFINE FILE) used for the lookahead process and final tables. The maximum number of records used is 200, and the maximum record length is 224 words. The file is created with the statement

   ```
   DEFINE FILE 5(200,224,U,IC)  .
   ```

4. NSCR (03): an unformatted sequential file used to store information for each group on which a split attempt has been made. There are up to nine records per group, with a maximum record length of 126 words.

5. NSETUP (04): an unformatted sequential file used to store the input parameters (local filter and label, I/O parameters, internal recode statements, predictor information, and computation parameters).

The input and output units necessary to run AID3 are given below.

| Unit | Function |
|---|---|
| FT01F001 | control card input |
| FT03F001 | NSCR |
| FT04F001 | NSETUP |
| FT05F001 | CUNIT |

| | | |
|---|---|---|
| FT06F001 | printer | |
| FT07F001 | formatted input data file | omit if using OSIRIS data files |
| FT08F001 | formatted output data file | |
| ISR01 | AUNIT | |
| ISR02 | BUNIT | |
| DICTIN | input dictionary | omit if using formatted data files |
| DATAIN | input data file | |
| DICTOUT | output dictionary | |
| DATAOUT | output data file | |

## C. Execution without OSIRIS

The AID3 subprograms are written in FORTRAN IV level G, however they use OSIRIS subroutines[1] some of which are written in IBM assembler, in particular the OSIRIS input and output, filter, and recode routines.

With the exception of the OSIRIS program DIRECT which reads and writes the direct access scratch files ISR01 and ISR02, the code using the OSIRIS options may be deleted from the program and a comparable feature will still be available. For example,

| OSIRIS | AID3 |
|---|---|
| OSIRIS data sets | Formatted data sets |
| Multivariate Recode | Internal Recode |
| Global/local filter or missing data | Subset selector filter |

Running the program without OSIRIS will, however, be less efficient.

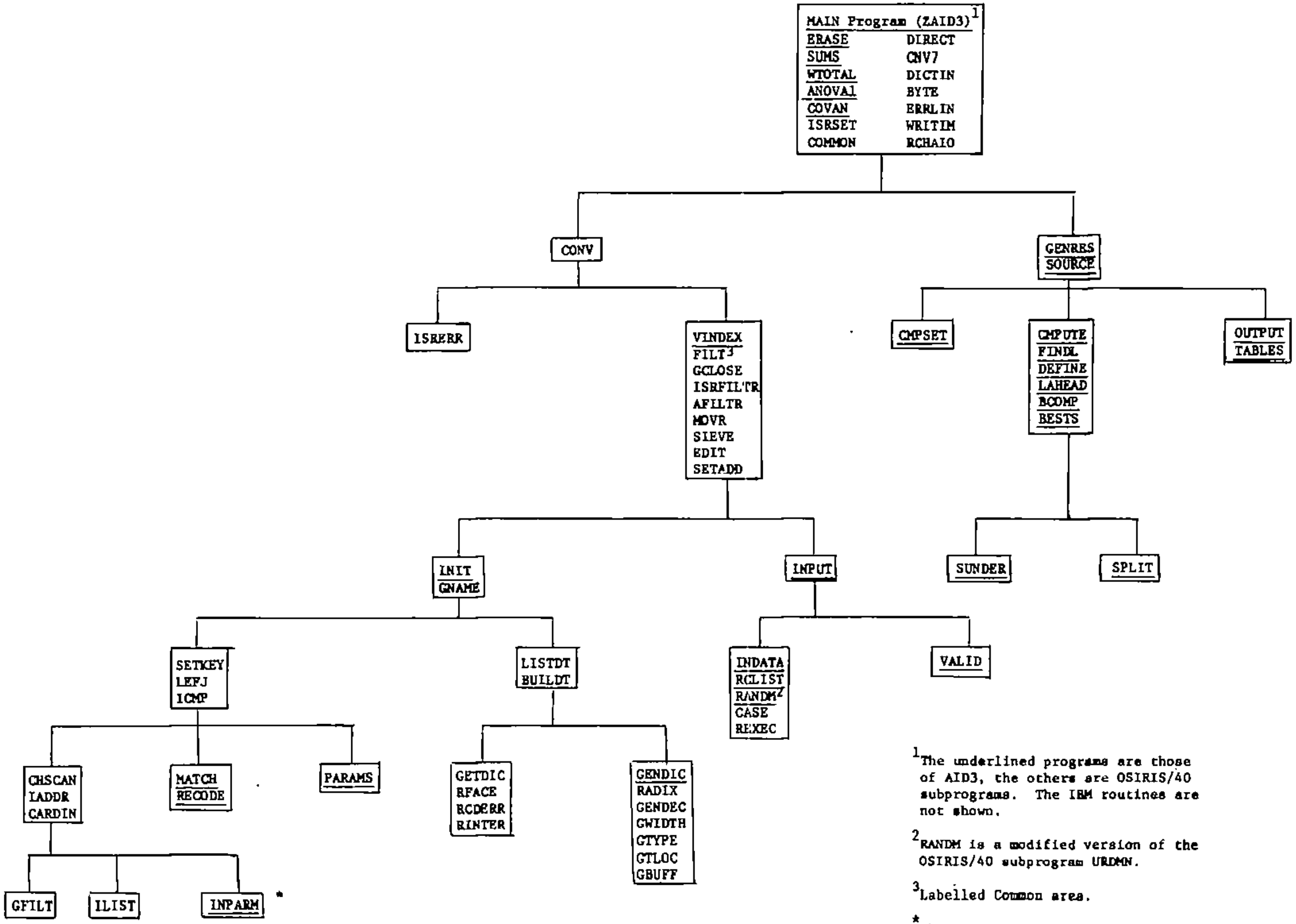
The subroutine DIRECT can be replaced with FORTRAN IV DEFINE FILES. This is tedious, since DIRECT is used throughout the program, i.e., during all modes, input, compute, and output.

Table A4.1 gives a list of all the AID3, OSIRIS, and IBM programs called by each of the AID3 subroutines. Also included are the labelled common areas used by each AID3 subroutine.

[1] See the OSIRIS Subroutine Manual

## Diagram 1

## Overlay Structure



[1] The underlined programs are those of AID3, the others are OSIRIS/40 subprograms. The IBM routines are not shown.

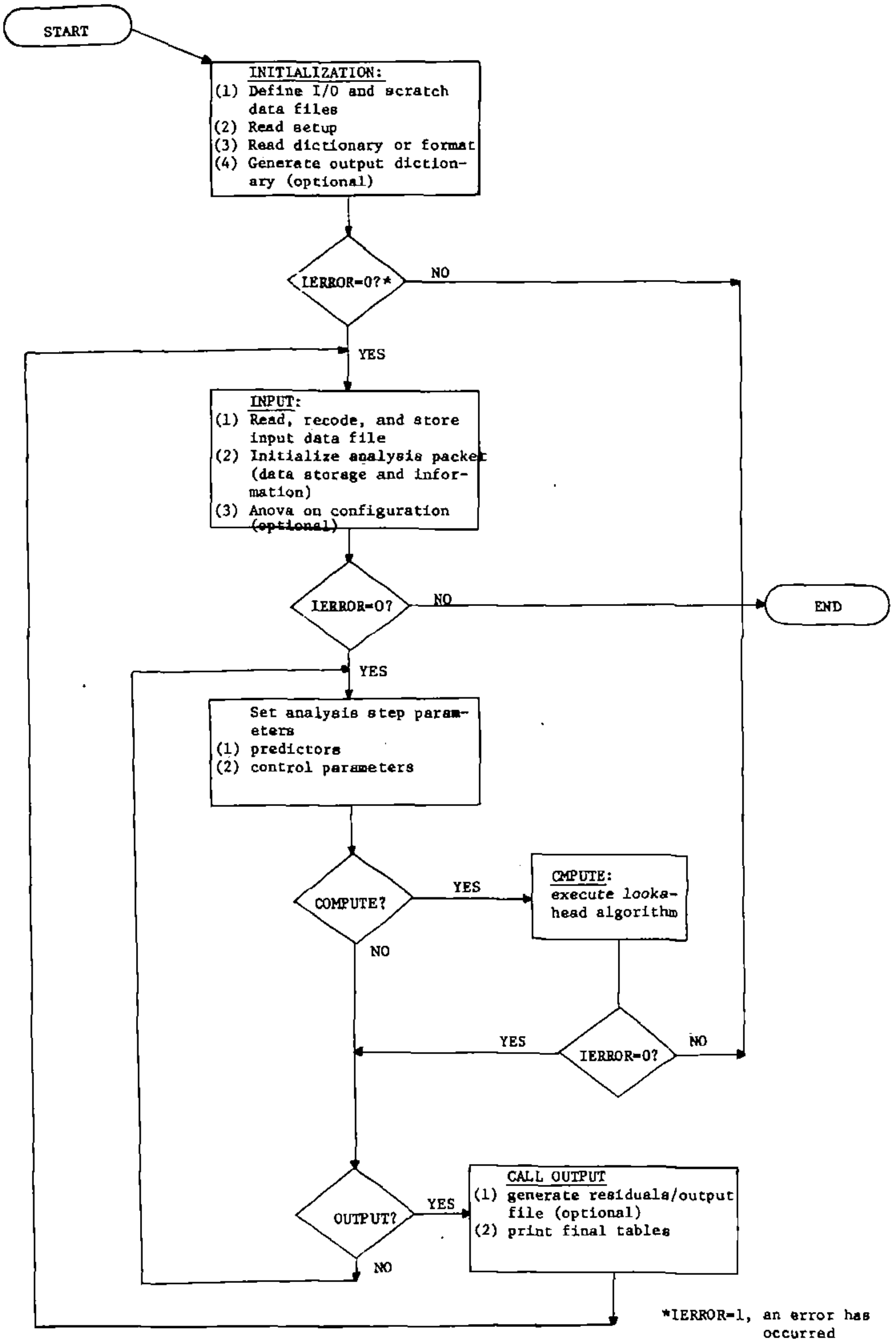
[2] RANDM is a modified version of the OSIRIS/40 subprogram URDMN.

[3] Labelled Common area.

*This is the longest path.

Diagram II

AID3 Program Structure


START
INITIALIZATION:
(1) Define I/O and scratch data files
(2) Read setup
(3) Read dictionary or format
(4) Generate output dictionary (optional)
IERROR=0?*
NO
YES
INPUT:
(1) Read, recode, and store input data file
(2) Initialize analysis packet (data storage and information)
(3) Anova on configuration ~~(optional)~~
IERROR=0?
NO
END
YES
Set analysis step parameters
(1) predictors
(2) control parameters
COMPUTE?
YES
CMPUTE:
execute lookahead algorithm
NO
YES
IERROR=0?
NO
OUTPUT?
YES
CALL OUTPUT
(1) generate residuals/output file (optional)
(2) print final tables
NO

*IERROR=1, an error has occurred
=-1, normal termination

## TABLE A4.1 AID3 SUBPROGRAMS REFERENCING TABLE

| | AID3 | OSIRIS | FORTRAN | LABELLED COMMON |
|---|---|---|---|---|
| MAIN(ZAID3 | INIT,INPUT,CMPSET, CMPUTE,OUTPUT | ISRSET* | DIOCS#,IBCOM# | LABELS,DSTORE,OPTION,PREDS,SPLITC,ELIGC,SYMTRY,RANKC |
| ERASE | — | — | — | — |
| SUMS | — | — | — | PREDS |
| WTOTAL | — | DIRECT* | — | DSTORE,PREDS |
| ANOVA1 | COVAN | — | IBCOM# | DSTORE,OPTION,PREDS,GROUPS,SPLITC,FACTOR |
| COVAN | — | — | IBCOM# | OPTION,FACTOR |
| VINDEX | — | — | — | VLIST |
| INIT | INPARM,RECODE, PARAMS,GENDIC | GFILT*,ILIST,SETKEY, GETDIC* | IBCOM# | LABELS,VLIST,DSTORE,OPTION,ELIGC,DEPEND,YREAD,FILT |
| INPARM | ERASE,VINDEX | LEFJ*,ISRFILTR*, SETKEY,GNAME* | IBCOM#,FRXPI#, FIXPI#,MINO | LABELS,VLIST,DSTORE,OPTION,PREDS,SPLITC,ELIGC,SYMTRY,RANKC,DEPEND, YREAD,FILT |
| RECODE | VINDEX,MATCH | — | IBCOM# | OPTION,RECORD |
| MATCH | — | — | — | |
| PARAMS | VINDEX | SETKEY,GNAME* | IBCOM# | LABELS,VLIST,DSTORE,OPTION,PREDS,SPLITC,ELIGC,SYMTRY,RANKC,DEPEND, YREAD |
| GENDIC | — | GNAME*,GWIDTH*,GNDEC*, RCHAIO*,RADIX*,CNV7*, LISTDT,MOVE* | IBCOM#,FIXPI#,ALOG10 | LABELS,VLIST,OPTION,PREDS,DEPEND,YREAD,FILT |
| INPUT | ERASE,ANOVA1,INDATA, VALID | SETADD* | IBCOM# | LABELS,VLIST,DSTORE,OPTION,PREDS,GROUPS,SPLITC,SYMTRY,RANKC, DEPEND,YREAD,FILT,FACTOR |
| INDATA | ERASE,RCLIST | DIRECT*,CASE*,GCLOSE* | IBCOM#,SQRT | LABELS,VLIST,DSTORE,OPTION,PREDS,GROUPS,SPLITC,DEPEND,YREAD |
| RCLIST | VINDEX,RANDM | — | ALOG,ALOG10,ARSIN IBCOM#,FRXPI#,SQRT | VLIST,OPTION,RECORD |
| VALID | ERASE,SUMS,WTOTAL | DIRECT*,SIEVE*,CNV7*, RCHAIO* | IBCOM# | VLIST,DSTORE,OPTION,PREDS,GROUPS,SPLITC,DEPEND,YREAD,FILT,FACTOR |
| GENRES | — | DIRECT*,RCHAIO*,CNV7* | IBCOM# | VLIST,DSTORE,OPTION,PREDS,GROUPS,SPLITC,DEPEND,YREAD |
| SOURCE | — | — | — | GROUPS,SPLITC,FACTOR |
| CMPSET | ERASE | — | IBCOM# | VLIST,OPTION,PREDS,SPLITC,ELIGC,SYMTRY,RANKC,DEPEND,FACTOR |
| CMPUTE | SOURCE,GENRES,FINDL, DEFINE,LAHEAD | — | IBCOM# | LABELS,VLIST,DSTORE,OPTION,PREDS,GROUPS,SPLITC,ELIGC,SYMTRY,RANK, DEPEND,FACTOR,BESTS |
| FINDL | — | — | — | GROUPS |
| DEFINE | GENRES,FINDL,SUNDER | — | IBCOM# | VLIST,DSTORE,OPTION,PREDS,GROUPS,SPLITC,SYMTRY,RANKC,DEPEND,FACTOR, BESTS |
| SUNDER | ERASE,SUMS,WTOTAL | DIRECT* | IBCOM# | VLIST,DSTORE,OPTION,PREDS,GROUPS,SPLITC,RANKC,DEPEND,FACTOR |
| LAHEAD | ERASE,FINDL,SUNDER, SPLIT,BCOMP | DIRECT* | IBCOM#,SQRT,MINO, MAXO | VLIST,DSTORE,OPTION,PREDS,GROUPS,SPLITC,ELIGC,SYMTRX,RANKC,BESTS |
| SPLIT | ERASE | — | IBCOM# | VLIST,OPTION,PREDS,GROUPS,SPLITC,ELIGC,DEPEND |
| BCOMP | — | — | MINO,MAXO | PREDS,ELIGC,RANKC,BESTS |
| OUTPUT | SOURCE,GENRES, ANOVA1,TABLES | DIRECT*,RCHAIO* | IBCOM# | VLIST,DSTORE,OPTION,GROUPS,SPLITC,DEPEND,YREAD,FACTOR |
| TABLES | ERASE,COVAN | DIRECT* | IBCOM#,SQRT,MINO,AMAX1 | VLIST,DSTORE,OPTION,PREDS,GROUPS,SPLITC,ELIGC,DEPEND,FACTOR |

* denotes a 360 assembly language routine

## Chart 1 - STORAGE AREA "AUNIT"[1]

NA Records of Length = NELEM + NV $\leq$ 307 words

NV $\leq$ 300, *the total number of variables accessed (input + generated)*

$$\underline{k^{\text{th}}\ \text{record}}\ :\ \overbrace{\begin{array}{|c|c|c|c|c|c|c|}\hline n_k & w_k & y_k & y_k^2 & x_k & x_k^2 & z_k \\ \hline\end{array}}^{\text{NELEM}^2}\overbrace{\begin{array}{|c|c|c|c|}\hline \cdot\ \ \cdot\ \ \cdot & v_k(i) & \cdot\ \ \cdot\ \ \cdot & v_k(\text{NV}) \\ \hline\end{array}}^{\text{NV}\emptyset\text{UT}}$$

$n_k$ = number of observations in $k^{\text{th}}$ record ($\equiv$1)

$w_k$ = weight variable

$y_k$ = weighted dependent variable ($wy'$)

$y_k^2$ = weighted square of the dependent variable ($wy'^2$)

$$\left.\begin{array}{l} x_k = \text{weighted independent variable } (wx') \\ x_k^2 = \text{weighted square of the independent variable } (wx'^2) \\ z_k = \text{weighted cross product } (wy'x') \end{array}\right\}\ \text{For slopes/regression analysis only}$$

$v_k(i)$ = value of variable i in $k^{\text{th}}$ record, i=1, ..., NV

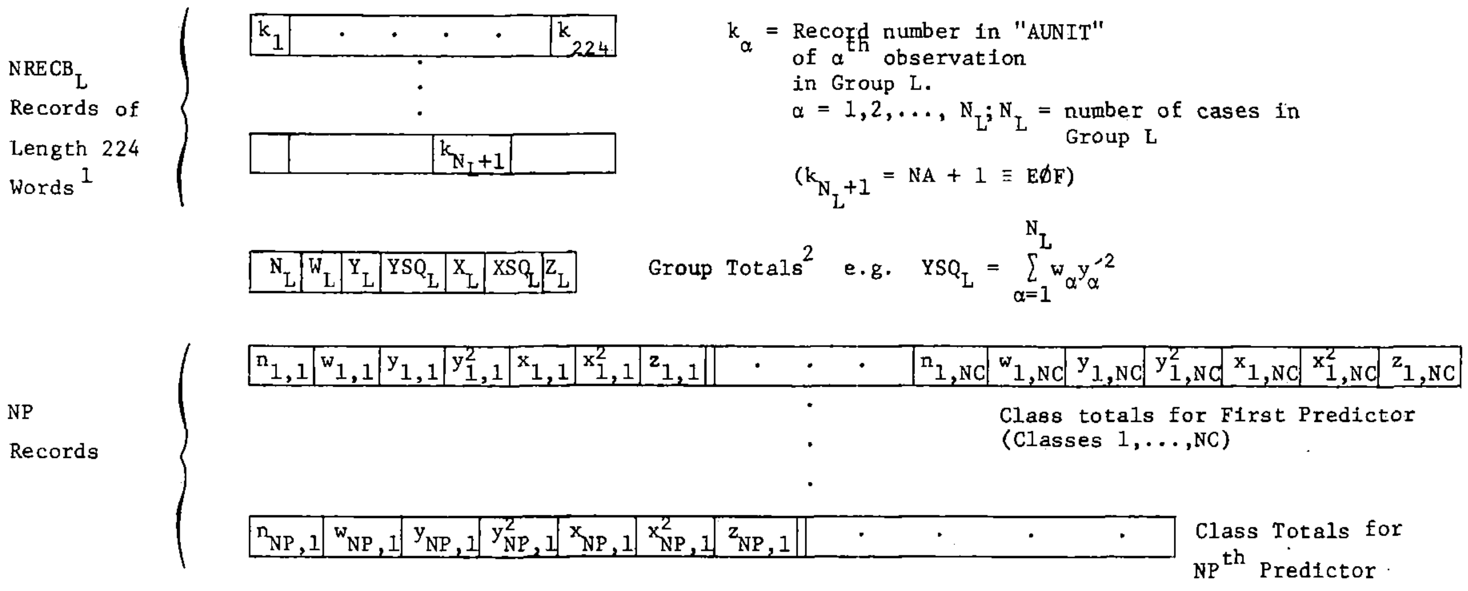
(e.g.) for predictors, class to which that predictor belongs in configuration

[1] The file contains 1 record per observation plus 2 final records containing the missing data 1 and missing data 2 codes respectively (NA + 2 records total).

[2] Space for all 7 elements must be allotted regardless of the analysis type.

# Chart 2 - STORAGE AREA "BUNIT"

For each Group L: $NRECB_L + NP + 1$ RECORDS (NP = number of predictors)



$k_\alpha$ = Record number in "AUNIT" of $\alpha^{th}$ observation in Group L.
$\alpha = 1,2,\ldots, N_L$; $N_L$ = number of cases in Group L

($k_{N_L+1} = NA + 1 \equiv$ EØF)

Group Totals[2] e.g. $YSQ_L = \sum_{\alpha=1}^{N_L} w_\alpha y_\alpha'^2$

Class totals for First Predictor (Classes 1,...,NC)

Class Totals for $NP^{th}$ Predictor

[1] $NRECB_L = (N_L + 1)/224$

[2] x, $x^2$ and z elements are generated for slopes or regression analyses only; however, the space must be allocated regardless.

# Appendix V
# Examples of Output

It is difficult to demonstrate all the options available with this program. An example of a pre-set tree was given earlier in chapter II. We present here three runs, the first two done as a pair.

The first run is an analysis of house values for a national sample of homeowners, with extensive recoding, a one-step lookahead, a premium for symmetry, and one variable ranked 0 so that its splits are suppressed.

The second run is an analysis of the residuals from the first run, without lookahead or symmetry premiums, but with three ranks of predictors.

The third run deals with the dominance of income in explaining house value by searching for different regressions (of house value on income), rather than merely different mean house values. It is, of course, still dominated by differences in level, not slope (income elasticity), and the overall regression (income effect) accounts for more of the variance than the subgroup differences in regression.

The three figures which follow summarize the main results, derivable from the three Group Summary Tables in the output. The data are weighted to offset oversampling among low income families and minority group members. Comments have been added on the computer print-out, but a few overall notes may be appropriate here. Neither the lookahead nor the symmetry premium made any substantial difference. Splits on groups 2, 4 and 7 of the first housevalue analysis were altered, but subsequent splits were made in each case on the predictor that would have been used earlier without the lookahead. Since the symmetry premium only operates when it is the second of a pair that is being split and looks only at the identical split (predictor and subclass division), it is understandable that it would require substantial losses in explanatory power to achieve symmetry.

The ranking in the second run also made little difference. Group 5 was split on race, reducing the error variance by less than commuting time would have, but a later split was made of the white group using commuting time. Since commuting time is highly correlated with more basic things like age (retired) and city size and distance from center, it was purposely kept out of the analysis until the very end by the ranking.

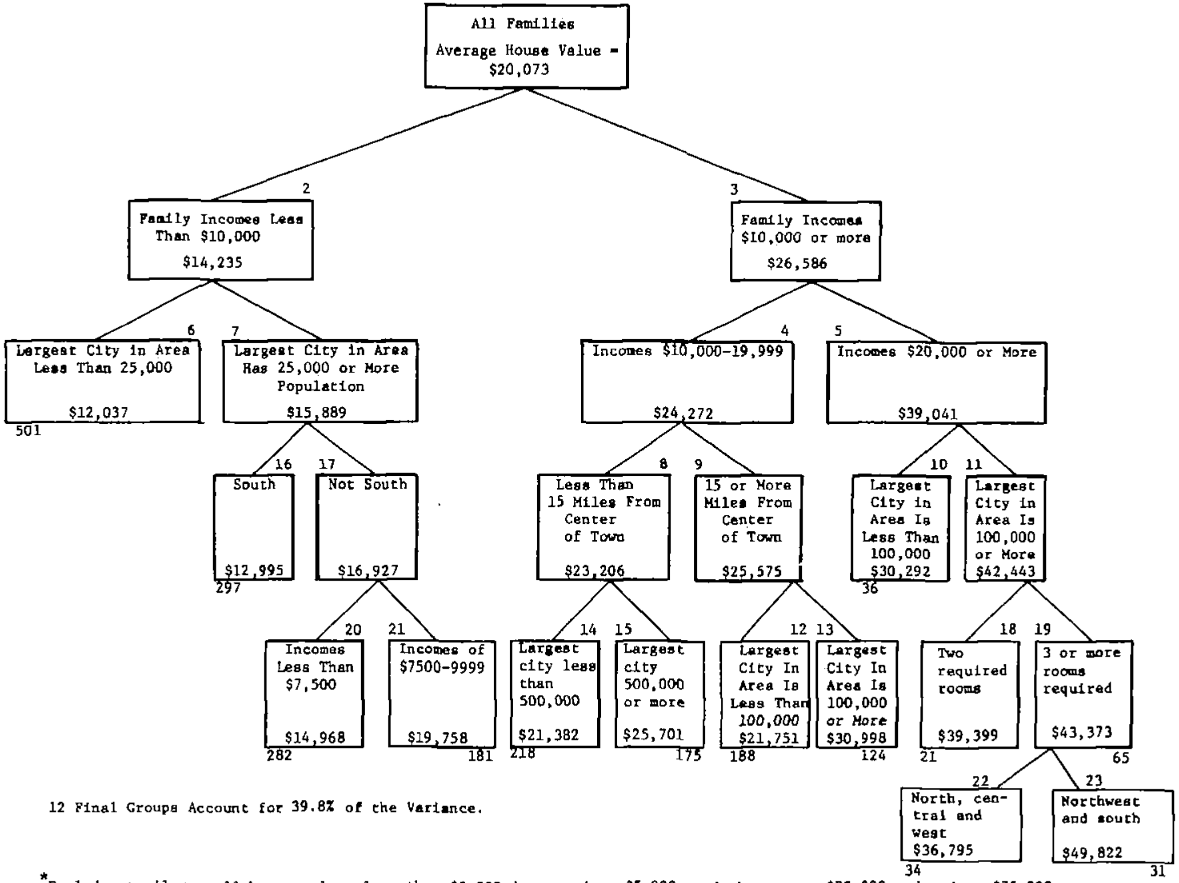
The third--regression--run selects groups with different regression (levels or slopes), but the print-out also gives the subgroup regressions for each subclass of each predictor for each subgroup developed. Table A5-1 gives examples for three predictors.

These runs took extensive computer time both because of the elaborate recode even of data filtered out and not used, and because of the lookahead and the printing of a lot of detailed output, and because of the large sample.

For further details, see the comments added to the computer print-out which follows.

Figure A5-1

House Value (1970) by Family and Location Factors*

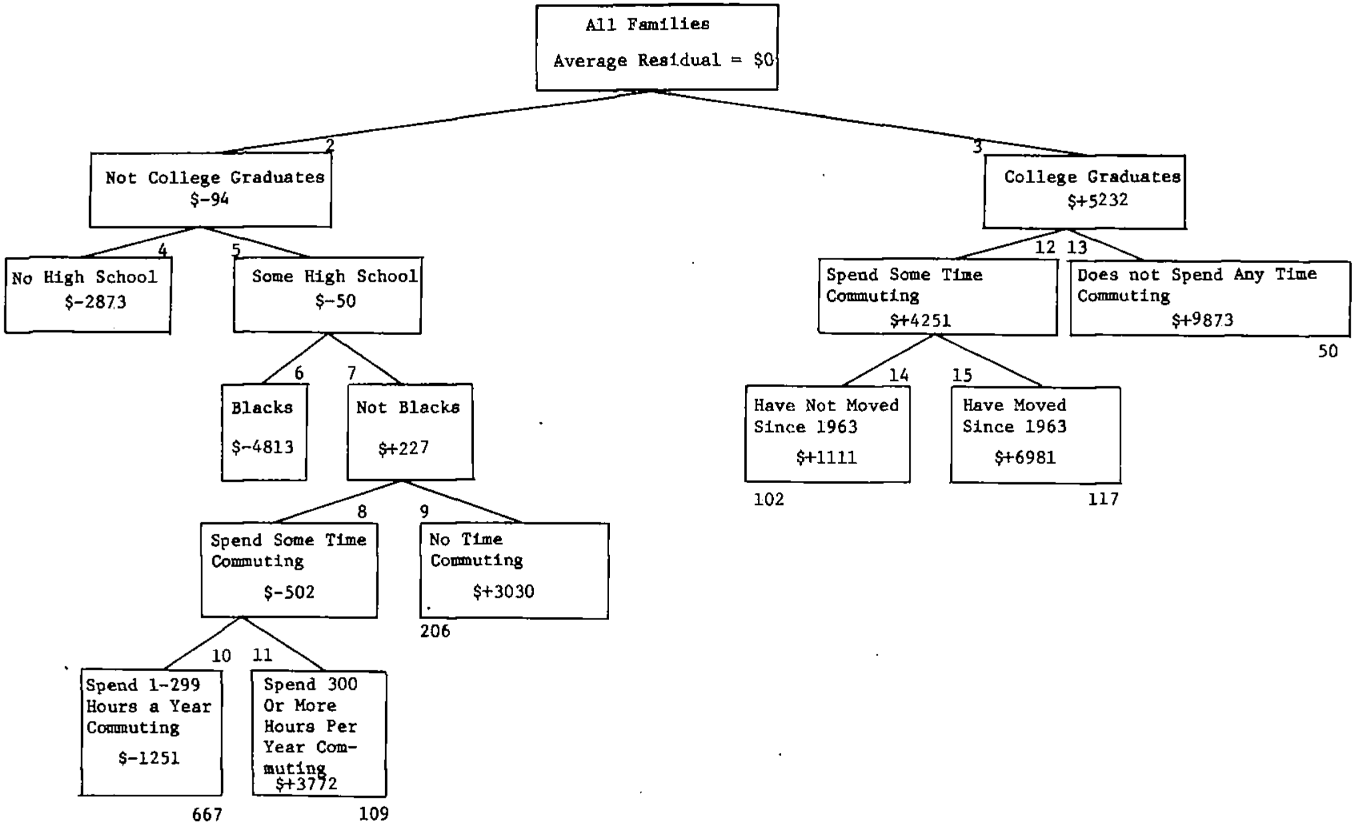

All Families
Average House Value = $20,073
2
Family Incomes Less Than $10,000
$14,235
3
Family Incomes $10,000 or more
$26,586
6
Largest City in Area Less Than 25,000
$12,037
501
7
Largest City in Area Has 25,000 or More Population
$15,889
4
Incomes $10,000-19,999
$24,272
5
Incomes $20,000 or More
$39,041
16
South
$12,995
297
17
Not South
$16,927
8
Less Than 15 Miles From Center of Town
$23,206
9
15 or More Miles From Center of Town
$25,575
10
Largest City in Area Is Less Than 100,000
$30,292
36
11
Largest City in Area Is 100,000 or More
$42,443
20
Incomes Less Than $7,500
$14,968
282
21
Incomes of $7500-9999
$19,758
181
14
Largest city less than 500,000
$21,382
218
15
Largest city 500,000 or more
$25,701
175
12
Largest City In Area Is Less Than 100,000
$21,751
188
13
Largest City In Area Is 100,000 or More
$30,998
124
18
Two required rooms
$39,399
21
19
3 or more rooms required
$43,373
65
22
North, central and west
$36,795
34
23
Northwest and south
$49,822
31

12 Final Groups Account for 39.8% of the Variance.

*Excludes trailers; all house values less than $5,000 increased to $5,000; and those over $75,000 reduced to $75,000.

Figure A5-2

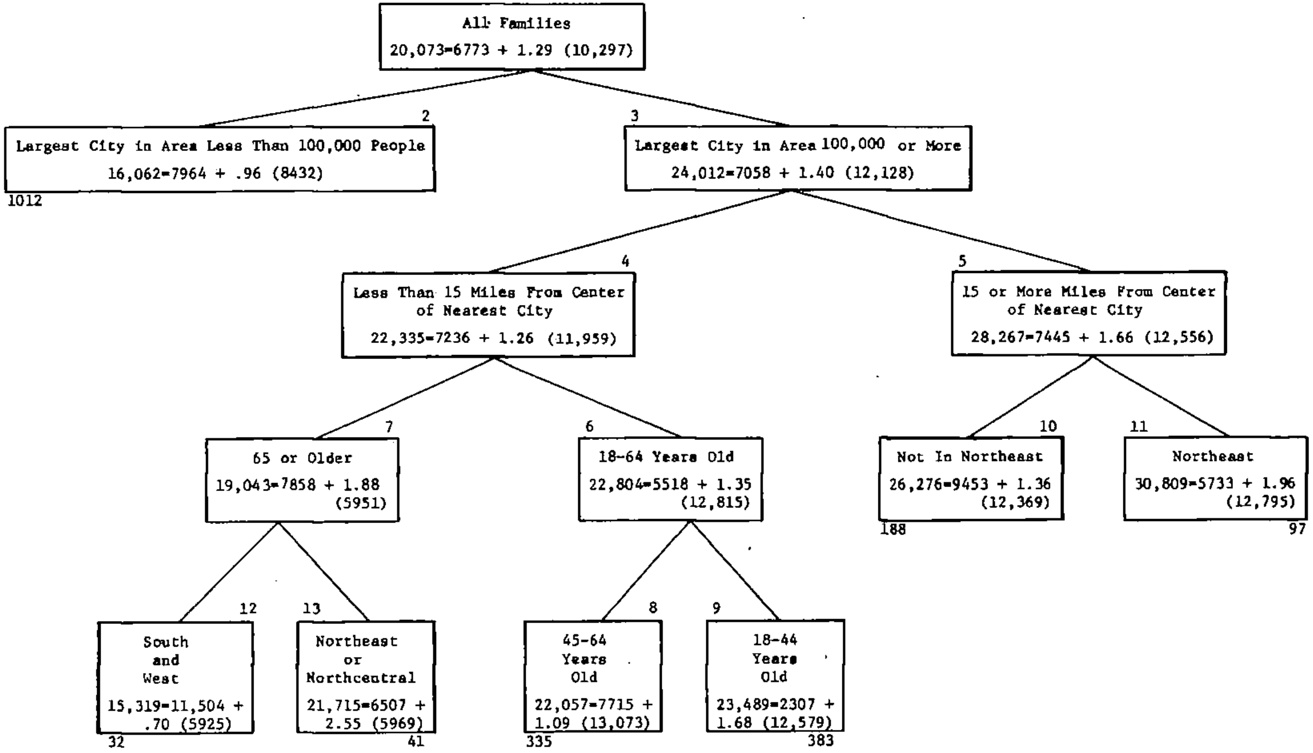
Residuals of House Value Related to Factors Affecting Price or Demand or Inertia



8 Final groups account for 11.6% of the residual variance or an additional 7.0% of the original variance

Figure A5-3

Regressions of House Value on Income* for Home Owners



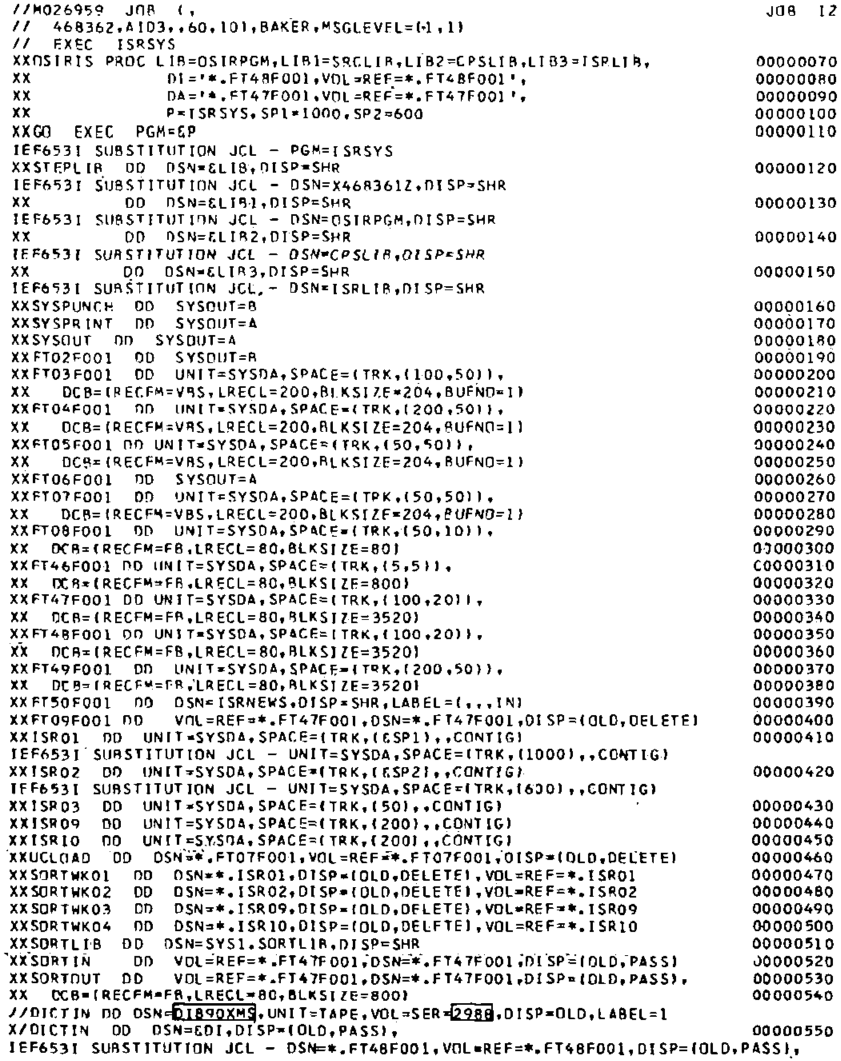
*Data Read:

$$\text{Average House Value} = \text{House Value At Zero Income} + \left(\text{Increase in Value Per Dollar Increase In Income}\right)\left(\text{Average Income}\right)$$

Table A5-1

REGRESSIONS OF HOUSE VALUE ON INCOME FOR SUBCLASSES OF THREE PREDICTORS
(for all home owners)

| | Average House Value | Marginal Increment (Slope of Regression Line on Income) | Average Income | Number of Cases |
|---|---|---|---|---|
| "Required" Number of Rooms (family/size and structure) | | | | |
| 2 | $17,620 | 1.07 | $ 7,956 | 661 |
| 3 | 21,220 | 1.46 | 10,900 | 557 |
| 4 | 22,560 | 1.50 | 12,390 | 486 |
| 5 | 19,990 | 1.59 | 11,560 | 225 |
| 6 or more | 19,980 | 1.38 | 12,280 | 159 |
| Race | | | | |
| White | 20,690 | 1.24 | 10,560 | 1626 |
| Black | 11,780 | 0.89 | 8,872 | 406 |
| Other | 19,870 | 1.25 | 9,594 | 56 |
| Size of Largest City in Area | | | | |
| SMSA's | | | | |
| 500,000 or more | 25,830 | 1.46 | 12,630 | 633 |
| 100,000-499,999 | 21,540 | 1.24 | 11,450 | 443 |
| 50,000-99,999 | 18,550 | .93 | 10,280 | 222 |
| Not SMSA's | | | | |
| 25,000-49,999 | 18,460 | .87 | 9,486 | 144 |
| 10,000-24,999 | 15,780 | 1.08 | 8,460 | 200 |
| Less than 10,000 | 13,940 | .86 | 7,004 | 446 |



```
//M026959  JOB  (,                                                             JOB  12
//  468362,AID3,,60,101,BAKER,MSGLEVEL=(1,1)
//  EXEC  ISRSYS
XXOSIRIS PROC LIB=OSIRPGM,LIB1=SRCLIB,LIB2=CPSLIB,LIB3=ISPLIB,                00000070
XX              DI='*.FT48F001,VOL=REF=*.FT48F001',                            00000080
XX              DA='*.FT47F001,VOL=REF=*.FT47F001',                            00000090
XX              P=ISRSYS,SP1=1000,SP2=600                                      00000100
XXGO  EXEC  PGM=&P                                                             00000110
IEF653I SUBSTITUTION JCL - PGM=ISRSYS
XXSTEPLIB  DD  DSN=&LIB,DISP=SHR                                               00000120
IEF653I SUBSTITUTION JCL - DSN=X468361Z,DISP=SHR
XX          DD  DSN=&LIB1,DISP=SHR                                             00000130
IEF653I SUBSTITUTION JCL - DSN=OSIRPGM,DISP=SHR
XX          DD  DSN=&LIB2,DISP=SHR                                             00000140
IEF653I SUBSTITUTION JCL - DSN=CPSLIB,DISP=SHR
XX          DD  DSN=&LIB3,DISP=SHR                                             00000150
IEF653I SUBSTITUTION JCL,- DSN=ISRLIB,DISP=SHR
XXSYSPUNCH  DD  SYSOUT=B                                                       00000160
XXSYSPRINT  DD  SYSOUT=A                                                       00000170
XXSYSOUT  DD  SYSOUT=A                                                         00000180
XXFT02F001  DD  SYSOUT=B                                                       00000190
XXFT03F001  DD  UNIT=SYSDA,SPACE=(TRK,(100,50)),                               00000200
XX   DCB=(RECFM=VBS,LRECL=200,BLKSIZE=204,BUFNO=1)                             00000210
XXFT04F001  DD  UNIT=SYSDA,SPACE=(TRK,(200,50)),                               00000220
XX   DCB=(RECFM=VBS,LRECL=200,BLKSIZE=204,BUFNO=1)                             00000230
XXFT05F001 DD UNIT=SYSDA,SPACE=(TRK,(50,50)),                                  00000240
XX   DCB=(RECFM=VBS,LRECL=200,BLKSIZE=204,BUFNO=1)                             00000250
XXFT06F001  DD  SYSOUT=A                                                       00000260
XXFT07F001  DD  UNIT=SYSDA,SPACE=(TRK,(50,50)),                                00000270
XX   DCB=(RECFM=VBS,LRECL=200,BLKSIZE=204,BUFNO=1)                             00000280
XXFT08F001  DD  UNIT=SYSDA,SPACE=(TRK,(50,10)),                                00000290
XX  DCB=(RECFM=FB,LRECL=80,BLKSIZE=80)                                         00000300
XXFT46F001 DD UNIT=SYSDA,SPACE=(TRK,(5,5)),                                    C0000310
XX  DCB=(RECFM=FB,LRECL=80,BLKSIZE=800)                                        00000320
XXFT47F001 DD UNIT=SYSDA,SPACE=(TRK,(100,20)),                                 00000330
XX  DCB=(RECFM=FB,LRECL=80,BLKSIZE=3520)                                       00000340
XXFT48F001 DD UNIT=SYSDA,SPACE=(TRK,(100,20)),                                 00000350
XX  DCB=(RECFM=FB,LRECL=80,BLKSIZE=3520)                                       00000360
XXFT49F001  DD  UNIT=SYSDA,SPACE=(TRK,(200,50)),                               00000370
XX  DCB=(RECFM=FB,LRECL=80,BLKSIZE=3520)                                       00000380
XXFT50F001  DD  DSN=ISRNEWS,DISP=SHR,LABEL=(,,,IN)                             00000390
XXFT09F001 DD   VOL=REF=*.FT47F001,DSN=*.FT47F001,DISP=(OLD,DELETE)            00000400
XXISR01  DD  UNIT=SYSDA,SPACE=(TRK,(&SP1),,CONTIG)                             00000410
IEF653I SUBSTITUTION JCL - UNIT=SYSDA,SPACE=(TRK,(1000),,CONTIG)
XXISR02  DD  UNIT=SYSDA,SPACE=(TRK,(&SP2),,CONTIG)                             00000420
IEF653I SUBSTITUTION JCL - UNIT=SYSDA,SPACE=(TRK,(600),,CONTIG)
XXISR03  DD  UNIT=SYSDA,SPACE=(TRK,(50),,CONTIG)                               00000430
XXISR09  DD  UNIT=SYSDA,SPACE=(TRK,(200),,CONTIG)                              00000440
XXISR10  DD  UNIT=SYSDA,SPACE=(TRK,(200),,CONTIG)                              00000450
XXUCLOAD  DD  DSN=*.FT07F001,VOL=REF=*.FT07F001,DISP=(OLD,DELETE)              00000460
XXSORTWK01  DD  DSN=*.ISR01,DISP=(OLD,DELETE),VOL=REF=*.ISR01                  00000470
XXSORTWK02  DD  DSN=*.ISR02,DISP=(OLD,DELETE),VOL=REF=*.ISR02                  00000480
XXSORTWK03  DD  DSN=*.ISR09,DISP=(OLD,DELETE),VOL=REF=*.ISR09                  00000490
XXSORTWK04  DD  DSN=*.ISR10,DISP=(OLD,DELFTE),VOL=REF=*.ISR10                  00000500
XXSORTLIB  DD  DSN=SYS1.SORTLIB,DISP=SHR                                       00000510
XXSORTIN    DD  VOL=REF=*.FT47F001,DSN=*.FT47F001,DISP=(OLD,PASS)              00000520
XXSORTOUT  DD   VOL=REF=*.FT47F001,DSN=*.FT47F001,DISP=(OLD,PASS),             00000530
XX  DCB=(RECFM=FB,LRECL=80,BLKSIZE=800)                                        00000540
//DICTIN DD DSN=DI890XMS,UNIT=TAPE,VOL=SER=2988,DISP=OLD,LABEL=1
X/DICTIN  DD  DSN=&DI,DISP=(OLD,PASS),                                         00000550
IEF653I SUBSTITUTION JCL - DSN=*.FT48F001,VOL=REF=*.FT48F001,DISP=(OLD,PASS),
```

*Only the cards (lines) beginning with a slash (/) are actually prepared and constitute the job control language (JCL).*

*Name and number of the dictionary file*

```
XX   DCB=BUFNO=1                                                          00000560
//DATAIN DD DSN=DA890XHS,UNIT=TAPE,VOL=SER=2988,DISP=OLD,LABEL=2
X/DATAIN   DD  DSN=&DA,DISP=(OLD,PASS),                                   00000570
IEF653I SUBSTITUTION JCL - DSN=*.FT47F001,VOL=REF=*.FT47F001,DISP=(OLD,PASS),
XX   DCB=BUFNO=1                                                          00000580
XXDICTOUT  DD  VOL=REF=*.FT48F001.DSN=*.FT48F001,DISP=(OLD,PASS),         00000590
XX   DCB=BUFNO=1                                                          00000600
XXDATAOUT    DD  VOL=REF=*.FT47F001,DSN=*.FT47F001,DISP=(OLD,PASS),       00000610
XX   DCB=BUFNO=1                                                          00000620
XXFT01F001 DD UNIT=SYSDA,SPACE=(TRK,(50,10)),DCB=(RECFM=F,BLKSIZE=80)     00000630
XXSYSIN DD DSN=*.FT01F001,DISP=(OLD,DELETE),VOL=REF=*.FT01F001            00000640
//SYSUDUMP DD SYSOUT=A
//SETUP DD *
//
IEF236I ALLOC. FOR M026959  GO
IEF237I 132   ALLOCATED TO STEPLIB
IEF237I 133   ALLOCATED TO
IEF237I 132   ALLOCATED TO
IEF237I 132   ALLOCATED TO
IEF237I 080   ALLOCATED TO SYSPUNCH
IEF237I 0E0   ALLOCATED TO SYSPRINT
IEF237I 0E1   ALLOCATED TO SYSOUT
IEF237I 081   ALLOCATED TO FT02F001
IEF237I 130   ALLOCATED TO FT03F001
IEF237I 131   ALLOCATED TO FT04F001
IEF237I 132   ALLOCATED TO FT05F001
IEF237I 0E2   ALLOCATED TO FT06F001
IEF237I 130   ALLOCATED TO FT07F001
IEF237I 131   ALLOCATED TO FT08F001
IEF237I 132   ALLOCATED TO FT46F001
IEF237I 130   ALLOCATED TO FT47F001
IEF237I 131   ALLOCATED TO FT48F001
IEF237I 132   ALLOCATED TO FT49F001
IEF237I 132   ALLOCATED TO FT50F001
IEF237I 130   ALLOCATED TO FT09F001
IEF237I 130   ALLOCATED TO ISR01
IEF237I 131   ALLOCATED TO ISR02
IEF237I 132   ALLOCATED TO ISR03
IEF237I 130   ALLOCATED TO ISR09
IEF237I 130   ALLOCATED TO ISR10
IEF237I 130   ALLOCATED TO UCLOAD
IEF237I 130   ALLOCATED TO SORTWK01
IEF237I 131   ALLOCATED TO SORTWK02
IEF237I 130   ALLOCATED TO SORTWK03
IEF237I 130   ALLOCATED TO SORTWK04
IEF237I 133   ALLOCATED TO SORTLIB
IEF237I 130   ALLOCATED TO SORTIN
IEF237I 130   ALLOCATED TO SORTOUT
IEF237I 281   ALLOCATED TO DICTIN
IEF237I 281   ALLOCATED TO DATAIN
IEF237I 131   ALLOCATED TO DICTOUT
IEF237I 130   ALLOCATED TO DATAOUT
IEF237I 132   ALLOCATED TO FT01F001
IEF237I 132   ALLOCATED TO SYSIN
IEF237I 0E3   ALLOCATED TO SYSUDUMP
IEF237I 040   ALLOCATED TO SETUP
IEC209I M026959  2988   281 TR=000,TW=000,FG=000,CL=000,N=000,SIO=00046
IEC209I M026959  2988   281 TP=000,TW=000,FG=000,CL=000,N=000,SIO=04660
IEF285I   X468361Z                                    KEPT
IEF285I   VOL SER NOS= ISRA  .
IEF285I   OSIRPGM                                     KEPT
```

*Name and number of the data file*

```
IEF285I   VOL SER NOS= MFT1  .
IEF285I   CPSLIB                                        KEPT
IEF285I   VOL SER NOS= ISRA  .
IEF285I   ISRLIB                                        KEPT
IEF285I   VOL SER NOS= ISRA  .
IEF285I   SYS73160.T091518.RF000.M026959.R0000253       DELETED
IEF285I   VOL SER NOS= PJK001.
IEF285I   SYS73160.T091518.RF000.M026959.R0000254       DELETED
IEF285I   VOL SER NOS= ISRB  .
IEF285I   SYS73160.T091518.RF000.M026959.R0000255       DELETED
IEF285I   VOL SER NOS= ISRA  .
IEF285I   SYS73160.T091518.RF000.M026959.R0000256       DELETED
IEF285I   VOL SER NOS=       .
IEF285I   SYS73160.T091518.RF000.M026959.R0000257       DELETED
IEF285I   VOL SER NOS= PJK001.
IEF285I   SYS73160.T091518.RF000.M026959.R0000258       DELETED
IEF285I   VOL SER NOS= ISRB  .
IEF285I   SYS73160.T091518.RF000.M026959.R0000259       DELETED
IEF285I   VOL SER NOS= ISRA  .
IEF285I   SYS73160.T091518.RF000.M026959.R0000260       DELETED
IEF285I   VOL SER NOS= PJK001.
IEF285I   SYS73160.T091518.RF000.M026959.R0000261       DELETED
IEF285I   VOL SER NOS= ISRB  .
IEF285I   SYS73160.T091518.RF000.M026959.R0000262       DELETED
IEF285I   VOL SER NOS= ISRA  .
IEF285I   ISRNEWS                                       KEPT
IEF285I   VOL SER NOS= ISRA  .
IEF283I   SYS73160.T091518.RF000.M026959.R0000260       NOT DELETED 8
IEF283I   VOL SER NOS= PJK001 1.
IEF285I   SYS73160.T091518.RF000.M026959.R0000263       DELETED
IEF285I   VOL SER NOS= PJK001.
IEF285I   SYS73160.T091518.RF000.M026959.R0000264       DELETED
IEF285I   VOL SER NOS= ISRB  .
IEF285I   SYS73160.T091518.RF000.M026959.R0000265       DELETED
IEF285I   VOL SER NOS= ISRA  .
IEF285I   SYS73160.T091518.RF000.M026959.R0000266       DELETED
IEF285I   VOL SER NOS= PJK001.
IEF285I   SYS73160.T091518.RF000.M026959.R0000267       DELETED
IEF285I   VOL SER NOS= PJK001.
IEF283I   SYS73160.T091518.RF000.M026959.R0000257       NOT DELETED 8
IEF283I   VOL SER NOS= PJK001 1.
IEF283I   SYS73160.T091518.RF000.M026959.R0000263       NOT DELETED 8
IEF283I   VOL SER NOS= PJK001 1.
IEF283I   SYS73160.T091518.RF000.M026959.R0000264       NOT DELETED 8
IEF283I   VOL SER NOS= ISRB   1.
IEF283I   SYS73160.T091518.RF000.M026959.R0000266       NOT DELETED 8
IEF283I   VOL SER NOS= PJK001 1.
IEF283I   SYS73160.T091518.RF000.M026959.R0000267       NOT DELETED 8
IEF283I   VOL SER NOS= PJK001 1.
IEF285I   SYS1.SORTLIB                                  KEPT
IEF285I   VOL SER NOS= MFT1  .
IEF285I   SYS73160.T091518.RF000.M026959.R0000260       PASSED
IEF285I   VOL SER NOS= PJK001.
IEF285I   SYS73160.T091518.RF000.M026959.R0000260       PASSED
IEF285I   VOL SER NOS= PJK001.
IEF285I   DI890XMS                                      KEPT
IEF285I   VOL SER NOS= 2988  .
IEF285I   DA890XMS                                      KEPT
IEF285I   VOL SER NOS= 2988  .
IEF285I   SYS73160.T091518.RF000.M026959.R0000261       PASSED
IEF285I   VOL SER NOS= ISRB  .
```

```
IEF285I   SYS73160.T091518.RF000.M026959.R0000260       PASSED
IEF285I   VOL SER NOS= PJK001.
IEF285I   SYS73160.T091518.RF000.M026959.R0000268       DELETED
IEF285I   VOL SER NOS= ISRA  .
IEF283I   SYS73160.T091518.RF000.M026959.R0000268       NOT DELETED 8
IEF283I   VOL SER NOS= ISRA   1.
IEF285I   SYS73160.T091518.RF000.M026959.R0000270       DELETED
IEF285I   VOL SER NOS=        .
ISP011I STEP GO        EXECUTION TIME =  2049.72 SEC.
ISP013I PARTITION   2: SIZE= 104, LWM=F0F000,  HWM=F0F000  , CORE ALLOCATED= 104, CORE USED=  96
IEF285I   SYS73160.T091518.RF000.M026959.R0000260       KEPT
IEF285I   VOL SER NOS= PJK001.
IEF285I   SYS73160.T091518.RF000.M026959.R0000260       KEPT
IEF285I   VOL SER NOS= PJK001.
IEF285I   SYS73160.T091518.RF000.M026959.R0000261       KEPT
IEF285I   VOL SER NOS= ISRB  .
IEF285I   SYS73160.T091518.RF000.M026959.R0000260       KEPT
IEF285I   VOL SER NOS= PJK001.
ISP012I TOT. M026959  EXECUTION TIME =  2049.72 SEC.
ISP016I TIME OF DAY = 13.59.00, DATE = 73.160
```

```
                      INSTITUTE FOR SOCIAL RESEARCH MONITOR SYSTEM      02/05/73
                 *****     FASTER VERSIONS OF TABLES, MOC, AND REGRESSN NOW AVAILABLE BY SPECIFYING              *****
                 *****                                                                                           *****
                 *****                 //   EXEC OSIRIS,LIB=X485321X,LIB1=OSIRPGM                                *****
                 *****                                                                                           *****
                 *****     SAVINGS ARE 10-45% FOR TABLES, 45% FOR MOC AND 20% FOR REGRESSN                       *****
                                                                                                                   This listing of the
*****TIME IS  12:37: 0                                                                                             70 cards is produced
                                                                                                                   by the OSIRIS monitor
*****LISTING OF SET-UP FOLLOWS:

           CARD                1         2         3         4         5         6         7         8
            NO.       12345678901234567890123456789012345678901234567890123456789012345678901234567890
              1       $RUN AID3
              2         INCLUDE V1264=1 AND V1109=0-1 AND V542=0-1 AND V1499=1-9*
              3       MTR 45, PROJECT 468070,AID3
              4       *                                                                                This variable list gives
              5       V101,V542,V603,V1009,V1109,V1122,V1146,V1168,V1240,V1250,V1274,V1276,V1365,      all the variables to be
              6       V1370,V1490,V1498,V1499,V1506,V1572,V1609,V1719,V1720,V1264,V1485*               read off the input data
              7       HOUSE VALUE TRUNCATED TO 5000-75000 BY SQUEEZING EXTREME CASES  807MTR45 MORGAN file .
              8       YVAR=1122 WEIG=1609 RECODE MOOP=NONE TABL=(BSS,ELIG,PRED,MEAN)  Means analysis, weighted dependent variable,
              9       RESI=(2005,0,8) RESN='HOUSE VALUE RESIDUALS'*                   residuals but no output file, print all
             10           2   IFV1719   LT      3000          V2001    =         1     tables.
             11               OR        IN      3000      4999                     2
             12                                 5000      7499                     3                      Create a 7-category
             13                                 7500      9999                     4                      code from a 5-digit
             14                                10000     14999                     5                      income field.
             15                                15000     19999                     6
             16                         GE     20000                               7
             17           3   IFV1365   EQ         1          V2002    =         1            MARRIED   Creates one classifi-
             18                                                     GOTO         4                      cation from two
             19               IFV1240   EQ         1          V2002    =         2            SNGL MAN  others.
             20              ALT                              V2002    =         3            SNGL SMN
             21           4    T                              V1720  MPY       100V1720
             22                                               V2003  DIVV1720     V1719
             23               IFV2003   LT        -5          V2003    =         1            BRACKET   Integer arithmetic
             24               OR        IN        -4         4                   2             PERCENT  requires multiplying
             25                                    5         9                   3              CHANGE  by 100 before divid-
             26                                   10        19                   4               IN     ing and bracketing.
             27                         GE        20                             5             INCOME
             28           5   IFV1274   EQ         1          V2004    =         1
             29                                                     GOTO         6            HOW
             30               IFV 603   EQ         1          V2004    =         2              LONG    Creates one classi-
             31                                                     GOTO         6             LIVED    fication from three.
             32               IFV 101   IN         7         8V2004    =         3             HERE     using "GOTO" to avoid
             33                                                     GOTO         6                      recoding a case twice.
             34               IFV 101   IN         4         6V2004    =         4
             35              ALT                              V2004    =         5
             36           6   IFV1168   IN         7         9V1168    =         6            TRUNCATE
             37           7   IFV1009   EQ         1          V1009    =         2              NINE    Truncating
             38           8   IFV1009   GT         6          V1009    =         6               CODES  predictor
             39           9   IFV1370   NE         1          V1370    =         0                      classifications.
             40          10   IFV1490  OUT         1         2V1490    =         3
             41          11   IFV1250  OUT         1         3V1250    =         5

           CARD                1         2         3         4         5         6         7         8
            NO.       12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

```
CARD             1         2         3         4         5         6         7         8
 NO.    12345678901234567890123456789012345678901234567890123456789012345678901234567890
 42        12  IFV1122   GT      75000          V1122    =     75000
 43            ORV1122   LT       5000          V1122    =      5000
 44        13  IFV1719   GT      25000          V1719    =     25000
 45        14  IFV1276   GT          5          V1276    =         5
 46        15  IFV1146   IN          1        99V1146    =         1            BRACKET
 47            OR                  100       149                   2             ANNUAL
 48                                150       199                   3              HOURS
 49                                200       299                   4            COMMUTNG
 50                      GE        300                             5
 51           END
 52     PRED=2001 MAXC=7        PREN='3-YR AVE $ INC'*
 53     PRED=(1168,1009) MAXC=6*
 54     PRED=2002 MAXC=3 PREN='SEX & MAR STATUS'*
 55     PRED=1506*
 56     PRED=1498 MAXC=5*
 57     PRED=1572 MAXC=4 F*
 58     PRED=1490 MAXC=3 F RANK=0 END*
 59     LOOK=(1,1) REDU=(.6,.12) MIN=3 RANK=ALL TRACE=BEST SYMM=30.
 60     SECOND STAGE OF 807NTR45 MORGAN
 61     YVAR=2005 WEIG=1609 TABL=(ELIG,MEAN)*
 62     PRED=1276 MAXC=5 RANK=2*
 63     PRED=2004 PREN='HOW LONG LIVD HERE' MAXC=5 RANK=2*
 64     PRED=1485*
 65     PRED=1490 F MAXC=3*
 66     PRED=1250 MAXC=5*
 67     PRED=2003 PREN='% CHANGE IN INCOME' MAXC=5*
 68     PRED=1370 MAXC=1*
 69     PRED=1146 MAXC=6 RANK=3 END*
 70     REDU=.6 MIN=3 RANK=UP RANGE=(2,2,2,2)*
```

*First analysis strategy specified predictors: whether maintained subclass order and rank.*

* *1-step lookahead, simple ranking, 30% premium for symmetry, print best split in trace, minimum group size=3.*

*Range ranking with preference UP, no lookahead*

*A card with /* after this indicates the end of the setup.*

```
          AID3: OSIRIS SEARCHING FOR STRUCTURE - JULY 1973

THE FILTER IS:

  INCLUDE V1264=1 AND V1109=0-1 AND V542=0-1 AND V1499=1-9*

          MTR 45, PROJECT 468070,AID3

*

THE VARIABLE LIST IS:

V101,V542,V603,V1009,V1109,V1122,V1146,V1168,V1240,V1250,V1274,V1276,V1365,

V1370,V1490,V1498,V1499,V1506,V1572,V1609,V1719,V1720,V1264,V1485*
```

*Execution of the program begins with a listing and interpretation of the setup.*

```
      VAR.    TYPE    VARIABLE NAME              TLOC   WIDTH  NODEC    RESP.   MDCODE1  MDCODE2   REFNO   ID   TSEQNO

T      101      0    WHEN MOVED IN 5:36           270     1      0        1              0000009                00000
T      542      0    CHANGE IN FU COMP  15:25    1070     1      0        1                                     00000
T      603      0    MOVED SINCE SPRNG6816-38    1149     1      0        1              0000009                00000
T     1009      0    BKT AGE HEAD 9V1009         1831     1      0        1                                     00000
T     1109      0    CHANGE IN FU COMP 21:18     2021     1      0        1                                     00000
T     1122      0    HOUSE VALUE     21:38-42    2041     5      0        1                                     00000
T     1146      0    HRS HEAD TRVL WK22:30-32    2103     3      0        1                                     00000
T     1168      0    # REQUIRED ROOMS 23:12      2154     1      0        1                                     00000
T     1240      0    SEX OF HEAD     26:40       2351     1      0        1                                     00000
T     1250      0    PUB TRANSP GOOD    26:52    2363     1      0        1              0000009                00000
T     1264      0    OWN OR RENT?       26:67    2378     1      0        1                                     00000
T     1274      0    MOVED SINCE SPRING 26:77    2388     1      0        1              0000009                00000
T     1276      0    MIGHT MOVE         26:79    2390     1      0        1              0000009                00000
T     1365      0    MARITAL STATUS     29:10    2514     1      0        1                                     00000
T     1370      0    EXPECT CHILDREN ?  29:18    2521     1      0        1              0000009                00000
T     1485      0    EDUCATION OF HEAD  31:43    2657     1      0        1              0000009                00000
T     1490      0    RACE               31:48    2662     1      0        1              0000009                00000
T     1498      0    DIST TO CNTR SMSA  31:58    2672     1      0        1              0000009                00000
T     1499      0    TYPE OF STRUCTURE  31:59    2673     1      0        1              0000009                00000
T     1506      0    LRGST PLAC/SMSA PSU31:66    2680     1      0        1                                     00000
T     1572      0    CURRENT REGION 0V472        2838     1      0        1     0000009                         00000
T     1609      0    WEIGHT             0V509    2892     2      0        1                              COR    00000
T     1719      0    MEAN MONEY INCOME           3179     5      0        1                        .     COR    00000
T     1720      0    SLOPE MONEY INCOME          3184     6      0        1                              COR    00000


         HOUSE VALUE TRUNCATED TO 5000-75000 BY SQUEEZING EXTREME CASES  807MTR45 MORGAN

YVAR=1122 WEIG=1609 RECODE MDOP=NONE TABL=(BSS,ELIG,PRED,MEAN)

RESI=(2305,0,8) RESN='HOUSE VALUE RESIDUALS'*

  NO  LOG TEST  REL A OPERAND B OPERAND RES    OP C OPERAND D OPERAND  TEXT
   7   IFV1719   LT       3000          0V2001    =         1         0
```

*Using the input variable list the dictionary file is read for a description of each variable.*

```
   0  OR      0  IN       3000      4999    0              2         0
   0          0           5000      7499    0              3         0
   0          0           7500      9999    0              4         0
   0          0          10000     14999    0              5         0
   0          0          15000     19999    0              6         0
   0          0  GE      20000         0    0              7         0
   3    IFV1365  EQ          1         0V2002    =         1         0  MARRIED
   0          0                0         0    0 GOTO         4         0
   0    IFV1240  EQ          1         0V2002    =         2         0  SNGL MAN
   0 ALT      0                0         0V2002    =         3         0  SNGL SMN
   4    T     0                0         0V1720 MPY       100V      1720
   0          0                0         0V2003 DIVV     1720V      1719
   0    IFV2003  LT         -5         0V2003    =         1         0  BRACKET
   0  OR      0  IN         -4         4    0              2         0   PERCENT
   0          0                5         9    0              3         0    CHANGE
   0          0               10        19    0              4         0      IN
   0          0  GE          20         0    0              5         0   INCOME
   5    IFV1274  EQ          1         0V2004    =         1         0
   0          0                0         0    0 GOTO         6         0  HOW
   0    IFV 603  EQ          1         0V2004    =         2         0    LONG
   0          0               ,0         0    0 GOTO         6         0   LIVED
   0    IFV 101  IN          7         8V2004    =         3         0   HERE
   0          0                0         0    0 GOTO         6         0
   0    IFV 101  IN          4         6V2004    =         4         0
   0 ALT      0                0         0V2004    =         5         0
   6    IFV1168  IN          7         9V1168    =         6         U  TRUNCATE
   7    IFV1009  EQ          1         0V1009    =         2         0   NINE
   8    IFV1009  GT          6         0V1009    =         6         0    CODES
   9    IFV1370  NE          1         0V1370    =         0         0
  10    IFV1490  OUT         1         2V1490    =         3         0
  11    IFV1250  OUT         1         3V1250    =         5         0
  12    IFV1122  GT      75000         0V1122    =     75000         0
   0    ORV1122  LT       5000         0V1122    =      5000         0
  13    IFV1719  GT      25000         0V1719    =     25000         0
  14    IFV1276  GT          5         0V1276    =         5         0
  15    IFV1146  IN          1        99V1146    =         1         0  BRACKET
   0  OR      0              100       149    0              2         0   ANNUAL
   0          0              150       199    0              3         0    HOURS
   0          0              200       299    0              4         0  COMMUTNG
   0          0  GE         300         0    0              5         0
   0 END      0                0         0    0              0         0

PRED=2001 MAXC=7          PREN='3-YR AVE $ INC'*

PRED=(1168,1009) MAXC=6*

PRED=2002 MAXC=3 PREN='SEX & MAR STATUS'*

PRED=1506*

PRED=1498 MAXC=5*

PRED=1572 MAXC=4 F*

PRED=1490 MAXC=3 F RANK=0 END*

LOOK=(1,1) REDU=(.6,.12) MIN=3 RANK=ALL TRACE=BEST SYMM=30.    *

COMPUTE     SPECIFIED

OUTPUT      SPECIFIED
```

```
         SECOND STAGE OF 807NTR45 MORGAN

YVAR=2005 WEIG=1609 TABL=(ELIG,MEAN)*

PRED=1276 MAXC=5 RANK=2*

PRED=2004 PREN='HOW LONG LIVD HERE' MAXC=5 RANK=2*

PRED=1485*

PRED=1490 F MAXC=3*

PRED=1250 MAXC=5*

PRED=2003 PREN='% CHANGE IN INCOME' MAXC=5*

PRED=1370 MAXC=1*

PRED=1146 MAXC=6 RANK=3 END*

REDU=.6 MIN=3 RANK=UP RANGE=(2,2,2,2)*

COMPUTE      SPECIFIED

OUTPUT      SPECIFIED


THE COMPLETE VARIABLE LIST IS:

   101    542    603   1009   1109   1122   1146   1168   1240   1250   1274   1276   1365   1370   1490   1498   1499   1506   1572   1609

  1719   1720   1264   1485   2005   2001   2002   2003   2004
```

*These variables, appended to the input variable list, are variables generated using the residual and recode options.*

```
HOUSE VALUE TRUNCATED TO 5000-75000 BY SQUEEZING EXTREME CASES  807MTR45 MORGAN

      2088 OBSERVATIONS READ AFTER GLOBAL FILTER

Y AVERAGE =              2.007342E 04
STANDARD DEVIATION =  1.278252E 04
BOUNDARIES =  -4.383920E 04   8.398600E 04




     2088 CASES INCLUDED IN THE ANALYSIS
             0 FILTERED (LOCAL/SUBSET SELECTOR)
             0 MISSING DATA CASES
             0 OUTLIERS INCLUDED
             0 INVALID PREDICTOR VALUES

      2088 SAMPLE OBSERVATIONS - WITH TOTALS
           WEIGHTS =                  8.157500E 04
           DEPENDENT VARIABLE (Y) =   1.637489D 09     AVERAGE =   2.007342E 04
           Y-SQUARED =                4.619239D 13    VARIANCE =   1.633931E 08


                         STAGE  1 OF THE ANALYSIS
                        BEST SPLIT BASED ON MEANS
                   1-STEP LOOKAHEAD WITH 1 FORCED SPLITS

SPLITTING CRITERIA -
    MAXIMUM NUMBER OF SPLITS =  25
    MINIMUM # OBSERVATIONS IN A GROUP =  3
    %AGE OF TOTAL SS N SPLITS MUST EXPLAIN =  0.6(N=1),  0.1(N=2),
    PREMIUM FOR SYMMETRY = 30.0
    PRINT   CASES OUTSIDE  5.0 STANDARD DEVIATIONS OF PARENT GROUP MEAN

       8     RANKED PREDICTORS SPECIFIED
PREDICTOR RANK PREFERENCE   AT
   SPLIT ATTEMPT RANGE -  0 RANKS UP,  0 RANKS DOWN
   ELIGIBILITY RANGE -    0 RANKS UP,  0 RANKS DOWN
           PREDICTOR            VARIABLE NUMBER   TYPE   MAX CLASS    RANK
      1    3-YR AVE $ INC                V2001     M        7          1
      2    # REQUIRED ROOMS 23:12        V1168     M        6          1
      3    BKT AGE HEAD SV1009           V1009     M        6          1
      4    SEX & MAR STATUS              V2002     M        3          1
      5    LRGST PLAC/SMSA PSU31:66      V1506     M        9          1
      6    DIST TO CNTR SMSA  31:58      V1498     M        5          1
      7    CURRENT REGION OV472          V1572     F        4          1
      8    RACE                 31:48    V1490     F        3          0

    WEIGHTED Y VARIABLE 1122  HOUSE VALUE     21:38-42 SCALED BY  1.0E 00
    RESIDUAL-HOUSE VALUE RESIDUALS    V2005- SCALE FACTOR  1.0E 00
             GROUP NO V   0, PREDICTED VALUE V   0- SCALE FACTOR  2.5E 08

             1 CANDIDATES -    GROUP       SS
                                 1   1.332240E 13

               ATTEMPT SPLIT ON GROUP   1 WITH SS = 1.332240E 13
```

```
                    LOOKAHEAD TENTATIVE PARTITION

SPLIT ATTEMPT ON GROUP   1 WITH N =       2088, SS =  1.332240E 13
    BEST SPLIT ON PREDICTOR 2001 =  3.102057E 12 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1168 =  2.726633E 11 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1009 =  4.380196E 11 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 2002 =  3.802388E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  1.288775E 12 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1498 =  6.633711E 11 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1572 =  5.695026E 11 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1490 =  4.048007E 11 AFTER CLASS  2
```

*Group 1 (the whole selected data set) is searched for the best split on each predictor in turn.*

```
TENTATIVE SPLIT 1. SPLIT GROUP   1 ON PREDICTOR 2001 WITH BSS = 3.102057E 12
    GROUP   2 WITH  1261 OBSERVATIONS FROM  4 CLASSES =  1  2  3  4
              W= 4.30130E 04 Y= 6.12273D 08 YSQ= 1.17260D 13 X=
    GROUP   3 WITH   827 OBSERVATIONS FROM  3 CLASSES =  5  6  7
              W= 3.85620E 04 Y= 1.02522D 09 YSQ= 3.44664D 13 X=

SPLIT ATTEMPT ON GROUP   3 WITH N =        827, SS =  7.209825E 12
    BEST SPLIT ON PREDICTOR 2001 =  1.111591F 12 AFTER CLASS  6
    BEST SPLIT ON PREDICTOR 1168 =  1.474717E 10 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1009 =  2.999766E 10 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 2002 =  3.170894E 09 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  4.843247E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1498 =  6.205892E 10 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1572 =  3.152271E 11 AFTER CLASS  4
```

*Partition group 1 on the first predictor (V2001) and try for the best second split on the child with the largest remaining variance.*

```
1-STEP LOOKAHEAD TO SPLIT GROUP    1, TOTAL BSS =  4.213648E 12
        1. SPLIT GROUP   1 ON PREDICTOR 2001, BSS =  3.102057E 12
        2. SPLIT GROUP   3 ON PREDICTOR 2001, BSS =  1.111591F 12
```

*The best total BSS for the two splits, the first on V2001.*

```
TENTATIVE SPLIT 1. SPLIT GROUP   1 ON PREDICTOR 1168 WITH BSS = 2.726633E 11
    GROUP   2 WITH   661 OBSERVATIONS FROM  1 CLASSES =  2
              W= 2.91370E 04 Y= 5.13416D 08 YSQ= 1.29566D 13 X=
    GROUP   3 WITH  1427 OBSERVATIONS FROM  4 CLASSES =  3  4  5  6
              W= 5.24380E 04 Y= 1.12407D 09 YSQ= 3.32358D 13 X=

SPLIT ATTEMPT ON GROUP   3 WITH N =       1427, SS =  9.139882E 12
    BEST SPLIT ON PREDICTOR 2001 =  2.232880E 12 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 1168 =  2.751463E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1009 =  1.484448E 11 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 2002 =  1.507097F 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  8.852666F 11 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1498 =  3.935432E 11 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1572 =  4.414253E 11 AFTER CLASS  4


1-STEP LOOKAHEAD TO SPLIT GROUP    1, TOTAL BSS =  2.505543E 12
        1. SPLIT GROUP   1 ON PREDICTOR 1168, BSS =  2.726633E 11
        2. SPLIT GROUP   3 ON PREDICTOR 2001, BSS =  2.232880E 12
```

*Repeat the process making the first split on the second predictor (V1168).*

```
TENTATIVE SPLIT 1. SPLIT GROUP   1 ON PREDICTOR 1009 WITH BSS = 4.380196E 11
    GROUP   2 WITH  1759 OBSERVATIONS FROM  4 CLASSES =  2  3  4  5
              W= 6.60020E 04 Y= 1.39918D 09 YSQ= 4.07739D 13 X=
    GROUP   3 WITH   329 OBSERVATIONS FROM  1 CLASSES =  6
              W= 1.55730E 04 Y= 2.38312D 08 YSQ= 5.41854D 12 X=

SPLIT ATTEMPT ON GROUP   2 WITH N =       1759, SS =  1.111268E 13
    BEST SPLIT ON PREDICTOR 2001 =  2.477005E 12 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 1168 =  8.328210E 10 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1009 =  8.709053E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 2002 =  1.683258F 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  9.942010E 11 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1498 =  4.563906E 11 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1572 =  4.739396E 11 AFTER CLASS  3


1-STEP LOOKAHEAD TO SPLIT GROUP   1, TOTAL BSS =  2.915025E 12
        1. SPLIT GROUP   1 ON PREDICTOR 1009, BSS =  4.380196E 11
        2. SPLIT GROUP   2 ON PREDICTOR 2001, BSS =  2.477005E 12
TENTATIVE SPLIT 1. SPLIT GROUP   1 ON PREDICTOR 2002 WITH BSS = 3.802388E 11
    GROUP   2 WITH  1660 OBSERVATIONS FROM  1 CLASSES =  1
              W= 6.66840E 04 Y= 1.40661D 09 YSQ= 4.10219D 13 X=
    GROUP   3 WITH   428 OBSERVATIONS FROM  2 CLASSES =  2  3
              W= 1.48910E 04 Y= 2.30879D 08 YSQ= 5.17054D 12 X=

SPLIT ATTEMPT ON GROUP   2 WITH N =      1660, SS =  1.135127E 13
    BEST SPLIT ON PREDICTOR 2001 =  2.501147E 12 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1168 =  1.402407E 11 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1009 =  2.172649E 11 AFTER CLASS  5

SEX & MAR STATUS         CONSTANT - NO SPLIT
    BEST SPLIT ON PREDICTOR 1506 =  1.216348E 12 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1498 =  5.137016E 11 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1572 =  5.588994E 11 AFTER CLASS  3


1-STEP LOOKAHEAD TO SPLIT GROUP   1, TOTAL BSS =  2.881386E 12
        1. SPLIT GROUP   1 ON PREDICTOR 2002, BSS =  3.802388E 11
        2. SPLIT GROUP   2 ON PREDICTOR 2001, BSS =  2.501147E 12
TENTATIVE SPLIT 1. SPLIT GROUP   1 ON PREDICTOR 1506 WITH BSS = 1.288775E 12
    GROUP   2 WITH  1076 OBSERVATIONS FROM  2 CLASSES =  1  2
              W= 4.11590E 04 Y= 9.88315D 08 YSQ= 3.15359D 13 X=
    GROUP   3 WITH  1012 OBSERVATIONS FROM  4 CLASSES =  3  4  5  6
              W= 4.04160E 04 Y= 6.49174D 08 YSQ= 1.46565D 13 X=

SPLIT ATTEMPT ON GROUP   2 WITH N =      1076, SS =  7.804341E 12
    BEST SPLIT ON PREDICTOR 2001 =  1.737969E 12 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 1168 =  1.188162E 11 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1009 =  1.722349E 11 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 2002 =  3.119723E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  1.848010E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1498 =  2.937691E 11 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1572 =  2.787367E 11 AFTER CLASS  2


1-STEP LOOKAHEAD TO SPLIT GROUP   1, TOTAL BSS =  3.026744E 12
        1. SPLIT GROUP   1 ON PREDICTOR 1506, BSS =  1.288775E 12
        2. SPLIT GROUP   2 ON PREDICTOR 2001, BSS =  1.737969E 12
TENTATIVE SPLIT 1. SPLIT GROUP   1 ON PREDICTOR 1498 WITH BSS = 6.633711E 11
    GROUP   2 WITH  1304 OBSERVATIONS FROM  3 CLASSES =  1  2  3
              W= 5.07440E 04 Y= 1.13140D 09 YSQ= 3.39227D 13 X=
    GROUP   3 WITH   784 OBSERVATIONS FROM  2 CLASSES =  4  5
              W= 3.08310E 04 Y= 5.06088D 08 YSQ= 1.22696D 13 X=

SPLIT ATTEMPT ON GROUP   2 WITH N =      1304, SS =  8.696735E 12
    BEST SPLIT ON PREDICTOR 2001 =  1.873763E 12 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 1168 =  1.296543E 11 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1009 =  2.204862E 11 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 2002 =  2.072489E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  4.940051E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1498 =  2.342267E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1572 =  3.481105E 11 AFTER CLASS  2
```

```
1-STEP LOOKAHEAD TO SPLIT GROUP   1, TOTAL BSS =  2.537134E 12
        1. SPLIT GROUP   1 ON PREDICTOR 1498, BSS =  6.633711E 11
        2. SPLIT GROUP   2 ON PREDICTOR 2001, BSS =  1.873763E 12
TENTATIVE SPLIT 1. SPLIT GROUP   1 ON PREDICTOR 1572 WITH BSS = 5.695026E 11
    GROUP   2 WITH   790 OBSERVATIONS FROM  1 CLASSES =  3
              W= 2.37430E 04 Y= 3.78693D 08 YSQ= 9.16820D 12 X=
    GROUP   3 WITH  1298 OBSERVATIONS FROM  3 CLASSES =  2  4  1
              W= 5.78320E 04 Y= 1.25880D 09 YSQ= 3.70242D 13 X=

SPLIT ATTEMPT ON GROUP   3 WITH N =      1298, SS =  9.624702E 12
    BEST SPLIT ON PREDICTOR 2001 =  2.012377E 12 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1168 =  2.224659E 11 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1009 =  3.248908E 11 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 2002 =  3.620859E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  8.785892E 11 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1498 =  3.664312E 11 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1572 =  2.575974E 11 AFTER CLASS  4


1-STEP LOOKAHEAD TO SPLIT GROUP   1, TOTAL BSS =  2.581879E 12
        1. SPLIT GROUP   1 ON PREDICTOR 1572, BSS =  5.695026E 11
        2. SPLIT GROUP   3 ON PREDICTOR 2001, BSS =  2.012377E 12
```

```
                ***** PARTITION OF GROUP    1 *****
FROM ELIGIBLE PREDICTORS AROUND THE CURRENT RANK  1,     0 UP AND  0 DOWN

      MAXIMUM ELIGIBLE BSS AT EACH STEP         MAXIMUM TOTAL BSS (LOOKAHEAD)
    1.SPLIT    1 ON V2001 BSS= 3.10206E 12    SPLIT   1 ON V2001 BSS = 3.10206E 12
    2.SPLIT    3 ON V2001 BSS= 1.11159E 12    SPLIT   3 ON V2001 BSS = 1.11159E 12
    PE(2)= 1.599E 10, TOTAL= 4.21365E 12                     TOTAL= 4.21365E 12
        PREDICTOR 2001 HAS RANK  1

SPLIT GROUP    1 ON 3-YR AVE $ INC            V2001
    GROUP    2 WITH  1261 OBSERVATIONS FROM  4 CLASSES =  1  2  3  4
               W= 4.30130E 04 Y= 6.12273D 08 YSQ= 1.17260D 13 X=
    GROUP    3 WITH   827 OBSERVATIONS FROM  3 CLASSES =  5  6  7
               W= 3.85620E 04 Y= 1.02522D 09 YSQ= 3.44664D 13 X=

             2 CANDIDATES -    GROUP       SS
                                   2   3.010512E 12
                                   3   7.209825E 12

                ATTEMPT SPLIT ON GROUP   3 WITH SS = 7.209825E 12
```

```
                    LOOKAHEAD TENTATIVE PARTITION

SPLIT ATTEMPT ON GROUP   3 WITH N =        827, SS =  7.209825E 12
    BEST SPLIT ON PREDICTOR 2001 =  1.111591E 12 AFTER CLASS  6
    BEST SPLIT ON PREDICTOR 1168 =  1.474717E 10 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1009 =  2.999766E 10 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 2002 =  3.170894E 09 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  4.843247E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1498 =  6.205892E 10 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1572 =  3.152271E 11 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1490 =  9.516037E 10 AFTER CLASS  2
TENTATIVE SPLIT 1. SPLIT GROUP   3 ON PREDICTOR 2001 WITH BSS = 1.111591E 12
    GROUP   4 WITH   705 OBSERVATIONS FROM  2 CLASSES =  5  6
              W= 3.25190E 04 Y= 7.89293D 08 YSQ= 2.32347D 13 X=
    GROUP   5 WITH   122 OBSERVATIONS FROM  1 CLASSES =  7
              W= 5.04300E 03 Y= 2.35924D 08 YSQ= 1.12317D 13 X=

SPLIT ATTEMPT ON GROUP   4 WITH N =        705, SS =  4.077182E 12
    BEST SPLIT ON PREDICTOR 2001 =  2.028198E 11 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 1168 =  1.491495E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1009 =  1.496528E 10 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 2002 =  1.962934E 09 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  2.657847E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1498 =  4.518104E 10 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1572 =  1.709934E 11 AFTER CLASS  4


1-STEP LOOKAHEAD TO SPLIT GROUP   3, TOTAL BSS =  1.377376E 12
        1. SPLIT GROUP   3 ON PREDICTOR 2001, BSS =  1.111591E 12
        2. SPLIT GROUP   4 ON PREDICTOR 1506, BSS =  2.657847E 11
TENTATIVE SPLIT 1. SPLIT GROUP   3 ON PREDICTOR 1168 WITH BSS = 1.474717E 10
    GROUP   4 WITH   186 OBSERVATIONS FROM  1 CLASSES =  2
              W= 8.99300E 03 Y= 2.28740D 08 YSQ= 7.33278D 12 X=
    GROUP   5 WITH   641 OBSERVATIONS FROM  4 CLASSES =  3  4  5  6
              W= 2.95790E 04 Y= 7.96477D 08 YSQ= 2.71336D 13 X=

SPLIT ATTEMPT ON GROUP   5 WITH N =        641, SS =  5.686833E 12
    BEST SPLIT ON PREDICTOR 2001 =  9.491542E 11 AFTER CLASS  6
    BEST SPLIT ON PREDICTOR 1168 =  1.956223E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1009 =  2.937651E 10 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 2002 =  9.730785E 08 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  3.869833E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1498 =  5.836793E 10 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1572 =  3.034998E 11 AFTER CLASS  4


1-STEP LOOKAHEAD TO SPLIT GROUP   3, TOTAL BSS =  9.639014E 11
        1. SPLIT GROUP   3 ON PREDICTOR 1168, BSS =  1.474717E 10
        2. SPLIT GROUP   5 ON PREDICTOR 2001, BSS =  9.491542E 11
TENTATIVE SPLIT 1. SPLIT GROUP   3 ON PREDICTOR 1009 WITH BSS = 2.999766E 10
    GROUP   4 WITH   428 OBSERVATIONS FROM  2 CLASSES =  2  3
              W= 1.95570E 04 Y= 5.36948D 08 YSQ= 1.85477D 13 X=
    GROUP   5 WITH   399 OBSERVATIONS FROM  3 CLASSES =  4  5  6
              W= 1.90050E 04 Y= 4.88269D 08 YSQ= 1.59187D 13 X=

SPLIT ATTEMPT ON GROUP   4 WITH N =        428, SS =  3.805554E 12
    BEST SPLIT ON PREDICTOR 2001 =  8.456975E 11 AFTER CLASS  6
    BEST SPLIT ON PREDICTOR 1168 =  3.475086E 10 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 1009 =  3.297352E 10 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 2002 =  6.239027E 08 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1506 =  4.029384E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1498 =  2.723362E 10 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1572 =  3.254444E 11 AFTER CLASS  2


1-STEP LOOKAHEAD TO SPLIT GROUP   3, TOTAL BSS =  8.756952E 11
       1. SPLIT GROUP   3 ON PREDICTOR 1009, BSS =  2.999766E 10
       2. SPLIT GROUP   4 ON PREDICTOR 2001, BSS =  8.456975E 11
TENTATIVE SPLIT 1. SPLIT GROUP   3 ON PREDICTOR 2002 WITH BSS = 3.170894E 09
    GROUP   4 WITH   782 OBSERVATIONS FROM  1 CLASSES =  1
              W= 3.65570E 04 Y= 5.74368D 08 YSQ= 3.27634D 13 X=
    GROUP   5 WITH    45 OBSERVATIONS FROM  2 CLASSES =  2  3
              W= 2.00500E 03 Y= 5.08483D 07 YSQ= 1.70305D 12 X=

SPLIT ATTEMPT ON GROUP   4 WITH N =       782, SS =  6.793158E 12
    BEST SPLIT ON PREDICTOR 2001 =  1.033711E 12 AFTER CLASS  6
    BEST SPLIT ON PREDICTOR 1168 =  1.726376E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1009 =  2.568592E 10 AFTER CLASS  3

SEX & MAR STATUS          CONSTANT - NO SPLIT
    BEST SPLIT ON PREDICTOR 1506 =  4.706512E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1498 =  4.860359E 10 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1572 =  3.118717E 11 AFTER CLASS  4


1-STEP LOOKAHEAD TO SPLIT GROUP   3, TOTAL BSS =  1.036882E 12
       1. SPLIT GROUP   3 ON PREDICTOR 2002, BSS =  3.170894E 09
       2. SPLIT GROUP   4 ON PREDICTOR 2001, BSS =  1.033711E 12
TENTATIVE SPLIT 1. SPLIT GROUP   3 ON PREDICTOR 1506 WITH BSS = 4.843247E 11
    GROUP   4 WITH   326 OBSERVATIONS FROM  1 CLASSES =  1
              W= 1.51140E 04 Y= 4.68540D 08 YSQ= 1.77001D 13 X=
    GROUP   5 WITH   501 OBSERVATIONS FROM  5 CLASSES =  2  3  4  5  6
              W= 2.34480E 04 Y= 5.56676D 08 YSQ= 1.67663D 13 X=

SPLIT ATTEMPT ON GROUP   5 WITH N =       501, SS =  3.550351E 12
    BEST SPLIT ON PREDICTOR 2001 =  4.462341E 11 AFTER CLASS  6
    BEST SPLIT ON PREDICTOR 1168 =  7.574913E 09 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1009 =  1.474403E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 2002 =  1.212888E 10 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1506 =  4.996884E 10 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1498 =  1.218131E 10 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1572 =  1.538995E 10 AFTER CLASS  4


1-STEP LOOKAHEAD TO SPLIT GROUP   3, TOTAL BSS =  9.305588E 11
       1. SPLIT GROUP   3 ON PREDICTOR 1506, BSS =  4.843247E 11
       2. SPLIT GROUP   5 ON PREDICTOR 2001, BSS =  4.462341E 11
TENTATIVE SPLIT 1. SPLIT GROUP   3 ON PREDICTOR 1498 WITH BSS = 6.205892E 10
    GROUP   4 WITH   155 OBSERVATIONS FROM  1 CLASSES =  1
              W= 7.03500E 03 Y= 1.68139D 08 YSQ= 4.91865D 12 X=
    GROUP   5 WITH   672 OBSERVATIONS FROM  4 CLASSES =  2  3  4  5
              W= 3.15270E 04 Y= 8.57077D 08 YSQ= 2.95478D 13 X=

SPLIT ATTEMPT ON GROUP   5 WITH N =       672, SS =  6.247688E 12
    BEST SPLIT ON PREDICTOR 2001 =  9.680789E 11 AFTER CLASS  6
    BEST SPLIT ON PREDICTOR 1168 =  1.565314E 10 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1009 =  2.117285E 10 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 2002 =  3.523215E 08 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1506 =  4.780164E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1498 =  7.677254E 10 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1572 =  2.813204E 11 AFTER CLASS  4
```

```
1-STEP LOOKAHEAD TO SPLIT GROUP    3, TOTAL BSS =  1.030138E 12
        1. SPLIT GROUP    2 ON PREDICTOR 1498, BSS =  6.205892E 10
        2. SPLIT GROUP    5 ON PREDICTOR 2001, BSS =  9.680789E 11
TENTATIVE SPLIT 1. SPLIT GROUP    3 ON PREDICTOR 1572 WITH BSS = 3.152271E 11
    GROUP    4 WITH    631 OBSERVATIONS FROM  3 CLASSES =  3  2  4
              W= 2.82770E 04 Y= 7.03020D 08 YSQ= 2.18480D 13 X=
    GROUP    5 WITH    196 OBSERVATIONS FROM  1 CLASSES =  1
              W= 1.02850E 04 Y= 3.22196D 08 YSQ= 1.26184D 13 X=

SPLIT ATTEMPT ON GROUP    4 WITH N =        631, SS =  4.369614E 12
    BEST SPLIT ON PREDICTOR 2001 =  6.060654E 11 AFTER CLASS  6
    BEST SPLIT ON PREDICTOR 1168 =  6.520046E 09 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1009 =  2.945135E 10 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 2002 =  1.551892E 08 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1506 =  1.326627E 11 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1498 =  2.222981E 10 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1572 =  1.726376E 10 AFTER CLASS  3


1-STEP LOOKAHEAD TO SPLIT GROUP    3, TOTAL BSS =  9.212925E 11
        1. SPLIT GROUP    3 ON PREDICTOR 1572, BSS =  3.152271E 11
        2. SPLIT GROUP    4 ON PREDICTOR 2001, BSS =  6.060654E 11
```

```
                   ***** PARTITION OF GROUP    3 *****
FROM ELIGIBLE PREDICTORS AROUND THE CURRENT RANK  1,     0 UP AND  0 DOWN

      MAXIMUM ELIGIBLE BSS AT EACH STEP        MAXIMUM TOTAL BSS (LOOKAHEAD)
    1.SPLIT    3 ON V2001 BSS= 1.11159E 12     SPLIT   3 ON V2001 BSS = 1.11159E 12
    2.SPLIT    4 ON V1506 BSS= 2.65785E 11     SPLIT   4 ON V1506 BSS = 2.65785E 11
    PE(2)= 1.599E 10, TOTAL= 1.37738E 12                   TOTAL= 1.37738E 12
        PREDICTOR 2001 HAS RANK  1

SPLIT GROUP    3 ON 3-YR AVE $ INC            V2001
    GROUP    4 WITH    705 OBSERVATIONS FROM  2 CLASSES =  5  6
               W= 3.25190E 04 Y= 7.89293D 08 YSQ= 2.32347D 13 X=
    GROUP    5 WITH    122 OBSERVATIONS FROM  1 CLASSES =  7
               W= 6.04300E 03 Y= 2.35924D 08 YSQ= 1.12317D 13 X=

              3 CANDIDATES -     GROUP       SS
                                    2    3.010512E 12
                                    4    4.077182E 12
                                    5    2.021067E 12

               ATTEMPT SPLIT ON GROUP    2 WITH SS = 3.010512E 12
```

```
                     LOOKAHEAD TENTATIVE PARTITION

SPLIT ATTEMPT ON GROUP    2 WITH N =       1261, SS =  3.010512E 12
    BEST SPLIT ON PREDICTOR 2001 =  1.887342E 11 AFTER CLASS  3

SYMMETRIC SPLIT INELIGIBLE - RESULTANT GROUP TOO SMALL
    BEST SPLIT ON PREDICTOR 1168 =  1.054448E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1009 =  2.628780E 09 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 2002 =  1.279263E 09 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  1.563909E 11 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1498 =  8.093644E 10 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1572 =  1.372607E 11 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1490 =  9.628654E 10 AFTER CLASS  2
TENTATIVE SPLIT 1. SPLIT GROUP    2 ON PREDICTOR 2001 WITH BSS = 1.887342E 11

EXTREME CASES LYING OUTSIDE THE INTERVAL (-2.761231E 04, 5.608148E 04)
        Y          W          WY            V1122
    60000        53.0     3.18000E 06      60000
    60000        48.0     2.88000E 06      60000
    60000        53.0     3.18000E 06      60000
    75000        24.0     1.80000E 06      75000
    GROUP    6 WITH    899 OBSERVATIONS FROM  3 CLASSES =  1  2  3
               W= 2.89780E 04 Y= 3.70246D 08 YSQ= 6.46847D 12 X=
    GROUP    7 WITH    362 OBSERVATIONS FROM  1 CLASSES =  4
               W= 1.40350E 04 Y= 2.42026D 08 YSQ= 5.25749D 12 X=

SPLIT ATTEMPT ON GROUP    6 WITH N =        899, SS =  1.737908E 12
    BEST SPLIT ON PREDICTOR 2001 =  6.386772E 10 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1168 =  5.869928E 09 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1009 =  3.208643E 09 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 2002 =  1.898237E 10 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1506 =  6.039798E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1498 =  4.410311E 10 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1572 =  6.607497E 10 AFTER CLASS  2


1-STEP LOOKAHEAD TO SPLIT GROUP    2, TOTAL BSS =  2.548092E 11
        1. SPLIT GROUP    2 ON PREDICTOR 2001, BSS =  1.887342E 11
        2. SPLIT GROUP    6 ON PREDICTOR 1572, BSS =  6.607497E 10
TENTATIVE SPLIT 1. SPLIT GROUP    2 ON PREDICTOR 1168 WITH BSS = 1.054448E 10

EXTREME CASES LYING OUTSIDE THE INTERVAL (-2.761231E 04, 5.608148E 04)
        Y          W          WY            V1122
    60000        53.0     3.18000E 06      60000
    60000        48.0     2.88000E 06      60000
    60000        53.0     3.18000E 06      60000
    75000        24.0     1.80000E 06      75000
    GROUP    6 WITH  1028 OBSERVATIONS FROM  3 CLASSES =  2  3  4
               W= 3.85740E 04 Y= 5.55564D 08 YSQ= 1.06816D 13 X=
    GROUP    7 WITH   233 OBSERVATIONS FROM  2 CLASSES =  5  6
               W= 4.43900E 03 Y= 5.67081D 07 YSQ= 1.04432D 12 X=

SPLIT ATTEMPT ON GROUP    6 WITH N =       1028, SS =  2.680100E 12
    BEST SPLIT ON PREDICTOR 2001 =  1.853190E 11 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1168 =  3.248488E 09 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1009 =  4.081058E 09 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 2002 =  3.002073E 09 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  1.318050E 11 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1498 =  6.848250E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1572 =  1.070942E 11 AFTER CLASS  2
```

```
1-STEP LOOKAHEAD TO SPLIT GROUP    2, TOTAL BSS =  1.958635E 11
        1. SPLIT GROUP    2 ON PREDICTOR 1168, BSS =  1.054448E 10
        2. SPLIT GROUP    6 ON PREDICTOR 2001, BSS =  1.853190E 11
TENTATIVE SPLIT 1. SPLIT GROUP    2 ON PREDICTOR 1009 WITH BSS = 2.628780E 09

EXTREME CASES LYING OUTSIDE THE INTERVAL (-2.761231E 04, 5.608148E 04)
        Y          W          WY           V1122
    60000       53.0     3.18000E 06      60000
    60000       48.0     2.88000E 06      60000
    60000       53.0     3.18000E 06      60000
    75000       24.0     1.80000E 06      75000
    GROUP    6 WITH    965 OBSERVATIONS FROM  4 CLASSES =  2  3  4  5
                W= 2.90270E 04 Y= 4.18168D 08 YSQ= 7.92797D 12 X=
    GROUP    7 WITH    296 OBSERVATIONS FROM  1 CLASSES =  6
                W= 1.39860E 04 Y= 1.94104D 08 YSQ= 3.79799D 12 X=

SPLIT ATTEMPT ON GROUP    6 WITH N =       965, SS =  1.903773E 12
    BEST SPLIT ON PREDICTOR 2001 =  1.086272E 11 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1168 =  1.004641E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1009 =  1.227882E 09 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 2002 =  7.344226E 09 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1506 =  9.267105E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1498 =  4.313632E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1572 =  9.653715E 10 AFTER CLASS  3


1-STEP LOOKAHEAD TO SPLIT GROUP    2, TOTAL BSS =  1.112560E 11
        1. SPLIT GROUP    2 ON PREDICTOR 1009, BSS =  2.628780E 09
        2. SPLIT GROUP    6 ON PREDICTOR 2001, BSS =  1.086272E 11
TENTATIVE SPLIT 1. SPLIT GROUP    2 ON PREDICTOR 2002 WITH BSS = 1.279263E 09

EXTREME CASES LYING OUTSIDE THE INTERVAL (-2.761231E 04, 5.608148E 04)
        Y          W          WY           V1122
    60000       53.0     3.18000E 06      60000
    60000       48.0     2.88000E 06      60000
    60000       53.0     3.18000E 06      60000
    75000       24.0     1.80000E 06      75000
    GROUP    6 WITH    878 OBSERVATIONS FROM  1 CLASSES =  1
                W= 3.01270E 04 Y= 4.32242D 08 YSQ= 8.25847D 12 X=
    GROUP    7 WITH    383 OBSERVATIONS FROM  2 CLASSES =  2  3
                W= 1.28860E 04 Y= 1.80030D 08 YSQ= 3.46749D 12 X=

SPLIT ATTEMPT ON GROUP    6 WITH N =       878, SS =  2.056949E 12
    BEST SPLIT ON PREDICTOR 2001 =  1.656110E 11 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1168 =  1.324876E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1009 =  2.674917E 09 AFTER CLASS  3

SEX & MAR STATUS           CONSTANT - NO SPLIT
    BEST SPLIT ON PREDICTOR 1506 =  1.138795E 11 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1498 =  6.668734E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1572 =  1.174730E 11 AFTER CLASS  3


1-STEP LOOKAHEAD TO SPLIT GROUP    2, TOTAL BSS =  1.668903E 11
        1. SPLIT GROUP    2 ON PREDICTOR 2002, BSS =  1.279263E 09
        2. SPLIT GROUP    6 ON PREDICTOR 2001, BSS =  1.656110E 11
TENTATIVE SPLIT 1. SPLIT GROUP    2 ON PREDICTOR 1506 WITH BSS = 1.563909E 11

EXTREME CASES LYING OUTSIDE THE INTERVAL (-2.761231E 04, 5.608148E 04)
        Y          W          WY           V1122
    60000       53.0    3.18000E 06      60000
    60000       48.0    2.88000E 06      60000
    60000       53.0    3.18000E 06      60000
    75000       24.0    1.80000E 06      75000
    GROUP    6 WITH    760 OBSERVATIONS FROM  4 CLASSES =  1  2  3  4
               W= 2.45430E 04 Y= 3.89958D 08 YSQ= 7.98458D 12 X=
    GROUP    7 WITH    501 OBSERVATIONS FROM  2 CLASSES =  5  6
               W= 1.84700E 04 Y= 2.22315D 08 YSQ= 3.74138D 12 X=

SPLIT ATTEMPT ON GROUP    6 WITH N =       760, SS =  1.788643E 12
    BEST SPLIT ON PREDICTOR 2001 =  1.108387E 11 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1168 =  7.764705E 09 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 1009 =  3.647996E 09 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 2002 =  2.099249E 09 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  1.671116E 10 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1498 =  1.743782E 10 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1572 =  7.373901E 10 AFTER CLASS  3


1-STEP LOOKAHEAD TO SPLIT GROUP    2, TOTAL BSS =  2.672296E 11
        1. SPLIT GROUP    2 ON PREDICTOR 1506, BSS =  1.563909E 11
        2. SPLIT GROUP    6 ON PREDICTOR 2001, BSS =  1.108387E 11
TENTATIVE SPLIT 1. SPLIT GROUP    2 ON PREDICTOR 1498 WITH BSS = 8.093644E 10

EXTREME CASES LYING OUTSIDE THE INTERVAL (-2.761231E 04, 5.608148E 04)
        Y          W         WY            V1122
    60000       53.0    3.18000E 06      60000
    60000       48.0    2.88000E 06      60000
    60000       53.0    3.18000E 06      60000
    75000       24.0    1.80000E 06      75000
    GROUP    6 WITH    673 OBSERVATIONS FROM  3 CLASSES =  1  2  3
               W= 2.13770E 04 Y= 3.33794D 08 YSQ= 6.71694D 12 X=
    GROUP    7 WITH    588 OBSERVATIONS FROM  2 CLASSES =  4  5
               W= 2.16360E 04 Y= 2.78479D 08 YSQ= 5.00902D 12 X=

SPLIT ATTEMPT ON GROUP    6 WITH N =       673, SS =  1.504882E 12
    BEST SPLIT ON PREDICTOR 2001 =  7.171736E 10 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1168 =  2.382365E 09 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1009 =  4.610589E 09 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 2002 =  2.707423E 09 AFTER CLASS  2
    BEST SPLIT ON PREDICTOR 1506 =  7.177608E 10 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1498 =  3.282777E 10 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1572 =  7.081243E 10 AFTER CLASS  2


1-STEP LOOKAHEAD TO SPLIT GROUP    2, TOTAL BSS =  1.527125E 11
        1. SPLIT GROUP    2 ON PREDICTOR 1498, BSS =  8.093644E 10
        2. SPLIT GROUP    6 ON PREDICTOR 1506, BSS =  7.177608E 10
TENTATIVE SPLIT 1. SPLIT GROUP    2 ON PREDICTOR 1572 WITH BSS = 1.372607E 11

EXTREME CASES LYING OUTSIDE THE INTERVAL (-2.761231E 04, 5.608148E 04)
        Y          W         WY            V1122
    60000       53.0    3.18000E 06      60000
    60000       48.0    2.88000E 06      60000
    60000       53.0    3.18000E 06      60000
    75000       24.0    1.80000E 06      75000
    GROUP    6 WITH    597 OBSERVATIONS FROM  1 CLASSES =  3
               W= 1.54950E 04 Y= 1.83677D 08 YSQ= 2.98483D 12 X=
    GROUP    7 WITH    664 OBSERVATIONS FROM  3 CLASSES =  2  1  4
               W= 2.75180E 04 Y= 4.28595D 08 YSQ= 8.74114D 12 X=
```

*With the $15.64 \times 10^{10}$ from the first split on city size and $11.08 \times 10^{10}$ from a second split on income, the total power is greater than any pair of splits, therefore group 2 will be split on city size (V1506).*

```
SPLIT ATTEMPT ON GROUP    7 WITH N =       664, SS =  2.065736E 12
    BEST SPLIT ON PREDICTOR 2001 =  1.186411E 11 AFTER CLASS  3
    BEST SPLIT ON PREDICTOR 1168 =  2.715812E 09 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 1009 =  2.517631E 09 AFTER CLASS  5
    BEST SPLIT ON PREDICTOR 2002 =  5.719982E 09 AFTER CLASS  1
    BEST SPLIT ON PREDICTOR 1506 =  9.607997E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1498 =  6.120853E 10 AFTER CLASS  4
    BEST SPLIT ON PREDICTOR 1572 =  2.891553E 10 AFTER CLASS  2


1-STEP LOOKAHEAD TO SPLIT GROUP    2, TOTAL BSS =  2.559018E 11
        1. SPLIT GROUP    2 ON PREDICTOR 1572, BSS =  1.372607E 11
        2. SPLIT GROUP    7 ON PREDICTOR 2001, BSS =  1.186411E 11
```

```
                ***** PARTITION OF GROUP    2 *****
FROM ELIGIBLE PREDICTORS AROUND THE CURRENT RANK  1,     0 UP AND  0 DOWN

      MAXIMUM ELIGIBLE BSS AT EACH STEP          MAXIMUM TOTAL BSS (LOOKAHEAD)
    1.SPLIT    2 ON V2001 BSS= 1.88734E 11     SPLIT   2 ON V1506 BSS = 1.56391E 11
    2.SPLIT    6 ON V1572 BSS= 6.60750E 10     SPLIT   6 ON V2001 BSS = 1.10839E 11
    PF(2)= 1.599E 10, TOTAL= 2.54809E 11                     TOTAL= 2.67230E 11
        PREDICTOR 1506 HAS RANK  1

SYMMETRY SPLIT ON V2001 =  0.0
    SYMMETRIC / MAX SPLIT =  0.0 PERCENT

SPLIT GROUP    2 ON LRGST PLAC/SMSA PSU31:66 V1506

EXTREME CASES LYING OUTSIDE THE INTERVAL (-2.761231E 04, 5.608148E 04)
        Y          W          WY           V1122
    60000       53.0     3.18000E 06     60000
    60000       48.0     2.88000E 06     60000
    60000       53.0     3.18000E 06     60000
   .75000       24.0     1.80000E 06     75000
    GROUP    6 WITH    501 OBSERVATIONS FROM  2 CLASSES =   5  6
               W= 1.84700E 04 Y= 2.22315D OR YSQ= 3.74138D 12 X=
    GROUP    7 WITH    760 OBSERVATIONS FROM  4 CLASSES =  1  2  3  4
               W= 2.45430E 04 Y= 3.89957D OR YSQ= 7.98458D 12 X=

              4 CANDIDATES -     GROUP        SS
                                     6   1.065477E 12
                                     4   4.077182E 12
                                     5   2.021067E 12
                                     7   1.788649E 12

                ATTEMPT SPLIT ON GROUP   4 WITH SS = 4.077182E 12
```

*Omit the rest of the splitting process.*

```
                ***** PARTITION OF GROUP  22 *****
FROM ELIGIBLE PREDICTORS AROUND THE CURRENT RANK  1.     0 UP AND  0 DOWN

      MAXIMUM ELIGIBLE BSS AT EACH STEP         MAXIMUM TOTAL BSS (LOOKAHEAD)

GROUP  22 COULD NOT BE SPLIT


END OF STAGE 1 OF THE ANALYSIS.  12 FINAL GROUPS.  12 INELIGIBLE FOR SPLITTING.

                VARIATION EXPLAINED (BSS(0)/TSS) =  39.8%


                1-WAY ANALYSIS OF VARIANCE ON FINAL GROUPS

                SOURCE            SUM OF SQUARES         DF     MEAN SQUARE

                BETWEEN            5.307254E 12         12.    4.422711E 11
                ERROR              8.015149E 12     81524.    9.831658E 07
                TOTAL              1.332240E 13     81536.    1.633931E 08
```

*A summary of the variance within (error) and between the final groups. Note that with weighted data one cannot use the degrees of freedom, or calculate F-Tests.*

```
                                GROUP SUMMARY TABLE
                           23 GROUPS OF WHICH 12 ARE FINAL

GROUP   1 , N =     2088, SUM W =  3.157500E 04
    Y MEAN= 2.007342E 04, VARIANCE= 1.633931E 08,   SS(L)/TSS= 1.000, BSS/TSS= 0.233
    SPLIT ON 3-YR AVE $ INC            , BSS(L) =  3.102057E 12  INTO
           2 WITH CLASSES   1  2  3  4
           3 WITH CLASSES   5  6  7

GROUP   3 , N =      827, SUM W =  3.856200E 04
    Y MEAN= 2.658618E 04, VARIANCE= 1.871938E 08,   SS(L)/TSS= 0.541, BSS/TSS= 0.083
    SPLIT ON 3-YR AVE $ INC            , BSS(L) =  1.111591E 12  INTO
           4 WITH CLASSES   5  6
           5 WITH CLASSES   7

GROUP   2 , N =     1261, SUM W =  4.301300E 04
    Y MEAN= 1.423459E 04, VARIANCE= 7.004651E 07,   SS(L)/TSS= 0.226, BSS/TSS= 0.012
    SPLIT ON LRGST PLAC/SMSA PSU31:66, BSS(L) =  1.563909E 11  INTO
           6 WITH CLASSES   5  6
           7 WITH CLASSES   1  2  3  4

GROUP   4 , N =      705, SUM W =  3.251900E 04
    Y MEAN= 2.427173E 04, VARIANCE= 1.255571E 08,   SS(L)/TSS= 0.306, BSS/TSS= 0.003
    SPLIT ON DIST TO CNTR SMSA  31:58, BSS(L) =  4.518104E 10  INTO
           8 WITH CLASSES   1  2
           9 WITH CLASSES   3  4  5

GROUP   5 , N =      122, SUM W =  6.043000E 03
    Y MEAN= 3.904086E 04, VARIANCE= 3.372119E 08,   SS(L)/TSS= 0.152, BSS/TSS= 0.014
    SPLIT ON LRGST PLAC/SMSA PSU31:66, BSS(L) =  1.798738E 11  INTO
          10 WITH CLASSES   3  4  5  6
          11 WITH CLASSES   1  2

GROUP   9 , N =      312, SUM W =  1.462700E 04
    Y MEAN= 2.557518E 04, VARIANCE= 1.651233E 08,   SS(L)/TSS= 0.181, BSS/TSS= 0.023
    SPLIT ON LRGST PLAC/SMSA PSU31:66, BSS(L) =  3.032692E 11  INTO
          12 WITH CLASSES   3  4  5  6
          13 WITH CLASSES   1  2

GROUP   8 , N =      393, SUM W =  1.789200E 04
    Y MEAN= 2.320613E 04, VARIANCE= 9.102715E 07,   SS(L)/TSS= 0.122, BSS/TSS= 0.006
    SPLIT ON LRGST PLAC/SMSA PSU31:66, BSS(L) =  8.140305E 10  INTO
          14 WITH CLASSES   2  3  5  6
          15 WITH CLASSES   1

GROUP   7 , N =      760, SUM W =  2.454300E 04
    Y MEAN= 1.588874E 04, VARIANCE= 7.297443E 07,   SS(L)/TSS= 0.134, BSS/TSS= 0.006
    SPLIT ON CURRENT REGION OV472     , BSS(L) =  7.373901E 10  INTO
          16 WITH CLASSES   3
          17 WITH CLASSES   1  2  4

GROUP   6*, N =      501, SUM W =  1.847000E 04
    Y MEAN= 1.203654E 04, VARIANCE= 5.780226E 07,   SS(L)/TSS= 0.080, BSS/TSS= 0.0

GROUP  11 , N =       86, SUM W =  4.351000E 03
    Y MEAN= 4.244310E 04, VARIANCE= 3.295334E 08,   SS(L)/TSS= 0.106, BSS/TSS= 0.001
    SPLIT ON # REQUIRED ROOMS 23:12  , BSS(L) =  1.231448E 10  INTO
          18 WITH CLASSES   2
          19 WITH CLASSES   3  4  5  6

GROUP  10*, N =       36, SUM W =  1.692000E 03
```

*This is the main result - the split record from which the branching diagram can be made.*

```
    Y MEAN= 3.029196E 04, VARIANCE= 2.577897E 08,   SS(L)/TSS= 0.032, BSS/TSS= 0.0

GROUP  17 , N =      463, SUM W =  1.806100E 04
    Y MEAN= 1.692718E C4, VARIANCE= 7.825983E 07,   SS(L)/TSS= 0.106, BSS/TSS= 0.008
    SPLIT ON 3-YR AVE $ INC           , BSS(L) =  1.001768E 11  INTO
          20 WITH CLASSES   1  2  3
          21 WITH CLASSES   4

GROUP  16*, N =      297, SUM W =  6.482000E 03
    Y MEAN= 1.299527E 04, VARIANCE= 4.714030E 07,   SS(L)/TSS= 0.023, BSS/TSS= 0.0

GROUP  19 , N =       65, SUM W =  3.333000E 03
    Y MEAN= 4.337286E C4, VARIANCE= 3.608515E 08,   SS(L)/TSS= 0.089, BSS/TSS= 0.011
    SPLIT ON CURRENT REGION OV472     , BSS(L) =  1.413889E 11  INTO
          22 WITH CLASSES   2  4
          23 WITH CLASSES   3  1

GROUP  18*, N =       21, SUM W =  1.018000E 03
    Y MEAN= 3.939901E 04, VARIANCE= 2.275338E 08,   SS(L)/TSS= 0.017, BSS/TSS= 0.0

GROUP  12*, N =      188, SUM W =  8.578000E 03
    Y MEAN= 2.175148E 04, VARIANCE= 1.386882E 08,   SS(L)/TSS= 0.089, BSS/TSS= 0.0

GROUP  15*, N =      175, SUM W =  7.556000E 03
    Y MEAN=.2.570084E 04, VARIANCE= 1.244561E 09,   SS(L)/TSS= 0.070, BSS/TSS= 0.0

GROUP  13*, N =      124, SUM W =  6.049000E 03
    Y MEAN= 3.099752E 04, VARIANCE= 1.534795E 08,   SS(L)/TSS= 0.069, BSS/TSS= 0.0

GROUP  21*, N =      181, SUM W =  7.289000E 03
    Y MEAN= 1.975753E 04, VARIANCE= 9.629286E 07,   SS(L)/TSS= 0.053, BSS/TSS= 0.0

GROUP  14*, N =      218, SUM W =  1.033600E 04
    Y MEAN= 2.138240E C4, VARIANCE= 5.910378E 07,   SS(L)/TSS= 0.046, BSS/TSS= 0.0

GROUP  20*, N =      282, SUM W =  1.067200E 04
    Y MEAN= 1.496752E 04, VARIANCE= 5.666890E 07,   SS(L)/TSS= 0.045, BSS/TSS= 0.0

GROUP  23*, N =       31, SUM W =  1.683000E 03
    Y MEAN= 4.982159E 04, VARIANCE= 3.220828E 08,   SS(L)/TSS= 0.039, BSS/TSS= 0.0

GROUP  22*, N =       34, SUM W =  1.650000E 03
    Y MEAN= 3.673515E C4, VARIANCE= 3.236129E 09,   SS(L)/TSS= 0.039, BSS/TSS= 0.0
```

```
                                            100*BSS/TSS TABLE FOR 1-STEP LOOK-AHEAD
                                                  23 GROUPS,  8 PREDICTORS
                                            < INDICATES LESS THAN 2 SPLITS WERE MADE

PREDICTOR                                  GROUP NUMBER (* INDICATES THE GROUP IS FINAL)
              1     3     2     4     5     9     8     7    6*    11   10*    17   16*    19   18*   12*   15*   13*   21*   14*

  2001     31.6  10.3   1.9   2.7 *****<  2.0   1.1   1.2   0.3<*****<*****<  1.3   0.3<*****<*****<  0.0<  0.4<  0.5<*****<  0.2<
  1168     18.8   7.2   1.5   1.7   1.4   2.3   0.8   1.0   0.1<  1.2   0.1<  0.9   0.0<  1.2 *****<  0.2<  0.1<  0.2<  0.2<  0.0<
  1009     21.9   6.6   0.8   1.6   1.6   2.2   0.8   0.8   0.0<  0.9   0.2<  0.8   0.0<  1.1   0.3<  0.1<  0.0<  0.2<  0.5<  0.1<
  2002     21.6   7.8   1.3   1.9   1.4   2.2   0.7   0.7   0.0<  0.0<  0.0<  0.7   0.0<  0.0<  0.0<  0.0<  0.0<  0.0<  0.0<  0.1<
  1506     22.7   7.0   2.0   2.3   2.1   2.5   1.2   0.6   0.0<  1.0   0.3<  0.7   0.0<  0.9   0.3<  0.1<*****<  0.1<  0.1<  0.0<
  1498     19.0   7.7   1.1   2.6   1.9   1.4   0.5   1.1   0.1<  0.8   0.0<  0.9   0.0<  1.0   0.1<  0.0<  0.0<  0.3<  0.2<  0.1<
  1572     19.4   6.9   1.5   2.2   1.7   1.7   1.1   1.3   0.1<  1.0   0.0<  0.9 *****<  1.4   0.1<  0.0<  0.6<  0.4<  0.0<  0.0<
  1490      3.0<  0.7<  0.7<  0.5<  0.2<  0.0<  0.4<  0.6<  0.3<  0.3<  0.0<  0.2<  0.2<  0.0<  0.0<  0.0<  0.5<  0.0<  0.1<  0.1<

PREDICTOR                                  GROUP NUMBER (* INDICATES THE GROUP IS FINAL)
            20*   23*   22*
  2001     0.1<*****<*****<
  1168     0.0<  0.2<  0.2<
  1009     0.1<  0.1<  0.2<
  2002     0.1<  0.0<*****<
  1506     0.1<  0.0<  0.0<
  1498     0.1<  0.3<  0.3<
  1572     0.2<  0.0<  0.0<
  1490     0.1<*****<  0.0<
```

*Comparison of this table with the next one shows where the lookahead reveals more two-split power in selecting an inferior predictor for the first split -*
*See groups 2, 4, 7 for example.*

```
                                         100*RSS/TSS TABLE FOR 0-STEP LOOK-AHEAD
                                                 23 GROUPS,  8 PREDICTORS

PREDICTOR                            GROUP NUMBER (* INDICATES THE GROUP IS FINAL)
                1      3      2      4      5      9      8      7     6*     11    10*     17    16*     19    18*    12*    15*    13*    21*    14*

  2001       23.3    8.3    1.4    1.5******     1.0    0.7    0.8    0.3************      0.8    0.3************      0.0    0.4    0.5******     0.2
  1168        2.0    0.1    0.1    0.1    0.2    0.4    0.1    0.1    0.1    0.1    0.1    0.1    0.0    0.3******     0.2    0.1    0.2    0.2    0.0
  1009        3.3    0.2    0.0    0.1    1.0    0.0    0.1    0.0    0.0    0.6    0.2    0.0    0.0    0.3    0.3    0.1    0.0    0.2    0.5    0.1
  2002        2.9    0.0    0.0    0.0    0.1    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.1
  1506        9.7    3.6    1.2    2.0    1.4    2.3    0.6    0.1    0.0    0.0    0.3    0.1    0.0    0.1    0.3    0.1******     0.1    0.1    0.0
  1498        5.0    0.5    0.6    0.3    0.6    0.5    0.1    0.1    0.1    0.4    0.0    0.1    0.0    0.5    0.1    0.0    0.0    0.3    0.2    0.1
  1572        4.3    2.4    1.0    1.3    1.2    0.7    0.5    0.6    0.1    0.7    0.0    0.2******     1.1    0.1    0.0    0.6    0.4    0.0    0.0
  1490        3.0    0.7    0.7    0.5    0.2    0.0    0.4    0.6    0.3    0.3    0.0    0.2    0.2    0.0    0.0    0.0    0.5    0.0    0.1    0.1

PREDICTOR                            GROUP NUMBER (* INDICATES THE GROUP IS FINAL)
              20*    23*    22*
  2001        0.1************
  1168        0.0    0.2    0.2
  1009        0.1    0.1    0.2
  2002        0.1    0.0******
  1506        0.1    0.0    0.0
  1498        0.1    0.3    0.3
  1572        0.2    0.0    0.0
  1490        0.1******     0.0
```

```
                                        100*BSS/TSS TABLE FOR 0-STEP LOOK-AHEAD
                                              23 GROUPS,  8 PREDICTORS

                                        MAXIMUM BSS REGARDLESS OF ELIGIBILITY

PREDICTOR                                 GROUP NUMBER (* INDICATES THE GROUP IS FINAL)
              1      3      2      4      5      9      8      7      6*    11     10*    17     16*    19     18*    12*    15*    13*    21*    14*

  2001      23.3    8.3    1.4    1.5******     1.0    0.7    0.8    0.3*************    0.8    0.3*************    0.0    0.4    0.5******     0.2
  1168       2.0    0.1    0.1    0.1    0.2    0.4    0.1    0.1    0.1    0.1    0.1    0.1    0.0    0.3******     0.2    0.1    0.2    0.2    0.0
  1009       3.3    0.2    0.0    0.1    1.0    0.0    0.1    0.0    0.0    0.6    0.2    0.0    0.0    0.4    0.3    0.1    0.0    0.2    0.5    0.1
  2002       2.9    0.0    0.0    0.0    0.2    0.0    0.0    0.0    0.0    0.2    0.0    0.0    0.0    0.1    0.1    0.0    0.0    0.0    0.0    0.1
  1506       9.7    3.6    1.2    2.0    1.4    2.3    0.6    0.1    0.0    0.0    0.3    0.1    0.0    0.1    0.3    0.1******     0.1    0.1    0.0
  1498       5.0    0.5    0.6    0.3    0.6    0.5    0.1    0.1    0.1    0.4    0.0    0.1    0.0    0.5    0.1    0.0    0.0    0.3    0.2    0.1
  1572       4.3    2.4    1.0    1.3    1.2    0.7    0.5    0.6    0.1    0.7    0.0    0.2******     1.1    0.1    0.0    0.6    0.4    0.0    0.0
  1490       3.0    0.7    0.7    0.5    0.2    0.2    0.4    0.6    0.3    0.3    0.0    0.2    0.2    0.1    0.2    0.0    0.5    0.3    0.1    0.1

PREDICTOR                                 GROUP NUMBER (* INDICATES THE GROUP IS FINAL)
             20*    23*    22*
  2001       0.1*************
  1168       0.0    0.2    0.2
  1009       0.1    0.1    0.5
  2002       0.1    0.0******
  1506       0.1    0.0    0.0
  1498       0.1    0.3    0.3
  1572       0.2    0.0    0.0
  1490       0.1******     0.0
```

```
                                 PROFILE OF CLASS MEANS AND SLOPES


                        3-YR AVE $ INC      , ETA = 0.351
                                             GROUP
CLASS         1         3         2         4         5         9         8         7         6*       11
  1 N       256         0       256         0         0         0         0       115       141         0
    YMEAN  1.053E 04 0.0       1.053F 04 0.0       0.0       0.0       0.0       1.174E 04 9.661E 03 0.0
  2 N       257         0       257         0         0         0         0       143       114         0
    YMEAN  1.323E 04 0.0       1.323E 04 0.0       0.0       0.0       0.0       1.475E 04 1.164E 04 0.0
  3 N       386         0       386         0         0         0         0       245       141         0
    YMEAN  1.408E 04 0.0       1.408E 04 0.0       0.0       0.0       0.0       1.510E 04 1.265E 04 0.0
  4 N       362         0       362         0         0         0         0       257       105         0
    YMEAN  1.724E 04 C.0       1.724E 04 0.0       u.0       0.0       0.0       1.853E 04 1.442E 04 0.0
  5 N       497       497         0       497         0       231       266         0         0         0
    YMEAN  2.259E 04 2.259E 04 0.0       2.259E 04 0.0       2.372E 04 2.157E 04 0.0       0.0       0.0
  6 N       208       208         0       208         0        81       127         0         0         0
    YMEAN  2.799E 04 2.799E 04 0.0       2.799E 04 0.0       3.047E 04 2.636E 04 0.0       0.0       0.0
  7 N       122       122         0         0       122         0         0         0         0        86
    YMEAN  3.904E 04 3.904E 04 0.0       0.0       3.904E 04 0.0       0.0       0.0       0.0       4.244E 04


                        3-YR AVE $ INC      , ETA = 0.351
                                             GROUP
CLASS        10*       17        16*       19        18*       12*       15*       13*       21*       14*
  1 N         0        54        61         0         0         0         0         0         0         0
    YMEAN  0.0       1.296E 04 9.710E 03 0.0       0.0       0.0       0.0       0.0       0.0       0.0
  2 N         0        83        60         0         0         0         0         0         0         0
    YMEAN  0.0       1.636E 04 9.727E 03 0.0       0.0       0.0       0.0       0.0       0.0       0.0
  3 N         0       145       100         0         0         0         0         0         0         0
    YMEAN  0.0       1.505F 04 1.526E 04 0.0       0.0       0.0       0.0       0.0       0.0       0.0
  4 N         0       181        76         0         0         0         0         0       181         0
    YMEAN  0.0       1.576E 04 1.450E 04 0.0       0.0       0.0       0.0       0.0       1.976E 04 0.0
  5 N         0         0         0         0         0       153       112        78         0       154
    YMEAN  0.0       0.0       0.0       0.0       0.0       2.135E 04 2.346E 04 2.822E 04 0.0       2.038E 04
  6 N         0         0         0         0         0        35        63        46         0        64
    YMEAN  0.0       0.0       0.0       0.0       0.0       2.347E 04 2.910E 04 3.528E 04 0.0       2.371E 04
  7 N        36         0         0        65        21         0         0         0         0         0
    YMEAN  3.029E 04 0.0       0.0       4.337E 04 3.940E 04 0.0       0.0       0.0       0.0       0.0


                        3-YR AVE $ INC      , ETA = 0.351
                                             GROUP
CLASS        20*       23*       22*
  1 N        54         0         0
    YMEAN  1.296E 04 0.0       0.0
  2 N        83         0         0
    YMEAN  1.636E 04 0.0       0.0
  3 N       145         0         0
    YMEAN  1.505E 04 0.0       0.0
  7 N         0        31        34
    YMEAN  0.0       4.982E 04 3.680E 04


                        # REQUIRED ROOMS 23:12   , ETA = 0.024
                                             GROUP
CLASS         1         3         2         4         5         9         8         7         6*       11
  2 N       661       186       475       155        31        73        82       256       219        21
    YMEAN  1.762E 04 2.546E 04 1.413E 04 2.347E 04 3.546E 04 2.540E 04 2.182E 04 1.561E 04 1.236E 04 3.940E 04
  3 N       557       262       295       232        30       102       130       180       115        23
    YMEAN  2.122E 04 2.676E 04 1.481E 04 2.457E 04 4.352E 04 2.591E 04 2.353E 04 1.655E 04 1.230E 04 4.793E 04
  4  N         486         728         258         187          41          88          99         165          93          28
     YMEAN  2.256E 04   2.803E 04   1.456F 04   2.549E 04   3.843E 04   2.767E 04   2.357E 04   1.626E 04   1.208F 04   4.156E 04
  5  N         225          84         141          72          12          34          38          96          45           9
     YMEAN  1.999E 04   2.590F 04   1.308E 04   2.316E 04   3.981E 04   2.231E 04   2.403E 04   1.570E 04   9.273E 03   3.937E 04
  6  N         159          67          92          59           8          15          44          63          29           5
     YMEAN  1.998E 04   2.442E 04   1.207E 04   2.197E 04   3.780E 04   1.807E 04   2.367E 04   1.297E 04   1.029E 04   4.004E 04


                              # REQUIRED ROOMS 23:12  , ETA = 0.024
                                                GROUP
CLASS          10*          17         16*          19         18*         12*         15*         13*         21*         14*
  2  N          10         177          79           0          21          51          32          22          38          50
     YMEAN  2.707E 04   1.671E 04   1.236E 04   0.0         3.940E 04   2.310E 04   2.309E 04   3.024E 04   2.294E 04   2.100E 04
  3  N           7         107          73          23           0          65          52          37          48          78
     YMEAN  2.755E 04   1.756E C4   1.403E 04   4.793E 04   0.0         2.132E 04   2.585E 04   3.283E 04   2.002E 04   2.200E 04
  4  N          13          93          72          28           0          43          43          45          51          56
     YMEAN  3.080E 04   1.739E 04   1.326E 04   4.156E 04   0.0         2.349E 04   2.656E 04   3.138E 04   1.864F 04   2.118E 04
  5  N           3          55          41           9           0          22          20          12          29          18
     YMEAN  4.125E 04   1.698F 04   1.112E 04   3.937E 04   0.0         1.901E 04   2.714E 04   2.882E 04   1.760F 04   2.132E 04
  6  N           3          31          32           5           0           7          28           8          15          16
     YMEAN  3.430E 04   1.252E 04   1.398E 04   4.004E 04   0.0         1.431E 04   2.709E 04   2.229F 04   1.269E 04   2.011E 04


                              # REQUIRED ROOMS 23:12  , ETA = 0.024
                                                GROUP
CLASS          20*         23*         22*
  2  N         135           C           0
     YMEAN  1.491E C4   0.0         0.0
  3  N          59          12          11
     YMEAN  1.511E 04   5.154E 04   4.324F 04
  4  N          42          12          16
     YMEAN  1.519E 04   5.059F 04   3.366E 04
  5  N          26           4           5
     YMEAN  1.598E 04   5.015E 04   3.091E 04
  6  N          16           3           2
     YMEAN  1.232E 04   3.651E C4   4.497F 04


                              BKT AGE HEAD 9V1009     , ETA = 0.049
                                                GROUP
CLASS           1           3           2           4           5           9           8           7          6*          11
  2 .N         372         152         220         140          12          53          87         145          75           9
     YMEAN  1.987E 04   2.574E C4   1.404E 04   2.379E 04   4.556E 04   2.641E 04   2.207E 04   1.507E 04   1.235E 04   4.944E 04
  3  N         530         276         254         238          38         114         124         175          79          29
     YMEAN  2.370E 04   2.844E 04   1.502F 04   2.561E 04   4.415E 04   2.609E 04   2.513E 04   1.678E 04   1.181E 04   4.639E 04
  4  N         479         243         236         200          43          89         111         149          87          30
     YMEAN  2.134E 04   2.511E 04   1.413E 04   2.315E 04   3.384E 04   2.417E 04   2.234E 04   1.621E 04   1.127E 04   3.689E 04
  5  N         378         123         255          97          26          41          56         142         113          17
     YMEAN  1.908E 04   2.625E 04   1.442E 04   2.347E 04   3.639E 04   2.581E 04   2.183E 04   1.559E 04   1.303E 04   3.941E 04
  6  N         329          33         296          30           3          15          15         149         147           1
    .YMEAN  1.530E 04   2.786E C4   1.388E 04   2.647E 04   4.252E 04   2.643E 04   2.651E 04   1.590E 04   1.167E 04   7.500E 04


                              BKT AGE HEAD 9V1009     , ETA = 0.049
                                                GROUP
CLASS          10*          17         16*          19         18*         12*         15*         13*         21*         14*
  2  N           3          84          61           8           1          34          35          19          49          52
     YMEAN  3.191E 04   1.601E 04   1.271E 04   5.154E 04   3.000E 04   2.220E 04   2.385E 04   3.321E 04   1.772E 04   2.105E 04
  3  N           9         113          62          25           4          60          64          54          59          60
     YMEAN  3.628E 04   1.761E 04   1.322E 04   4.485E 04   5.657E 04   2.037E 04   2.756E 04   3.233E 04   1.927E 04   2.287E 04
  4  N          13          72          77          25           5          55          45          34          32          66
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      YMEAN  2.625E 04   1.756E 04   1.313E 04   3.654E 04   3.861E 04   2.117E 04   2.592E 04   2.853E 04   2.018E 04   1.997E 04
  5   N          9          85          57           6          11          25          25          16          29          31
      YMEAN  3.067E 04   1.644E 04   1.356E 04   4.805E 04   3.425E 04   2.285E 04   2.323E 04   2.999E 04   1.869E 04   2.073E 04
  6   N          2         109          40           1           0          14           6           1          12           9
      YMEAN  2.500E 04   1.705E 04   1.258E 04   7.500E 04   0.0         2.683E 04   2.687E 04   2.100E 04   3.046E 04   2.626E 04


                                 BKT AGE HEAD 9V1009      , ETA = 0.049
                                                           GROUP
CLASS           20*         23*         22*
  2   N         35           6           2
      YMEAN  1.243E 04   5.388F 04   4.300E 04
  3   N         54          13          12
      YMEAN  1.545E 04   4.755E C4   4.170E 04
  4   N         40           9          16
      YMEAN  1.462E 04   4.759E C4   2.942E 04
  5   N         56           3           3
      YMEAN  1.500E 04   5.797E C4   3.838E 04
  6   N         97           0           1
      YMEAN  1.542E 04   C.0         7.500E 04


                                 SEX & MAR STATUS         , ETA = 0.029
                                                           GROUP
CLASS            1           3           2           4           5           9           8           7           6*         11
  1   N        1660         782         878         664         118         297         367         501         377          84
      YMEAN  2.109E 04   2.665E 04   1.435E 04   2.433E 04   3.883E 04   2.569E 04   2.321E 04   1.609E 04   1.218E 04   4.209E 04
  2   N         82          17          65          14           3           7           7          41          24           1
      YMEAN  1.443E 04   2.296E C4   1.171F 04   1.916E 04   4.032E 04   2.052E 04   1.759E 04   1.161E 04   1.184E 04   5.500E 04
  3   N        346          28         318          27           1           8          19         218         100           1
      YMEAN  1.580E 04   2.686E C4   1.450F 04   2.546E 04   6.000E 04   2.596E 04   2.522E 04   1.630E 04   1.162E 04   6.000E 04


                                 SEX & MAR STATUS         , ETA * 0.029
                                                           GROUP
CLASS           10*         17          16*         19          18*         12*         15*         13*         21*         14*
  1   N         34         296         205          64          20         179         167         118         157         200
      YMEAN  3.016E 04   1.727E 04   1.309E 04   4.312E 04   3.863E 04   2.194E 04   2.583E 04   3.100E 04   1.978E 04   2.119E 04
  2   N          2          23          18           0           1           5           0           2           3           7
      YMEAN  3.258E 04   1.309E 04   8.208E 03   0.0         5.500E 04   1.582E 04   0.0         3.263E 04   1.814E 04   1.759E 04
  3   N          0         144          74           1           0           4           8           4          21          11
      YMEAN  0.0         1.686E 04   1.418F 04   6.000E 04   0.0         2.107E 04   2.187E 04   3.026E 04   1.984E 04   2.677E 04


                                 SEX & MAR STATUS         , ETA = 0.029
                                                           GROUP
CLASS           20*         23*         22*
  1   N        139          30          34
      YMEAN  1.428E 04   4.551E C4   3.680E 04
  2   N         20           0           0
      YMEAN  1.223E 04   C.0         0.0
  3   N        123           1           0
      YMEAN  1.636E 04   6.0C0E 04   0.0


                                 LRGST PLAC/SMSA PSU31:66, ETA = 0.123
                                                           GROUP
CLASS            1           3           2           4           5           9           8           7           6*         11
  1   N        633         326         307         266          60          91         175         307           0          60
      YMEAN  2.583E C4   3.100E 04   1.673E 04   2.800E 04   4.276E 04   3.187E 04   2.570E 04   1.673E 04   0.0         4.276E 04
  2   N        443         212         231         186          26          33         153         231           0          26
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     YMEAN   2.154E C4    2.542E 04    1.622F 04    2.306E 04    4.169E 04    2.849E 04    2.189E 04    1.622E 04    0.0          4.169E 04
  3  N         222           90          132           78           12           18           60          132            0            0
     YMEAN   1.855E 04    2.305F 04    1.463E 04    2.164E 04    3.195E 04    2.590E 04    2.035E 04    1.463E 04    0.0          0.0
  4  N         144           54           90           43           11           43            0           90            0            0
     YMEAN   1.846E 04    2.369E 04    1.495E 04    2.182E 04    3.082E 04    2.182E 04    0.0          1.495E 04    0.0          0.0
  5  N         200           60          140           52            8           51            1            0          140            0
     YMEAN   1.578E 04    2.248E 04    1.245E 04    2.068E 04    3.462E 04    2.069E 04    2.000E 04    0.0          1.245E 04    0.0
  6  N         446           85          361           80            5           76            4            0          361            0
     YMEAN   1.394E 04    2.101E 04    1.185E 04    2.120E 04    1.809E 04    2.141E 04    1.768E 04    0.0          1.185E 04    0.0


                        LRGST PLAC/SMSA PSU31:66, ETA = 0.123
                                                    GROUP
CLASS           10*          17           16*          19           18*          12*          15*          13*          21*          14*
  1  N           0          217           90           48           12            0          175           91           78            0
     YMEAN   0.0          1.760E 04    1.198E 04    4.224E 04    4.498E 04    0.0          2.570E 04    3.187E 04    1.980E 04    0.0
  2  N           0          121          110           17            9            0            0           33           57          153
     YMEAN   0.0          1.742E 04    1.389E 04    4.673E 04    3.192E 04    0.0          0.0          2.849E 04    2.120E 04    2.189E 04
  3  N          12           63           69            0            0           18            0            0           27           60
     YMEAN   3.195E 04    1.585E C4    1.261E 04    0.0          0.0          2.590E 04    0.0          0.0          1.728E 04    2.035E 04
  4  N          11           62           28            0            0           43            0            0           19            0
     YMEAN   3.082E 04    1.556E C4    1.276E 04    0.0          0.0          2.182F 04    0.0          0.0          1.954E 04    0.0
  5  N           8            0            0            0            0           51            0            0            0            1
     YMEAN   3.462E 04    0.0          0.0          0.0          0.0          2.069E 04    0.0          0.0          0.0          2.000E 04
  6  N           5            0            0            0            0           76            0            0            0            4
     YMEAN   1.809E 04    0.0          0.0          0.0          0.0          2.141E 04    0.0          0.0          0.0          1.768E 04


                        LRGST PLAC/SMSA PSU31:66, ETA * 0.123
                                                    GROUP
CLASS           20*          23*          22*
  1  N         139           21           27
     YMEAN   1.611E 04    4.899E 04    3.619E 04
  2  N          64           10            7
     YMEAN   1.407E 04    5.176E C4    3.914F 04
  3  N          36            0            0
     YMEAN   1.480E 04    0.0          0.0
  4  N          43            0            0
     YMEAN   1.379E 04    0.0          0.0


                        DIST TO CNTR SMSA  31:58, ETA = 0.072
                                                    GROUP
CLASS            1            3            2            4            5            9            8            7            6*          11
  1  N         386          155          231          136           19            0          136          214           17           15
     YMEAN   1.885E 04    2.390E 04    1.386F 04    2.218E 04    3.492E 04    0.0          2.218E 04    1.446E 04    8.984E 03    3.731E 04
  2  N         582          306          276          257           49            0          257          271            5           45
     YMEAN   2.280E 04    2.656E C4    1.670F 04    2.373E 04    4.035E 04    0.0          2.373E 04    1.681E 04    9.170E 03    4.085E 04
  3  N         336          170          166          144           26          144            0          127           39           20
     YMEAN   2.504E 04    3.104F 04    1.617E 04    2.797E 04    4.669E 04    2.797F 04    0.0          1.764E 04    1.247E 04    5.072E 04
  4  N         245           69          176           62            7           62            0           83           93            3
     YMEAN   1.739E 04    2.404E C4    1.392E 04    2.293E 04    3.361E 04    2.293E 04    0.0          1.481E 04    1.296E 04    3.702E 04
  5  N         539          127          412          106           21          106            0           65          347            3
     YMEAN   1.597E 04    2.516E 04    1.243E 04    2.387E 04    3.146E 04    2.387E 04    0.0          1.494E 04    1.198E 04    4.000E 04


                        DIST TO CNTR SMSA  31:58, ETA = 0.072
                                                    GROUP
CLASS           10*          17           16*          19           18*          12*          15*          13*          21*          14*
  1  N           4          115           99           12            3            0           57            0           37           79
     YMEAN   2.566E 04    1.533F 04    1.289E 04    3.422E 04    4.983E 04    0.0          2.522E 04    0.0          1.616E 04    2.023E 04
  2  N          4          182           89           32           13            0          118            0           84          139
     YMEAN  3.477E 04   1.766E 04    1.354E 04    4.301E 04    3.544E 04    0.0          2.592E 04    0.0          1.991E 04    2.203E 04
  3  N          6           70           57           15            5           42            0          102           27            0
     YMEAN  3.174E 04   1.953E C4    1.406E 04    5.281E 04    4.348E 04    2.133E 04    0.0          3.054E 04    2.262E 04    0.0
  4  N          4           52           31            3            0           47            0           15           22            0
     YMEAN  3.072E 04   1.622E 04    1.106E 04    3.702E 04    0.0          2.058E 04    0.0          2.935E 04    2.157E 04    0.0
  5  N         18           44           21            3            0           99            0            7           11            0
     YMEAN  2.973E 04   1.531E 04    1.116E 04    4.000E 04    0.0          2.250E 04    0.0          4.071E 04    1.820E 04    0.0


                                   DIST TO CNTR SMSA  31:58, ETA = 0.072
                                                           GROUP
CLASS          20*          23*          22*
  1  N         78            4            8
     YMEAN  1.494E 04   4.722E 04    2.804E 04
  2  N         98           15           17
     YMEAN  1.546E 04   4.535E 04    4.067E 04
  3  N         43            8            7
     YMEAN  1.731E 04   6.213E 04    4.055E 04
  4  N         30            1            2
     YMEAN  1.187E 04   5.000E 04    2.850E 04
  5  N         33            3            0
     YMEAN  1.432E 04   4.000E C4    0.0


                                   CURRENT REGION 0V472     , ETA = 0.063
                                                           GROUP
CLASS           1            3            2            4            5            9            8            7           6*           11
  1  N        364          196          168          164           32           79           85          152           16           28
     YMEAN  2.494E 04   3.133E 04    1.611E 04    2.812E 04    4.694E 04    2.963E 04    2.669E 04    1.624E 04    1.495E 04    4.862E 04
  2  N        617          288          329          242           46          105          137          185          144           33
     YMEAN  1.988E 04   2.526E 04    1.453E 04    2.319E 04    3.558E 04    2.443E 04    2.223E 04    1.629E 04    1.271E 04    3.799E 04
  3  N        790          193          597          168           25           88           80          297          300           12
     YMEAN  1.595E 04   2.364E 04    1.185E 04    2.137E 04    3.714E 04    2.281E 04    1.982E 04    1.300E 04    1.103E 04    4.297E 04
  4  N        317          150          167          131           19           40           91          126           41           13
     YMEAN  2.133E 04   2.557E 04    1.710E 04    2.422E 04    3.457E 04    2.503E 04    2.381F 04    1.890E 04    1.319E 04    3.755E 04


                                   CURRENT REGION 0V472     , ETA = 0.063
                                                           GROUP
CLASS          10*          17          16*           19          18*          12*          15*          13*          21*          14*
  1  N          4          152            0           24            4           36           48           43           55           37
     YMEAN  3.225E 04   1.624E 04    0.0          5.013E 04    3.982E 04    2.274E 04    2.986E 04    3.452E 04    1.992E 04    2.258E 04
  2  N         13          185            0           24            9           68           58           37           77           79
     YMEAN  2.907E 04   1.629E 04    0.0          3.627E 04    4.278E 04    2.221E 04    2.435E 04    2.851E 04    1.905E 04    2.080E 04
  3  N         13            0          297            7            5           64           25           24            0           55
     YMEAN  3.158E 04   0.0          1.300E 04    4.862E 04    3.502E 04    2.052E 04    1.684E 04    2.892E 04    0.0          2.084E 04
  4  N          6          126            0           10            3           20           44           20           49           47
     YMEAN  2.896E 04   1.890E 04    0.0          3.819E 04    3.504E 04    2.190E 04    2.618E 04    2.860E 04    2.078E 04    2.197E 04


                                   CURRENT REGION 0V472     , ETA = 0.063
                                                           GROUP
CLASS          20*          23*          22*
  1  N         97           24            0
     YMEAN  1.390E 04   5.013E 04    0.0
  2  N        108            0           24
     YMEAN  1.403E 04   0.0          3.627E 04
  3  N          0            7            0
     YMEAN  0.0         4.862E 04    0.0
  4  N         77            0           10
```

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    YMEAN    1.776E 04   0.0         3.819E 04

                              RACE               31:48, ETA = 0.030
                                                      GROUP
CLASS              1           3           2           4           5           9           8           7          6*          11
  1 N           1626         754         872         636         118         301         335         470         402          83
    YMEAN    2.069E 04   2.685E 04   1.470E 04   2.446E 04   3.945E 04   2.549E 04   2.357E 04   1.652E 04   1.246E 04   4.302E 04
  2 N            406          57         349          54           3           9          45         256          93           3
    YMEAN    1.178E 04   1.833E 04   9.653E 03   1.707E 04   2.758E 04   2.301E 04   1.551E 04   1.080E 04   7.418E 03   2.758E 04
  3 N             56          16          40          15           1           2          13          34           6           0
    YMEAN    1.987E 04   2.804E 04   1.533E 04   2.912E 04   2.700E 04   4.095E 04   2.582E 04   1.660E 04   8.772E 03   0.0

                              RACE               31:48, ETA = 0.030
                                                      GROUP
CLASS            10*          17         16*          19         18*         12*         15*         13*         21*         14*
  1 N             35         346         124          63          20         183         139         118         143         196
    YMEAN    3.038E 04   1.727E 04   1.396E 04   4.377E 04   4.050E 04   2.171E 04   2.645E 04   3.082E 04   2.016E 04   2.146E 04
  2 N              0          91         165           2           1           4          31           5          28          14
    YMEAN    0.0         1.231E 04   9.653E 03   3.186E 04   1.800E 04   2.123E 04   1.391E 04   2.651E 04   1.220E 04   1.690E 04
  3 N              1          26           8           0           0           1           5           1          10           8
    YMEAN    2.700E 04   1.686E 04   1.591E 04   0.0         0.0         2.800E 04   2.509E 04   5.500E 04   1.777E 04   2.609E 04

                              RACE               31:48, ETA = 0.030
                                                      GROUP
CLASS            20*         23*         22*
  1 N            203          31          32
    YMEAN    1.515E 04   4.982E 04   3.715E 04
  2 N             63           0           2
    YMEAN    1.235E 04   0.0         3.186E 04
  3 N             16           0           0
    YMEAN    1.635E 04   0.0         0.0
```

*This table gives much the same as the previous table but in a different more cumbersome format*
*We omit 21 pages of it.*

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                              PREDICTOR SUMMARY TABLE.  23 GROUPS

                     3-YR AVE $ INC           , ETA =

GROUP CLASS          N         W  PCT   PCT        SUM        PCT   PCT        MEAN         VARIANCE           SLOPE
  1                              GROUP TOTAL                 GROUP TOTAL
     1 Y           256    8805.0  10.8  10.8  9.271272E 07   5.7   5.7  1.052955E 04  4.233778E 07
     2 Y           257    7736.0   9.5   9.5  1.023791E 08   6.3   6.3  1.323411E 04  7.076661E 07
     3 Y           386   12437.0  15.2  15.2  1.751543E 08  10.7  10.7  1.408332E 04  6.077859E 07
     4 Y           362   14035.0  17.2  17.2  2.420264E 08  14.8  14.8  1.724449E 04  7.744030E 07
     5 Y           497   22398.0  27.5  27.5  5.060375E 08  30.9  30.9  2.259298E 04  1.189778E 08
     6 Y           208   10121.0  12.4  12.4  2.832548E 08  17.3  17.3  2.798684E 04  1.206158E 08
     7 Y           122    6043.0   7.4   7.4  2.359240E 08  14.4  14.4  3.904088E 04  3.372114E 08
       ETA = 0.351, ETA(NSTEP) = 0.316, ETA(0) = 0.233

GROUP CLASS          N         W  PCT   PCT        SUM        PCT   PCT        MEAN         VARIANCE           SLOPE
  3                              GROUP TOTAL                 GROUP TOTAL
     5 Y           497   22398.0  58.1  27.5  5.060375E 08  49.4  30.9  2.259298E 04  1.189778E 08
     6 Y           208   10121.0  26.2  12.4  2.832548E 08  27.6  17.3  2.798684E 04  1.206158E 08
     7 Y           122    6043.0  15.7   7.4  2.359240E 08  23.0  14.4  3.904088E 04  3.372114E 08
       ETA = 0.182, ETA(NSTEP) = 0.191, ETA(0) = 0.154

GROUP CLASS          N         W  PCT   PCT        SUM        PCT   PCT        MEAN         VARIANCE           SLOPE
  2                              GROUP TOTAL                 GROUP TOTAL
     1 Y           256    8805.0  20.5  10.8  9.271272E 07  15.1   5.7  1.052955E 04  4.233778E 07
     2 Y           257    7736.0  18.0   9.5  1.023791E 08  16.7   6.3  1.323411E 04  7.076661E 07
     3 Y           386   12437.0  28.9  15.2  1.751543E 08  28.6  10.7  1.408332E 04  6.077859E 07
     4 Y           362   14035.0  32.6  17.2  2.420264E 08  39.5  14.8  1.724449E 04  7.744030E 07
       ETA = 0.085, ETA(NSTEP) = 0.085, ETA(0) = 0.063

GROUP CLASS          N         W  PCT   PCT        SUM        PCT   PCT        MEAN         VARIANCE           SLOPE
  4                              GROUP TOTAL                 GROUP TOTAL
     5 Y           497   22398.0  69.9  27.5  5.060375E 08  64.1  30.9  2.259298E 04  1.189778E 08
     6 Y           208   10121.0  31.1  12.4  2.832548E 08  35.9  17.3  2.798684E 04  1.206158E 08
       ETA = 0.050, ETA(NSTEP) = 0.073, ETA(0) = 0.050

GROUP CLASS          N         W  PCT   PCT        SUM        PCT   PCT        MEAN         VARIANCE           SLOPE
  5                              GROUP TOTAL                 GROUP TOTAL
     7 Y           122    6043.0 100.0   7.4  2.359240E 08 100.0  14.4  3.904088E 04  3.372114E 08

GROUP CLASS          N         W  PCT   PCT        SUM        PCT   PCT        MEAN         VARIANCE           SLOPE
  9                              GROUP TOTAL                 GROUP TOTAL
     5 Y           231   10615.0  72.6  13.0  2.518379E 08  67.3  15.4  2.372472E 04  1.590032E 08
     6 Y            81    4012.0  27.4   4.9  1.222502E 08  32.7   7.5  3.047113E 04  1.500305E 08
       ETA = 0.055, ETA(NSTEP) = 0.110, ETA(0) = 0.055

GROUP CLASS          N         W  PCT   PCT        SUM        PCT   PCT        MEAN         VARIANCE           SLOPE
  8                              GROUP TOTAL                 GROUP TOTAL
     5 Y           266   11783.0  65.9  14.4  2.541996E 08  61.2  15.5  2.157342E 04  8.119678E 07
     6 Y           127    6109.0  34.1   7.5  1.610046E 08  38.8   9.8  2.635531E 04  9.559144E 07
       ETA = 0.057, ETA(NSTEP) = 0.089, ETA(0) = 0.057

GROUP CLASS          N         W  PCT   PCT        SUM        PCT   PCT        MEAN         VARIANCE           SLOPE
  7                              GROUP TOTAL                 GROUP TOTAL
     1 Y           115    3681.0  15.0   4.5  4.321102E 07  11.1   2.6  1.173893E 04  4.820251E 07
     2 Y           143    3964.0  16.2   4.9  5.848483E 07  15.0   3.6  1.475399E 04  6.323427E 07
     3 Y           245    7259.0  29.6   8.9  1.096388E 08  28.1   6.7  1.510385E 04  5.778098E 07
     4 Y           257    9639.0  39.3  11.8  1.786229E 08  45.8  10.9  1.853127E 04  8.392960E 07
       ETA = 0.078, ETA(NSTEP) = 0.087, ETA(0) = 0.062
```

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GROUP CLASS          N           W  PCT    PCT          SUM        PCT    PCT        MEAN          VARIANCE             SLOPE
  6*                                GROUP  TOTAL                   GROUP  TOTAL
      1 Y          141      5124.0  27.7    6.3  4.950168E 07  22.3    3.0  9.660746E 03  3.659565E 07
      2 Y          114      3772.0  20.4    4.6  4.389426E 07  19.7    2.7  1.163686E 04  7.425475E 07
      3 Y          141      5178.0  28.0    6.3  6.551544E 07  29.5    4.0  1.265265E 04  6.186522E 07
      4 Y          105      4396.0  23.8    5.4  6.340354E 07  28.5    3.9  1.442300E 04  5.214976E 07
        ETA = 0.053, ETA(NSTEP) = 0.038, ETA(0) = 0.038

GROUP CLASS          N           W  PCT    PCT          SUM        PCT    PCT        MEAN          VARIANCE             SLOPE
  11                                GROUP  TOTAL                   GROUP  TOTAL
      7 Y           86      4351.0 100.0    5.3  1.846700E 08 100.0   11.3  4.244312E 04  3.295327E 08

GROUP CLASS          N           W  PCT    PCT          SUM        PCT    PCT        MEAN          VARIANCE             SLOPE
  10*                               GROUP  TOTAL                   GROUP  TOTAL
      7 Y           36      1692.0 100.0    2.1  5.125400E 07 100.0    3.1  3.029196E 04  2.577897E 08

GROUP CLASS          N           W  PCT    PCT          SUM        PCT    PCT        MEAN          VARIANCE             SLOPE
  17                                GROUP  TOTAL                   GROUP  TOTAL
      1 Y           54      2301.0  12.7    2.8  2.981147E 07   9.8    1.8  1.295588E 04  4.735947E 07
      2 Y           83      3003.0  16.6    3.7  4.913746E 07  16.1    3.0  1.636279E 04  6.729133E 07
      3 Y          145      5368.0  29.7    6.6  8.078443E 07  26.4    4.9  1.504926E 04  5.267962E 07
      4 Y          181      7389.0  40.9    9.1  1.459888E 08  47.8    8.9  1.975759E 04  9.629045E 07
        ETA = 0.082, ETA(NSTEP) = 0.123, ETA(0) = 0.071

GROUP CLASS          N           W  PCT    PCT          SUM        PCT    PCT        MEAN          VARIANCE             SLOPE
  16*                               GROUP  TOTAL                   GROUP  TOTAL
      1 Y           61      1380.0  21.3    1.7  1.339954E 07  15.9    0.8  9.709813E 03  4.408869E 07
      2 Y           60       961.0  14.8    1.2  9.347370E 06  11.1    0.6  9.726711E 03  1.820810E 07
      3 Y          100      1891.0  29.2    2.3  2.885440E 07  34.3    1.8  1.525880E 04  7.308680E 07
      4 Y           76      2250.0  34.7    2.8  3.263403E 07  38.7    2.0  1.450401E 04  2.282747E 07
        ETA = 0.131, ETA(NSTEP) = 0.129, ETA(0) = 0.129

GROUP CLASS          N           W  PCT    PCT          SUM        PCT    PCT        MEAN          VARIANCE             SLOPE
  19                                GROUP  TOTAL                   GROUP  TOTAL
      7 Y           65      3333.0 100.0    4.1  1.445618E 08 100.0    8.8  4.337288E 04  3.608504E 08

GROUP CLASS          N           W  PCT    PCT          SUM        PCT    PCT        MEAN          VARIANCE             SLOPE
  18*                               GROUP  TOTAL                   GROUP  TOTAL
      7 Y           21      1018.0 100.0    1.2  4.010819E 07 100.0    2.4  3.939901E 04  2.275338E 08

GROUP CLASS          N           W  PCT    PCT          SUM        PCT    PCT        MEAN          VARIANCE             SLOPE
  12*                               GROUP  TOTAL                   GROUP  TOTAL
      5 Y          153      6943.0  80.9    8.5  1.482040E 08  79.4    9.1  2.134582E 04  1.530113E 08
      6 Y           35      1635.0  19.1    2.0  3.838011E 07  20.6    2.3  2.347407E 04  7.677040E 07
        ETA = 0.005, ETA(NSTEP) = 0.005, ETA(0) = 0.005

GROUP CLASS          N           W  PCT    PCT          SUM        PCT    PCT        MEAN          VARIANCE             SLOPE
  15*                               GROUP  TOTAL                   GROUP  TOTAL
      5 Y          112      4555.0  60.3    5.6  1.068696E 08  55.0    6.5  2.346204E 04  1.155645E 08
      6 Y           63      3001.0  39.7    3.7  8.732608E 07  45.0    5.3  2.909899E 04  1.204836E 08
        ETA = 0.061, ETA(NSTEP) = 0.061, ETA(0) = 0.061

GROUP CLASS          N           W  PCT    PCT          SUM        PCT    PCT        MEAN          VARIANCE             SLOPE
  13*                               GROUP  TOTAL                   GROUP  TOTAL
      5 Y           78      3672.0  60.7    4.5  1.036338E 08  55.3    6.3  2.822272E 04  1.411110E 08
      6 Y           46      2377.0  39.3    2.9  8.387005E 07  44.7    5.1  3.528399E 04  1.451202E 08
        ETA = 0.078, ETA(NSTEP) = 0.078, ETA(0) = 0.078

GROUP CLASS          N           W  PCT    PCT          SUM        PCT    PCT        MEAN          VARIANCE             SLOPE
  21*                               GROUP  TOTAL                   GROUP  TOTAL
      4 Y          181      7399.0 100.0    9.1  1.459888E 09 100.0    8.9  1.975759E 04  9.629045E 07
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GROUP CLASS          N           W  PCT   PCT           SUM        PCT   PCT          MEAN           VARIANCE            SLOPE
  14*                              GROUP TOTAL                    GROUP TOTAL
       5 Y         154      7228.0  69.9   8.9  1.473300E 08  66.7   9.0  2.038323E 04  5.639320E 07
       6 Y          64      3108.0  30.1   3.8  7.367850E 07  33.3   4.5  2.370608E 04  5.855110E 07
         ETA = 0.039, ETA(NSTEP) = 0.039, ETA(0) = 0.039

GROUP CLASS          N           W  PCT   PCT           SUM        PCT   PCT          MEAN           VARIANCE            SLOPE
  20*                              GROUP TOTAL                    GROUP TOTAL
       1 Y          54      2301.0  21.6   2.8  2.981149E 07  18.7   1.8  1.295588E 04  4.735947E 07
       2 Y          83      3003.0  28.1   3.7  4.913747E 07  30.8   3.0  1.636279E 04  6.729176E 07
       3 Y         145      5368.0  50.3   6.6  8.078445E 07  50.6   4.9  1.504926E 04  5.267962E 07
         ETA = 0.025, ETA(NSTEP) = 0.020, ETA(0) = 0.020

GROUP CLASS          N           W  PCT   PCT           SUM        PCT   PCT          MEAN           VARIANCE            SLOPE
  23*                              GROUP TOTAL                    GROUP TOTAL
       7 Y          31      1683.0 100.0   2.1  8.384981E 07 100.0   5.1  4.982163E 04  3.220803E 08

GROUP CLASS          N           W  PCT   PCT           SUM        PCT   PCT          MEAN           VARIANCE            SLOPE
  22*                              GROUP TOTAL                    GROUP TOTAL
       7 Y          34      1650.0 100.0   2.0  6.071200E 07 100.0   3.7  3.679515E 04  3.236129E 08
```

```
SECOND STAGE OF 807NTR45 MORGAN

      2088 OBSERVATIONS READ AFTER GLOBAL FILTER

Y AVERAGE =          -1.306527E-01
STANDARD DEVIATION =  9.914672E 03
BOUNDARIES =  -4.957349E 04   4.957323E 04


   1. ID =      53249,  Y =  5.324900E 04
   2. ID =      55242,  Y =  5.524200E 04
   3. ID =      53249,  Y =  5.324900E 04
```

*The second run on the residuals generated in the first run.*

```
     2088 CASES INCLUDED IN THE ANALYSIS
             0 FILTERED (LOCAL/SUBSET SELECTOR)
             0 MISSING DATA CASES
             3 OUTLIERS INCLUDED
             0 INVALID PREDICTOR VALUES

      2088 SAMPLE OBSERVATIONS - WITH TOTALS
           WEIGHTS =                 8.157500E 04
           DEPENDENT VARIABLE (Y) =  -1.065800D 04      AVERAGE =  -1.306527E-01
           Y-SQUARED =               8.015038D 12     VARIANCE =    9.830075E 07


                         STAGE  2 OF THE ANALYSIS
                        BEST SPLIT BASED ON MEANS
                  0-STEP LOOKAHEAD WITH 1 FORCED SPLITS

SPLITTING CRITERIA -
    MAXIMUM NUMBER OF SPLITS =  25
    MINIMUM # OBSERVATIONS IN A GROUP =  3
    %AGE OF TOTAL SS N SPLITS MUST EXPLAIN =  0.6(N=1),
    PRINT   CASES OUTSIDE  5.0 STANDARD DEVIATIONS OF PARENT GROUP MEAN
```

*Note three rank levels, using the last predictor only as a last resort.*

```
       8      RANKED PREDICTORS SPECIFIED
PREDICTOR RANK PREFERENCE     UP
   SPLIT ATTEMPT RANGE -   2 RANKS UP,  2 RANKS DOWN
   ELIGIBILITY RANGE -     2 RANKS UP,  2 RANKS DOWN
              PREDICTOR           VARIABLE NUMBER   TYPE     MAX CLASS     RANK
     1        MIGHT MOVE           26:79    V1276     M          5          2
     2        HOW LONG LIVD HERE            V2004     M          5          2
     3        EDUCATION OF HEAD  31:43      V1485     M          9          1
     4        RACE               31:48      V1490     F          3          1
     5        PUB TRANSP GOOD    26:52      V1250     M          5          1
     6        % CHANGE IN INCOME            V2003     M          5          1
     7        EXPECT CHILDREN ?  29:18      V1370     M          1          1
     8        HRS HEAD TRVL WK22:30-32      V1146     M          6          3
```

*Summary of the residual variables.*

*Range ranking UP: use low rank numbers first.*

```
    WEIGHTED Y VARIABLE 2005  AID3 Y-VARIABLE          SCALED BY  1.0E 00

             1 CANDIDATES -     GROUP        SS
                                  1     8.015038E 12

               ATTEMPT SPLIT ON GROUP    1 WITH SS = 8.015038E 12
```

```
                  LOOKAHEAD TENTATIVE PARTITION

SPLIT ATTEMPT ON GROUP    1 WITH N =      2088, SS =  8.015038E 12
SUMS- W = 8.157500F 04, Y =-1.065800F 04, YSQ = 8.015038E 12, X =

PREDICTOR  EDUCATION OF HEAD  31:43
  10 NON-EMPTY CLASSES   0  1  2  3  4  5  6  7  8  9
 PARTITION       N       WEIGHT        Y-MEAN  Y-VARIANCE        X-MEAN  X-VARIANCE          SLOPE          BSS
 BETWEEN  1    183 5.33200E 03 -4.20937E 03 3.95312E 07  0.0         0.0         0.0
     AND  2   1905 7.62430E 04  2.94239E 02 1.01103E 08  0.0         0.0         0.0                 1.01077F 11
 BETWEEN  2    642 2.18100E 04 -2.87333E 03 5.91161E 07  0.0         0.0         0.0
     AND  3   1446 5.97650E 04  1.04839E 03 1.08533E 08  0.0         0.0         0.0                 2.45753E 11
 BETWEEN  3   1006 3.42940E 04 -2.25760E 03 6.94271E 07  0.0         0.0         0.0
     AND  4   1082 4.72810E 04  1.63726E 03 1.12939E 08  0.0         0.0         0.0                 3.01530E 11
 BETWEEN  4   1368 4.94910E 04 -1.52803E 03 7.76312E 07  0.0         0.0         0.0
     AND  5    720 3.20840E 04  2.35673E 03 1.21165E 08  0.0         0.0         0.0                 2.93755E 11
 BETWEEN  5   1577 5.82270E 04 -1.27708E 03 7.95989E 07  0.0         0.0         0.0
     AND  6    511 2.33480E 04  3.18442E 03 1.30950E 08  0.0         0.0         0.0                 3.31725E 11
 BETWEEN  6   1819 6.91500E 04 -9.40164E 02 8.18338E 07  0.0         0.0         0.0
     AND  7    269 1.24250E 04  5.23152E 03 1.58187E 08  0.0         0.0         0.0                 4.01180E 11
 BETWEEN  7   1981 7.67610E 04 -3.13565E 02 9.24691E 07  0.0         0.0         0.0
     AND  8    107 4.81400E 03  4.99769E 03 1.66243E 08  0.0         0.0         0.0                 1.27786E 11
   BEST SPLIT ON PREDICTOR 1485 =  4.011800E 11 AFTER CLASS  6

PREDICTOR  RACE                  31:48
   3 NON-EMPTY CLASSES   2  1  3
 PARTITION       N       WEIGHT        Y-MEAN  Y-VARIANCE        X-MEAN  X-VARIANCE          SLOPE          BSS
 BETWEEN  2    406 5.49300E 03 -4.67897E 03 4.04610E 07  0.0         0.0         0.0
     AND  1   1682 7.60820E 04  3.37674E 02 1.00799E 08  0.0         0.0         0.0                 1.28932E 11
   BEST SPLIT ON PREDICTOR 1490 =  1.289322E 11 AFTER CLASS  2

PREDICTOR  PUB TRANSP GOOD     26:52
   3 NON-EMPTY CLASSES   1  3  5
 PARTITION       N       WEIGHT        Y-MEAN  Y-VARIANCE        X-MEAN  X-VARIANCE          SLOPE          BSS
   BEST SPLIT ON PREDICTOR 1250 =  1.294188E 10 AFTER CLASS  3

PREDICTOR  % CHANGE IN INCOME
   5 NON-EMPTY CLASSES   1  2  3  4  5
 PARTITION       N       WEIGHT        Y-MEAN  Y-VARIANCE        X-MEAN  X-VARIANCE          SLOPE          BSS
   BEST SPLIT ON PREDICTOR 2003 =  2.235174E 09 AFTER CLASS  4

PREDICTOR  EXPECT CHILDREN ?  29:18
   2 NON-EMPTY CLASSES   0  1
 PARTITION       N       WEIGHT        Y-MEAN  Y-VARIANCE        X-MEAN  X-VARIANCE          SLOPE          BSS
   BEST SPLIT ON PREDICTOR 1370 =  8.701179E 08 AFTER CLASS  0

PREDICTOR  MIGHT MOVE          26:79
   2 NON-EMPTY CLASSES   1  5
 PARTITION       N       WEIGHT        Y-MEAN  Y-VARIANCE        X-MEAN  X-VARIANCE          SLOPE          BSS
   BEST SPLIT ON PREDICTOR 1276 =  4.164543E 09 AFTER CLASS  1

PREDICTOR  HOW LONG LIVD HERE
   5 NON-EMPTY CLASSES   1  2  3  4  5
 PARTITION       N       WEIGHT        Y-MEAN  Y-VARIANCE        X-MEAN  X-VARIANCE          SLOPE          BSS
 BETWEEN  4    838 3.13780E 04  1.35179E 03 1.07215E 08  0.0         0.0         0.0
     AND  5   1250 5.01970E 04 -8.45209E 02 9.09477E 07  0.0         0.0         0.0                 9.31974E 10
   BEST SPLIT ON PREDICTOR 2004 =  9.319737E 10 AFTER CLASS  4

PREDICTOR  HRS HEAD TRVL WK22:30-32
   6 NON-EMPTY CLASSES   0  1  2  3  4  5
 PARTITION       N       WEIGHT        Y-MEAN  Y-VARIANCE        X-MEAN  X-VARIANCE          SLOPE          BSS
```

*Details of the trace only given for eligible splits.*

```
 BETWEEN  4  1839 7.28340E 04 -3.11383E 02 9.23840E 07  0.0          0.0          0.0
     AND  5   249 8.74100E 03  2.59336E 03 1.40612E 08  0.0          0.0          0.0            6.58498E 10
    BEST SPLIT ON PREDICTOR 1146 =  6.584982E 10 AFTER CLASS  4

EDUCATION OF HEAD  31:43 YIELDS MAXIMUM BSS =  4.011800E 11


0-STEP LOOKAHEAD TO SPLIT GROUP   1, TOTAL BSS =  4.011800E 11
         1. SPLIT GROUP   1 ON PREDICTOR 1485, BSS =  4.011800E 11
```

```
             ***** PARTITION OF GROUP    1 *****
FROM ELIGIBLE PREDICTORS AROUND THE CURRENT RANK  1,     2 UP AND  2 DOWN

      MAXIMUM ELIGIBLE BSS AT EACH STEP         MAXIMUM TOTAL BSS (LOOKAHEAD)
    1.SPLIT    1 ON V1485 BSS= 4.01180E 11     SPLIT    1 ON V1485 BSS = 4.01180E 11
    PE(1)= 4.809E 10, TOTAL= 4.01180E 11                      TOTAL= 4.01180E 11
        PREDICTOR 1485 HAS RANK  1

SPLIT GROUP    1 ON EDUCATION OF HEAD  31:43 V1485

EXTREME CASES LYING OUTSIDE THE INTERVAL (-4.957349E 04, 4.957323E 04)
       Y          W          WY            V2005
    53249       44.0     2.34256E 06      53249
    55242       24.0     1.32581E 06      55242
    53249       48.0     2.55595E 06      53249
    GROUP    2 WITH  1819 OBSERVATIONS FROM  7 CLASSES =  0  1  2  3  4  5  6
               W= 6.91500E 04 Y=-6.50124D 07 YSQ= 5.71682D 12 X=
    GROUP    3 WITH   269 OBSERVATIONS FROM  3 CLASSES =  7  8  9
               W= 1.24250E 04 Y= 6.50017D 07 YSQ= 2.29822D 12 X=

              2 CANDIDATES -     GROUP       SS
                                   2    5.655695E 12
                                   3    1.958161E 12

                ATTEMPT SPLIT ON GROUP   2 WITH SS = 5.655695E 12
```

*Omit the trace of split attempts on the remaining groups.*

```
                 ***** PARTITION OF GROUP    6 *****
FROM ELIGIBLE PREDICTORS AROUND THE CURRENT RANK  1,     2 UP AND  2 DOWN

      MAXIMUM ELIGIBLE BSS AT EACH STEP         MAXIMUM TOTAL BSS (LOOKAHEAD)

GROUP    6 COULD NOT BE SPLIT


END OF STAGE 2 OF THE ANALYSIS.   8 FINAL GROUPS.   8 INELIGIBLE FOR SPLITTING.

                VARIATION EXPLAINED (BSS(O)/TSS) =  11.6%


                1-WAY ANALYSIS OF VARIANCE ON FINAL GROUPS

                SOURCE            SUM OF SQUARES         DF    MEAN SQUARE

                BETWEEN             9.322081E 11          8.   1.165260E 11
                ERROR               7.082830E 12     81528.   8.687616E 07
                TOTAL               8.015038E 12     81536.   9.830075E 07
```

*Summary of variance explained. Mean squares (and F- or t-tests) are inappropriate here because of weighted data.*

$$\frac{1.023693 \times 10^{12}}{8.064064 \times 10^{12}} = 12.7\%$$

```
                              GROUP SUMMARY TABLE
                        15 GROUPS OF WHICH  8 ARE FINAL

GROUP    1 , N =      2088, SUM W =  8.157500E 04
    Y MEAN=-1.306527E-01, VARIANCE= 9.830075E 07,    SS(L)/TSS= 1.000, BSS/TSS= 0.050
    SPLIT ON EDUCATION OF HEAD  31:43, BSS(L) =  4.011800E 11  INTO
           2 WITH CLASSES   0  1  2  3  4  5  6
           3 WITH CLASSES   7  8  9

GROUP    2 , N =      1819, SUM W =  6.915000E 04
    Y MEAN=-9.401646E 02, VARIANCE= 8.183382E 07,    SS(L)/TSS= 0.706, BSS/TSS= 0.015
    SPLIT ON EDUCATION OF HEAD  31:43, BSS(L) =  1.190583E 11  INTO
           4 WITH CLASSES   0  1  2
           5 WITH CLASSES   3  4  5  6

GROUP    5 , N =      1177, SUM W =  4.734000E 04
    Y MEAN=-4.953375E 01, VARIANCE= 8.983806E 07,    SS(L)/TSS= 0.530, BSS/TSS= 0.008
    SPLIT ON RACE                31:48, BSS(L) =  6.239313E 10  INTO
           6 WITH CLASSES   2
           7 WITH CLASSES   1  3

GROUP    7 , N =       982, SUM W =  4.474100E 04
    Y MEAN= 2.271636E 02, VARIANCE= 9.112443E 07,    SS(L)/TSS= 0.508, BSS/TSS= 0.011
    SPLIT ON HRS HEAD TRVL WK22:30-32, BSS(L) =  9.139598E 10  INTO
           8 WITH CLASSES   1  2  3  4  5
           9 WITH CLASSES   0

GROUP    8 , N =       776, SUM W =  3.550600E 04
    Y MEAN=-5.017549E 02, VARIANCE= 8.285798E 07,    SS(L)/TSS= 0.367, BSS/TSS= 0.014
    SPLIT ON HRS HEAD TRVL WK22:30-32, BSS(L) =  1.136891E 11  INTO
          10 WITH CLASSES   1  2  3  4
          11 WITH CLASSES   5

GROUP   10*, N =       667, SUM W =  3.021100E 04
    Y MEAN=-1.250889E 03, VARIANCE= 7.198024E 07,    SS(L)/TSS= 0.271, BSS/TSS= 0.0

GROUP    3 , N =       269, SUM W =  1.242500E 04
    Y MEAN= 5.231527E 03, VARIANCE= 1.581866E 08,    SS(L)/TSS= 0.244, BSS/TSS= 0.007
    SPLIT ON HRS HEAD TRVL WK22:30-32, BSS(L) =  5.655285E 10  INTO
          12 WITH CLASSES   1  2  3  4  5
          13 WITH CLASSES   0

GROUP   12 , N =       219, SUM W =  1.025800E 04
    Y MEAN= 4.250957E 03, VARIANCE= 1.395635E 08,    SS(L)/TSS= 0.178, BSS/TSS= 0.011
    SPLIT ON HOW LONG LIVD HERE       , BSS(L) =  8.793706E 10  INTO
          14 WITH CLASSES   5
          15 WITH CLASSES   1  2  3  4

GROUP    4*, N =       642, SUM W =  2.181000E 04
    Y MEAN=-2.873336E 03, VARIANCE= 5.911613E 07,    SS(L)/TSS= 0.161, BSS/TSS= 0.0

GROUP    9*, N =       206, SUM W =  9.235000E 03
    Y MEAN= 3.029653E 03, VARIANCE= 1.135218E 08,    SS(L)/TSS= 0.130, BSS/TSS= 0.0

GROUP   15*, N =       117, SUM W =  5.487000E 03
    Y MEAN= 6.981141E 03, VARIANCE= 1.454259E 08,    SS(L)/TSS= 0.099, BSS/TSS= 0.0

GROUP   11*, N =       109, SUM W =  5.295000E 03
    Y MEAN= 3.772486E 03, VARIANCE= 1.244924E 08,    SS(L)/TSS= 0.081, BSS/TSS= 0.0

GROUP   14*, N =       102, SUM W =  4.771000E 03
```

*This summarizes the split record in enough detail for making the diagram preceding the examples.*

```
    Y MEAN= 1.111043E 03, VARIANCE= 1.155824E 08,   SS(L)/TSS= 0.068, BSS/TSS= 0.0

GROUP  13*, N =      50, SUM W =  2.167000E 03
    Y MEAN= 9.873273E 03, VARIANCE= 2.243776E 08,   SS(L)/TSS= 0.059, BSS/TSS= 0.0

GROUP   6*, N =     195, SUM W =  2.599000E 03
    Y MEAN=-4.812793E 03, VARIANCE= 4.412045E 07,   SS(L)/TSS= 0.014, BSS/TSS= 0.0
```

```
                                  100*BSS/TSS TABLE FOR 0-STEP LOOK-AHEAD
                                        15 GROUPS,  8 PREDICTORS
                                 < INDICATES LESS THAN 1 SPLITS WERE MADE

PREDICTOR                        GROUP NUMBER (* INDICATES THE GROUP IS FINAL)
              1      2      5      7      8     10*     3     12     4*     9*    15*    11*    14*    13*    6*
  1485       5.0    1.5    0.3    0.1    0.3    0.3    0.2    0.1    0.2    0.1    0.0    0.1    0.1    0.2    0.1
  1490       1.6    1.1    0.8    0.1    0.1    0.0    0.1    0.0    0.2    0.0    0.1    0.3    0.0    0.6 *****
  1250       0.2    0.1    0.1    0.0    0.0    0.0    0.3    0.1    0.0    0.0    0.3    0.0    0.1    0.4    0.0
  2003       0.0    0.1    0.2    0.2    0.2    0.1    0.5    0.5    0.0    0.1    0.2    0.3    0.4    0.2    0.1
  1370       0.0    0.0    0.0    0.0    0.0    0.0    0.1    0.0    0.0    0.0    0.0    0.3    0.2    0.3    0.0
  1276       0.1    0.0    0.0    0.0    0.0    0.1    0.1    0.0    0.1    0.0    0.1    0.0    0.0    0.0    0.0
  2004       1.2    0.4    0.2    0.4    0.5    0.2    0.5    1.1    0.0    0.1    0.0    0.6 *****    0.3    0.0
  1146       0.8    0.6    1.2    1.1    1.4    0.0    0.7    0.3    0.3 *****    0.5 *****    0.0 *****    0.0
```

⬭ *Split made of this group, see group summary table immediately preceding. Note group 5 splits on an inferior predictor because it had a lower rank number. The rows of this table will be ordered by rank whatever the order of the predictor cards.*

▢ *No split on this predictor here because of inferior rank.*

```
                                         100*BSS/TSS TABLE FOR 0-STEP LOOK-AHEAD
                                                 15 GROUPS,  8 PREDICTORS

                                          MAXIMUM BSS REGARDLESS OF ELIGIBILITY

PREDICTOR                                GROUP NUMBER (* INDICATES THE GROUP IS FINAL)
                     1     2     5     7     8   10*     3    12    4*    9*   15*   11*   14*   13*    6*
      1485         5.0   1.5   0.3   0.1   0.3   0.3   0.2   0.1   0.2   0.1   0.0   0.1   0.1   0.2   0.1
      1490         1.6   1.1   0.8   0.1   0.1   0.0   0.1   0.0   0.2   0.0   0.1   0.3   0.0   0.6******
      1250         0.2   0.1   0.1   0.0   0.0   0.0   0.3   0.1   0.0   0.0   0.3   0.0   0.1   0.4   0.0
      2003         0.0   0.1   0.2   0.2   0.2   0.1   0.5   0.5   0.0   0.1   0.2   0.3   0.4   0.2   0.1
      1370         0.0   0.0   0.0   0.0   0.0   0.0   0.1   0.0   0.0   0.0   0.0   0.3   0.2   0.3   0.0
      1276         0.1   0.0   0.0   0.0   0.0   0.1   0.1   0.0   0.1   0.0   0.1   0.0   0.0   0.0   0.0
      2004         1.2   0.4   0.2   0.4   0.5   0.2   0.5   1.1   0.0   0.1   0.0   0.6******   0.3   0.0
      1146         0.8   0.6   1.2   1.1   1.4   0.0   0.7   0.3   0.3******   0.5******   0.0******   0.0
```

```
                              PROFILE OF CLASS MEANS AND SLOPES


                      EDUCATION OF HEAD  31:43, ETA = 0.072
                                                GROUP
CLASS          1          2          5          7          8          10*          3          12          4*          9*
  0  N        68         68          0          0          0           0           0           0          68           0
     YMEAN -4.732E 03 -4.732E 03  0.0        0.0        0.0         0.0         0.0         0.0       -4.732E 03   0.0
  1  N       115        115          0          0          0           0           0           0         115           0
     YMEAN -3.888E 03 -3.888E 03  0.0        0.0        0.0         0.0         0.0         0.0       -3.888E 03   0.0
  2  N       459        459          0          0          0           0           0           0         459           0
     YMEAN -2.441E 03 -2.441E 03  0.0        0.0        0.0         0.0         0.0         0.0       -2.441E 03   0.0
  3  N       364        364        364        259        177         152           0           0           0          82
     YMEAN -1.182E 03 -1.182E 03 -1.182E 03 -6.607E 02 -2.080E 03  -2.804E 03   0.0         0.0         0.0        2.501E 03
  4  N       362        362        362        317        258         226           0           0           0          59
     YMEAN  1.183E 02  1.183E 02  1.183E 02  2.269E 02 -4.630E 02  -9.881E 02   0.0         0.0         0.0        3.189E 03
  5  N       209        209        209        184        153         128           0           0           0          31
     YMEAN  1.446E 02  1.446E 02  1.446E 02  2.915E 02 -1.548E 02  -1.205E 03   0.0         0.0         0.0        2.429E 03
  6  N       242        242        242        222        188         161           0           0           0          34
     YMEAN  8.558E 02  8.558E 02  8.558E 02  1.130E 03  5.504E 02  -2.957E 02   0.0         0.0         0.0        4.511E 03
  7  N       162          0          0          0          0           0         162         127           0           0
     YMEAN  5.379E 03  0.0        0.0        0.0        0.0         0.0        5.379E 03   4.619E 03   0.0         0.0
  8  N        93          0          0          0          0           0          93          82           0           0
     YMEAN  5.582E 03  0.0        0.0        0.0        0.0         0.0        5.582E 03   4.224E 03   0.0         0.0
  9  N        14          0          0          0          0           0          14          10           0           0
     YMEAN -4.791E 02  0.0        0.0        0.0        0.0         0.0       -4.791E 02  -9.809E 02   0.0         0.0


                      EDUCATION OF HEAD  31:43, ETA = 0.072
                                                GROUP
CLASS          15*        11*        14*        13*        6*
  3  N          0         25          0          0        105
     YMEAN  0.0        1.898E 03  0.0        0.0       -6.246E 03
  4  N          0         32          0          0         45
     YMEAN  0.0        3.098E 03  0.0        0.0       -2.171E 03
  5  N          0         25          0          0         25
     YMEAN  0.0        4.704E 03  0.0        0.0       -3.235E 03
  6  N          0         27          0          0         20
     YMEAN  0.0        5.353E 03  0.0        0.0       -6.704E 03
  7  N         71          0         56         35          0
     YMEAN  7.031E 03  0.0        1.554E 03  8.220E 03  0.0
  8  N         42          0         40         11          0
     YMEAN  6.962E 03  0.0        1.368E 03  1.611E 04  0.0
  9  N          4          0          6          4          0
     YMEAN  6.167E 03  0.0       -5.756E 03  2.775E 03  0.0


                      RACE               31:48, ETA = 0.017
                                                GROUP
CLASS          1          2          5          7          8          10*          3          12          4*          9*
  1  N      1626       1382        957        957        754         648         244         197         425         203
     YMEAN  3.016E 02 -6.526E 02  1.821E 02  1.821E 02 -5.664E 02  -1.251E 03   5.418E 03   4.265E 03  -2.614E 03   3.029E 03
  2  N       406        392        195          0          0           0          14          11         197           0
     YMEAN -4.679E 03 -4.990E 03 -4.813E 03  0.0        0.0         0.0        2.021E 02   4.160E 03  -5.169E 03   0.0
  3  N        56         45         25         25         22          19          11          11          20           3
     YMEAN  1.804E 03  1.139E 03  2.580E 03  2.580E 03  2.514E 03  -1.267E 03   4.000E 03   4.000E 03  -1.009E 03   3.101E 03

                      RACE               31:48, ETA = 0.017
                                                GROUP
CLASS          15*        11*        14*        13*        6*
```

```
  1  N           102          106           95           47            0
     YMEAN   7.280E 03    3.376E 03    9.627E 02    1.065E 04    0.0
  2  N             8            0            3            3          195
     YMEAN   4.866E 03    0.0          1.780E 03   -1.829E 04   -4.813E 03
  3  N             7            3            4            0            0
     YMEAN   3.226E 03    1.702E 04    5.494E 03    0.0          0.0


                             PUB TRANSP GOOD     26:52, ETA = 0.002
                                                     GROUP
CLASS                1            2            5            7            8          10*            3           12           4*           9*
  1  N           771          676          461          326          272          227           95           76          215           54
     YMEAN  -4.460E 02   -1.322E 03   -6.223E 02   -7.711E 01   -5.514E 02   -1.329E 03    4.290E 03    4.007E 03   -3.107E 03    2.377E 03
  3  N           128          110           75           66           55           48           18           17           35           11
     YMEAN  -6.662E 02   -8.901E 02   -4.151E 02   -4.004E 02   -1.209E 03   -1.905E 03    4.630E 02    3.702E 02   -2.047E 03    3.428E 03
  5  N          1207         1049          651          598          455          396          158          126          398          143
     YMEAN   3.878E 02   -6.392E 02    4.456E 02    5.919E 02   -2.145E 02   -1.069E 03    6.289E 03    4.935E 03   -2.758E 03    3.215E 03


                             PUB TRANSP GOOD     26:52, ETA = 0.002
                                                     GROUP
CLASS              15*          11*          14*          13*           6*
  1  N            37           45           39           19          135
     YMEAN   5.203E 03    3.038E 03    2.831E 03    5.546E 03   -5.146E 03
  3  N             8            7            9            1            9
     YMEAN   2.607E 02    3.392E 03    4.721E 02    2.032E 03   -1.907E 03
  5  N            72           59           54           32           53
     YMEAN   8.719E 03    5.220E 03    1.651E 00    1.191E 04   -4.312E 03


                             % CHANGE IN INCOME       , ETA = 0.001
                                                     GROUP
CLASS                1            2            5            7            8          10*            3           12           4*           9*
  1  N           331          304          166          140           80           65           27           18          138           60
     YMEAN  -1.208E 01   -6.275E 02    7.143E 02    1.103E 03    8.248E 02   -1.307E 03    7.483E 03    3.097E 03   -2.989E 03    1.462E 03
  2  N           433          369          238          210          165          148           64           48          131           45
     YMEAN   3.512E 02   -4.060E 02    6.856E 02    8.229E 02    1.326E 02   -4.030E 02    4.068E 03    3.894E 03   -2.702E 03    3.496E 03
  3  N           321          272          193          168          148          130           49           43           79           20
     YMEAN  -4.123E 02   -1.211E 03   -7.113E 02   -6.123E 02   -1.117E 03   -1.400E 03    3.497E 03    3.266E 03   -2.668E 03    3.094E 03
  4  N           512          440          308          261          223          192           72           66          132           38
     YMEAN  -3.069E 02   -1.186E 03   -3.022E 02   -1.134E 02   -1.062E 03   -1.588E 03    4.027E 03    2.953E 03   -3.555E 03    5.712E 03
  5  N           473          418          262          195          154          128           55           44          156           41
     YMEAN   1.471E 02   -1.377E 03   -8.006E 02   -2.577E 02   -9.846E 02   -1.778E 03    9.327E 03    8.481E 03   -2.595E 03    2.490E 03


                             % CHANGE IN INCOME       , ETA = 0.001
                                                     GROUP
CLASS              15*          11*          14*          13*           6*
  1  N             6           15           12            9           26
     YMEAN   1.417E 04    8.812E 03   -2.228E 03    1.603E 04   -5.125E 03
  2  N            24           17           24           16           28
     YMEAN   5.284E 03    4.279E 03    2.495E 03    4.612E 03   -3.403E 03
  3  N            17           18           26            6           25
     YMEAN   5.175E 03    8.076E 02    2.131E 03    6.063E 03   -3.041E 03
  4  N            40           31           26            6           47
     YMEAN   6.595E 03    2.070E 03   -2.515E 03    1.530E 04   -3.549E 03
  5  N            30           26           14           11           67
     YMEAN   8.567E 03    2.648E 03    8.246E 03    1.321E 04   -7.086E 03


                             EXPECT CHILDREN ?  29:18, ETA = 0.000
```

```
                                                                 GROUP
CLASS           1           2           5           7           8          10*           3          12           4*          9*
  0  N        1949        1713        1079         896         697         596         236         191         634         199
     YMEAN  -2.941E 01  -5.532E 02  -4.302E 00   2.725E 02  -4.815F 02  -1.160E 03   5.456E 03   4.237E 03  -2.855E 03   2.972E 03
  1  N         139         1C6          98          86          79          71          33          28           8           7
     YMEAN   3.641E 02  -7.540F 02  -5.809E 02  -2.227E 02  -6.746E 02  -1.976E 03   3.647E 03   4.347E 03  -5.111E 03   4.463E 03


                                        EXPECT CHILDREN ?  29:18, ETA = 0.000
                                                                 GROUP
CLASS          15*         11*         14*         13*          6*
  0  N          93         101          98          45         183
     YMEAN   7.170E 03   3.228E C3   1.468E 03   1.102E 04  -4.660F 03
  1  N          24           8           4           5          12
     YMEAN   6.271E 03   1.081F C4  -8.035E 03  -7.660E 02  -6.177E 03


                                        MIGHT MOVE         26:79, ETA = 0.001
                                                                 GROUP
CLASS           1           2           5           7           8          10*           3          12           4*          9*
  1  N         291         233         177         158         128         107          58          50          56          30
     YMEAN   5.331E 02  -4.320E 02  -1.662F 02  -1.494E 01  -9.853E 02  -2.182E 03   3.945E 03   3.485E 03  -1.362E 03   4.269E 03
  5  N        1797        1586        1000         824         648         560         211         169         586         176
     YMEAN  -9.587E 01  -1.023E C3  -2.747E 01   2.741F 02  -4.050E 02  -1.070E 03   5.595F 03   4.484E 03  -3.039E 03   2.819E 03


                                        MIGHT MOVE         26:79, ETA = 0.001
                                                                 GROUP
CLASS          15*         11*         14*         13*          6*
  1  N          29          21          21           8          19
     YMEAN   4.681E 03   4.822E C3   1.853E 03   7.162E 03  -4.312E 03
  5  N          88          88          81          42         176
     YMEAN   7.756E 03   3.525E 03   9.111E 02   1.038E 04  -4.870E 03


                                        HOW LONG LIVD HERE       , ETA = 0.014
                                                                 GROUP
CLASS           1           2           5           7           8          10*           3          12           4*          9*
  1  N         175         146         114          77          68          57          29          26          32           9
     YMEAN   2.661E 03   1.179E 03   1.667E 03   2.106E 03   1.888E 03   1.140E 03   8.120E 03   6.486E 03  -1.881E 03   3.840E 03
  2  N         148         124          96          80          69          61          24          21          28          11
     YMEAN   2.635E 02  -1.394E 03  -9.899E 02  -5.996E 02  -8.092E 02  -1.127E 03   7.247E 03   7.546E 03  -3.390E 03   6.921E 02
  3  N         186         155         114          91          79          66          31          27          41          12
     YMEAN   9.508E 02  -7.520E 02  -1.193E 02   2.726E 02   9.093E 01  -1.322E 03   7.178E 03   6.789E 03  -3.608E 03   1.571E 03
  4  N         329         277         199         171         149         124          52          43          78          22
     YMEAN   1.489E 03   3.349E 02   1.500E 03   2.055E 03   1.136E 03  -5.524E 02   6.355E 03   7.091E 03  -3.251E 03   8.630F 03
  5  N        1250        1117         654         563         411         359         133         102         463         152
     YMEAN  -8.452E 02  -1.433E C3  -5.996F 02  -4.496E 02  -1.521E 03  -1.867E 03   3.381E 03   1.111E 03  -2.783E 03   2.505E 03


                                        HOW LONG LIVD HERE       , ETA = 0.014
                                                                 GROUP
CLASS          15*         11*         14*         13*          6*
  1  N          26          11           0           3          37
     YMEAN   6.486E 03   6.048E 03   0.0         2.334E 04  -3.801E 03
  2  N          21           8           0           3          16
     YMEAN   7.546E 03   1.477E C3   0.0         5.280E 03  -7.904E 03
  3  N          27          13           0           4          23
     YMEAN   6.789E 03   6.941E 03   0.0         1.035E 04  -4.489E 03
  4  N          43          25           0           9          28
     YMEAN   7.091E 03   8.417E 03   0.0         2.249E 03  -7.545E 03
  5  N           0          52         102          31          91
     YMEAN   0.0         6.916E 02   1.111E 03   1.135E 04  -3.632E 03


                             HRS HEAD TRVL WK22:30-32, ETA = 0.017
                                                     GROUP
CLASS            1           2           5           7           8          10*          3          12           4*          9*
  0  N         577         527         233         206           0           0          50           0         294         206
     YMEAN   1.096E 03   2.055E 02   2.714E 03   3.030E 03   0.0         0.0         9.873E 03   0.0        -1.879E 03   3.030E 03
  1  N         422         354         265         234         234         234          68          68          89           0
     YMEAN  -7.592E 02  -1.771E 03  -1.432E 03  -1.223E 03  -1.223E 03  -1.223E 03   3.678E 03   3.678E 03  -3.041E 03   0.0
  2  N         286         245         177         150         150         150          41          41          68           0
     YMEAN  -1.567E 03  -2.184E 03  -1.575E 03  -1.489E 03  -1.489E 03  -1.489E 03   1.345E 03   1.345E 03  -4.185E 03   0.0
  3  N         240         211         154         126         126         126          29          29          57           0
     YMEAN  -9.706E 02  -1.775E 03  -1.053E 03  -7.397E 02  -7.397E 02  -7.397E 02   3.876E 03   3.876E 03  -4.440E 03   0.0
  4  N         314         266         204         157         157         157          48          48          62           0
     YMEAN  -7.612E 02  -2.187E 03  -1.743E 03  -1.469E 03  -1.469E 03  -1.469E 03   5.320E 03   5.320E 03  -3.978E 03   0.0
  5  N         249         216         144         109         109           0          33          33          72           0
     YMEAN   2.593E 03   1.442E 03   3.195E 03   3.772E 03   3.772E 03   0.0         7.881E 03   7.881E 03  -5.325E 03   0.0


                             HRS HEAD TRVL WK22:30-32, ETA = 0.017
                                                     GROUP
CLASS           15*         11*         14*         13*          6*
  0  N           0           0           0          50          27
     YMEAN   0.0         0.0         0.0         9.973E 03  -3.645E 03
  1  N          29           0          39           0          31
     YMEAN   6.172E 03   0.0         1.809E 03   0.0        -5.129E 03
  2  N          24           0          17           0          27
     YMEAN   2.695E 03   0.0        -5.414E 02   0.0        -3.065E 03
  3  N          16           0          13           0          28
     YMEAN   5.710E 03   0.0         1.599E 03   0.0        -6.085E 03
  4  N          29           0          19           0          47
     YMEAN   8.069E 03   0.0         1.274E 03   0.0        -6.781E 03
  5  N          19         109          14           0          35
     YMEAN   1.291E 04   3.772E 03   5.647E 02   0.0        -4.335E 03

..... END OF ANALYSIS .....
```

*The ETA (really $n^2$) is the fraction of the variance of group 1 accounted for by all the subclass means of that predictor. (The BSS's are for binary splits only).*

**APPENDIX 5 - Second Example, Run 3**

```
          INSTITUTE FOR SOCIAL RESEARCH MONITOR SYSTEM       02/05/73
     *****     FASTER VERSIONS OF TABLES, MDC, AND REGRESSN NOW AVAILABLE BY SPECIFYING              *****
     *****                                                                                           *****
     *****                    //   EXEC OSIRIS,LIB=X485321X,LIB1=OSIRPGM                             *****
     *****                                                                                           *****
     *****     SAVINGS ARE 10-45% FOR TABLES, 45% FOR MDC AND 20% FOR REGRESSN                       *****

*****TIME IS   0:32:45

*****LISTING OF SET-UP FOLLOWS:
```

*Most of this is identical to the first run so only alterations are noted.*

```
 CARD             1         2         3         4         5         6         7         8
  NO.    12345678901234567890123456789012345678901234567890123456789012345678901234567890
   1     $RUN AID3
   2       INCLUDE V1264=1 AND V1109=0-1 AND V542=0-1 AND V1499=1-9*
   3     MTR 46, PROJECT 468070,AID3
   4     *
   5     V101,V542,V603,V1009,V1109,V1122,V1146,V1168,V1240,V1250,V1274,V1276,V1365,
   6     V1370,V1490,V1498,V1499,V1506,V1572,V1609,V1719,V1720,V1264,V1485*
   7     HOUSE VALUE TRUNCATED TO 5000-75000 BY SQUEEZING EXTREME CASES  807MTR45 MORGAN
   8     YVAR=1122, WEIGHT=1609, XVAR=1719, ANAL=REGR,               RECODE, MDOP=NONE,
   9     TABLES=(ELIG,MEAN)*
  10         1   IFV1264   NE         1
  11             ORV1109   GT         1
  12             ORV 542   GT         1
  13             ORV1499   EQ         0          V1264    =         0
  14         2   IFV1719   LT      3000          V2001    *         1
  15             OR        IN      3000      4999                   2
  16                               5000      7499                   3
  17                               7500      9999                   4
  18                              10000     14999                   5
  19                              15000     19999                   6
  20                       GE     20000                             7
  21         3   IFV1365   EQ         1          V2002    =         1            MARRIED
  22                                                   GOTO         4
  23             IFV1240   EQ         1          V2002    =         2            SNGL MAN
  24            ALT                              V2002    =         3            SNGL SMN
  25         4    T                              V1720  MPY       100V1720
  26                                             V2003  DIVV1720     V1719
  27             IFV2003   LT        -5          V2003    =         1            BRACKET
  28             OR        IN        -4         4                   2             PERCENT
  29                                  5         9                   3              CHANGE
  30                                 10        19                   4               IN
  31                       GE        20                             5             INCOME
  32         5   IFV1274   EQ         1          V2004    *         1
  33                                                   GOTO         6            HOW
  34             IFV 603   EQ         1          V2004    =         2              LONG
  35                                                   GOTO         6             LIVED
  36             IFV 101   IN         7         8V2004    =         3             HERE
  37                                                   GOTO         6
  38             IFV 101   IN         4         6V2004    =         4
  39            ALT                              V2004    =         5
  40         6   IFV1168   IN         7         9V1168    =         6            TRUNCATE
  41         7   IFV1009   EQ         1          V1009    =         2             NINE

 CARD             1         2         3         4         5         6         7         8
  NO.    12345678901234567890123456789012345678901234567890123456789012345678901234567890
```

*Specify subgroup regressions (of house value on income) instead of means. The covariate (x) must be specified.*

*Delete*

```
CARD           1         2         3         4         5         6         7         8
 NO.  12345678901234567890123456789012345678901234567890123456789012345678901234567890
  42      8   IFV1009   GT         6          V1009    =         6              CODES
  43      9   IFV1370   NE         1          V1370    =         0
  44     10   IFV1490  OUT         1         2V1490    =         3
  45     11   IFV1250  OUT         1         3V1250    =         5
  46     12   IFV1122   GT     75000          V1122    =     75000
  47          ORV1122   LT      5000          V1122    *      5000
  48     13   IFV1719   GT     25000          V1719    =     25000
  49     14   IFV1276   GT         5          V1276    =         5
  50     15   IFV1146   IN         1        99V1146    =         1            BRACKET
  51          OR                 100       149                   2             ANNUAL
  52                             150       199                   3              HOURS
  53                             200       299                   4            COMMUTNG
  54                    GE       300                             5
  55         END
  56  PRED=2001 MAXC=7 RANK=0 PREN='3-YR AVE $ INC'*
  57  PRED=(1168,1009) MAXC=6*
  58  PRED=2002 MAXC=3 PREN='SEX & MAR STATUS'*
  59  PRED=1506*
  60  PRED=1498 MAXC=5*
  61  PRED=1572 MAXC=4 F*
  62  PRED=1430 MAXC=3 F RANK=0 END*
  63  MIN=3 REDU=.5 RANK=ALL TRACE=ALL* }  Omit lookahead and symmetry premiums
                                          and use a lower reducibility criterion
                                          because the main income effect is
                                          already removed.
```

```
     VAR.   TYPE   VARIABLE NAME             TLOC   WIDTH  NODEC   RESP.   MDCODE1  MDCODE2   REFNO   ID    TSEQNO

T     101     0   WHEN MOVED IN 5:36          270     1      0      1               0000009                 00000
T     542     0   CHANGE IN FU COMP  15:25   1070     1      0      1                                       00000
T     603     0   MOVED SINCE SPRNG6816-38   1149     1      0      1               0000009                 00000
T    1009     0   BKT AGE HEAD 9V1009        1831     1      0      1                                       00000
T    1109     0   CHANGE IN FU COMP 21:18    2021     1      0      1                                       00000
T    1122     0   HOUSE VALUE     21:38-42   2041     5      0      1                                       00000
T    1146     0   HRS HEAD TRVL WK22:30-32   2103     3      0      1                                       00000
T    1168     0   # REQUIRED ROOMS 23:12     2154     1      0      1                                       00000
T    1240     0   SEX OF HEAD     26:40      2351     1      0      1                                       00000
T    1250     0   PUB TRANSP GOOD    26:52   2363     1      0      1               0000009                 00000
T    1264     0   OWN OR RENT?       26:67   2378     1      0      1                                       00000
T    1274     0   MOVED SINCE SPRING 26:77   2388     1      0      1               0000009                 00000
T    1276     0   MIGHT MOVE         26:79   2390     1      0      1               0000009                 00000
T    1365     0   MARITAL STATUS     29:10   2514     1      0      1                                       00000
T    1370     0   EXPECT CHILDREN ?  29:18   2521     1      0      1               0000009                 00000
T    1485     0   EDUCATION OF HEAD  31:43   2657     1      0      1               0000009                 00000
T    1490     0   RACE               31:48   2662     1      0      1               0000009                 00000
T    1498     0   DIST TO CNTR SMSA  31:58   2672     1      0      1               0000009                 00000
T    1499     0   TYPE OF STRUCTURE  31:59   2673     1      0      1               0000009                 00000
T    1506     0   LRGST PLAC/SMSA PSU31:66   2680     1      0      1                                       00000
T    1572     0   CURRENT REGION OV472       2838     1      0      1      0000009                          00000
T    1609     0   WEIGHT             OV509   2892     2      0      1                                COR   00000
T    1719     0   MEAN MONEY INCOME          3179     5      0      1                                COR   00000
T    1720     0   SLOPE MONEY INCOME         3184     6      0      1                                COR   00000


         HOUSE VALUE TRUNCATED TO 5000-75000 BY SQUEEZING EXTREME CASES  807MTR45 MORGAN

YVAR=1122, WEIGHT=1609, XVAR=1719, ANAL=REGR,             RECODE, MDOP=NONE,

TABLES=(ELIG,MEAN)*

  NO  LOG TEST  REL A OPERAND B OPERAND RES    OP C OPERAND D OPERAND  TEXT
   1    IFV1264   NE          1          0    0              0          0
  0   ORV1109   GT        1       0     0                0         0
  0   ORV 542   GT        1       0     0                0         0
  0   ORV1499   EQ        0       0V1264      =          0         0
  2   IFV1719   LT     3000       0V2001      =          1         0
  0   OR      0 IN     3000    4999     0                2         0
  0           0        5000    7499     0                3         0
  0           0        7500    9999     0                4         0
  0           0       10000   14999     0                5         0
  0           0       15000   19999     0                6         0
  0           0 GE    20000       0     0                7         0
  3   IFV1365   EQ        1       0V2002      =          1         0   MARRIED
  0           0           0       0     0 GOTO           4         0
  0   IFV1240   EQ        1       0V2002      =          2         0   SNGL MAN
  0  ALT      0           0       0V2002      =          3         0   SNGL SMN
  4    T      0           0       0V1720  MPY         100V      1720
  0           0           0       0V2003  DIVV       1720V      1719
  0   IFV2003   LT       -5       0V2003      =          1         0   BRACKET
  0   OR      0 IN       -4       4     0                2         0    PERCENT
  0           0           5       9     0                3         0     CHANGE
  0           0          10      19     0                4         0      IN
  0           0 GE       20       0     0                5         0    INCOME
  5   IFV1274   EQ        1       0V2004      =          1         0
  0           0           0       0     0 GOTO           6         0   HOW
  0   IFV 503   EQ        1       0V2004      =          2         0     LONG
  0           0           0       0     0 GOTO           6         0    LIVED
  0   IFV 101   IN        7       8V2004      =          3         0     HERE
  0           0           0       0     0 GOTO           6         0
  0   IFV 101   IN        4       6V2004      =          4         0
  0  ALT      0           0       0V2004      =          5         0
  6   IFV1168   IN        7       9V1168      =          6         0   TRUNCATE
  7   IFV1009   EQ        1       0V1009      =          2         0    NINE
  8   IFV1009   GT        6       0V1009      =          6         0     CODES
  9   IFV1370   NE        1       0V1370      =          0         0
 10   IFV1490  OUT        1       2V1490      =          3         0
 11   IFV1250  OUT        1       3V1250      =          5         0
 12   IFV1122   GT    75000       0V1122      =      75000         0
  0   ORV1122   LT     5000       0V1122      =       5000         0
 13   IFV1719   GT    25000       0V1719      =      25000         0
 14   IFV1276   GT        5       0V1276      =          5         0
 15   IFV1146   IN        1      99V1146      =          1         0   BRACKET
  0   OR      0         100     149     0                2         0    ANNUAL
  0           0         150     199     0                3         0     HOURS
  0           0         200     299     0                4         0   COMMUTNG
  0           0 GE      300       0     0                5         0
  0  END      0           0       0     0                0         0

PRED=2001 MAXC=7 RANK=0 PREN='3-YR AVE $ INC'*

PRED=(1169,1009) MAXC=6*

PRED=2002 MAXC=3 PREN='SEX & MAR STATUS'*

PRED=1506*

PRED=1498 MAXC=5*

PRED=1572 MAXC=4 F*

PRED=1490 MAXC=3 F RANK=0 END*

MIN=3 REDU=.5 RANK=ALL TRACE=ALL*
```

```
COMPUTE     SPECIFIED

OUTPUT     SPECIFIED


THE COMPLETE VARIABLE LIST IS:

   101   542   603  1003  1109  1122  1146  1168  1240  1250  1274  1276  1365  1370  1490  1498  1499  1506  1572  1609
  1719  1720  1264  1485  2001  2002  2003  2004
```

```
HOUSE VALUE TRUNCATED TO 500C-75C00 BY SQUEEZING EXTREME CASES  807MTR45 MORGAN

      2088 OBSERVATIONS READ AFTER GLOBAL FILTER

Y AVERAGE =             2.C07342E C4
STANDARD DEVIATION =  1.278252E 04
BOUNDARIES =  -4.383920E 04      8.398600E 04



     2088 CASES INCLUDED IN THE ANALYSIS
             0 FILTERED (LOCAL/SUBSET SELECTOR)
             0 MISSING DATA CASES
             0 OUTLIERS INCLUDED
             0 INVALID PREDICTOR VALUES

      2088 SAMPLE OBSERVATIONS - WITH TOTALS
           WEIGHTS =                  8.157500E 04
           DEPENDENT VARIABLE (Y) =   1.637489D 09     AVERAGE =   2.007342E 04
           Y-SQUARED =                4.619239D 13    VARIANCE =   1.633931E 08
           COVARIATE (X) =            8.399546D 08     AVERAGE =   1.029671E 04
           X-SQUARED =                1.152288D 13    VARIANCE =   3.524958E 07
           CROSS-PRODUCTS (Z) =       2.057341D 13       SLOPE =   1.291759E 00


                          STAGE  1 OF THE ANALYSIS
                         BEST SPLIT BASED ON REGR'N
                    0-STEP LOOKAHEAD WITH 1 FORCED SPLITS

SPLITTING CRITERIA -
    MAXIMUM NUMBER OF SPLITS =  25
    MINIMUM # OBSERVATIONS IN A GROUP =  3
    %AGE OF TOTAL SS N SPLITS MUST EXPLAIN =  0.5(N=1),
    PRINT   CASES OUTSIDE  5.0 STANDARD DEVIATIONS OF PARENT GROUP MEAN

       8      RANKED PREDICTORS SPECIFIED
PREDICTOR RANK PREFERENCE    AT
   SPLIT ATTEMPT RANGE -  0 RANKS UP,  0 RANKS DOWN
   ELIGIBILITY RANGE -    0 RANKS UP,  0 RANKS DOWN
            PREDICTOR           VARIABLE NUMBER   TYPE   MAX CLASS     RANK
      1     3-YR AVE $ INC                V2001     M        7           0
      2     # REQUIRED ROOMS 23:12        V1168     M        6           1
      3     BKT AGE HEAD 5V1009           V1009     M        6           1
      4     SEX & MAR STATUS              V2002     M        3           1
      5     LRGST PLAC/SMSA PSU31:66      V1506     M        9           1
      6     DIST TO CNTR SMSA  31:58      V1498     M        5           1
      7     CURRENT REGION 0V472          V1572     F        4           1
      8     RACE                 31:48    V1490     F        3           0

    WEIGHTED Y VARIABLE 1122  HOUSE VALUE      21:38-42 SCALED BY  1.0E 00
    COVARIAT-MEAN MONEY INCOME         V1719- SCALE FACTOR  1.0E 00

            1 CANDIDATES -     GROUP       SS
                                  1   1.332240E 13

              ATTEMPT SPLIT ON GROUP   1 WITH SS = 1.332240E 13
```

```
                    LOOKAHEAD TENTATIVE PARTITION

SPLIT ATTEMPT ON GROUP   1 WITH N =      2088, SS =  1.332240E 13
SUMS- W = 8.157500E 04, Y = 1.637489E 09, YSQ = 4.619239E 13, X = 9.399544E 08, XSQ = 1.152288E 13, Z = 2.057340E 13

PREDICTOR  # REQUIRED ROOMS 23:12
   5 NON-EMPTY CLASSES    2  3  4  5  6
 PARTITION       N       WEIGHT         Y-MEAN  Y-VARIANCE         X-MEAN  X-VARIANCE           SLOPE             BSS
 BETWEEN  2    661 2.91370E 04   1.76208E 04 1.34392E 08   7.95602E 03 3.50509E 07   1.06818E 00
     AND  3   1427 5.24380E 04   2.14362E 04 1.74471E 08   1.15973E 04 3.06433E 07   1.47149E 00   1.17609E 11
 BETWEEN  3   1218 5.30230E 04   1.92428E 04 1.51947E 08   9.28299E 03 3.40712E 07   1.27265E 00
     AND  4    870 2.85520E 04   2.16159E 04 1.81204E 08   1.21793E 04 3.20263E 07   1.50884E 00   8.63691E 10
 BETWEEN  4   1704 7.11340E 04   2.00866E 04 1.60302E 08   1.00739E 04 3.52920E 07   1.27774E 00
     AND  5    384 1.04410E 04   1.99834E 04 1.84950E 08   1.18147E 04 3.24122E 07   1.50635E 00   6.63707E 10
 BETWEEN  5   1929 7.78800E 04   2.00779E 04 1.61590E 08   1.02025E 04 3.51569E 07   1.29543E 00
     AND  6    159 3.69500E 03   1.95796E 04 2.02590E 08   1.22818E 04 3.32970E 07   1.37642E 00   2.82361E 10
    BEST SPLIT ON PREDICTOR 1169 =  1.176083E 11 AFTER CLASS  2

PREDICTOR  BKT AGE HEAD 9V1009
   5 NON-EMPTY CLASSES    2  3  4  5  6
 PARTITION       N       WEIGHT         Y-MEAN  Y-VARIANCE         X-MEAN  X-VARIANCE           SLOPE             BSS
 BETWEEN  2    372 1.42350E 04   1.98748E 04 1.44940E 08   1.08257E 04 1.90342E 07   1.77756E 00
     AND  3   1716 6.73400E 04   2.01154E 04 1.67369E 08   1.01849E 04 3.86186E 07   1.24434E 00   8.29969E 10
 BETWEEN  3    902 3.34870E 04   2.20743E 04 1.75047E 08   1.18262E 04 2.52645E 07   1.72314E 00
     AND  4   1186 4.80880E 04   1.86800E 04 1.50677E 08   9.23166E 03 3.94637E 07   1.09833E 00   2.28220E 11
 BETWEEN  4   1381 5.10150E 04   2.18217E 04 1.70766E 08   1.21373E 04 2.76472E 07   1.50984E 00
     AND  5    707 3.05600E 04   1.71550E 04 1.37659E 08   7.22423E 03 3.28804E 07   1.14261E 00   1.43227E 11
 BETWEEN  5   1759 6.60020E 04   2.11990E 04 1.68465E 08   1.16022E 04 3.10754E 07   1.39050E 00
     AND  6    329 1.55730E 04   1.53029E 04 1.14113E 08   4.76387E 03 1.51379E 07   1.50738E 00   1.39603E 11
    BEST SPLIT ON PREDICTOR 1009 =  2.282205E 11 AFTER CLASS  3

PREDICTOR  SEX & MAR STATUS
   3 NON-EMPTY CLASSES    1  2  3
 PARTITION       N       WEIGHT         Y-MEAN  Y-VARIANCE         X-MEAN  X-VARIANCE           SLOPE             BSS
 BETWEEN  1   1660 6.66840E 04   2.10937E 04 1.70328E 08   1.14257E 04 3.24255E 07   1.36936E 00
     AND  2    428 1.48910E 04   1.55046E 04 1.07083E 08   5.24077E 03 1.66594E 07   1.34439E 00   8.38022E 10
 BETWEEN  2   1742 6.98820E 04   2.07886E 04 1.68766E 08   1.11953E 04 3.33723E 07   1.35647E 00
     AND  3    346 1.16930E 04   1.57993E 04 1.10312E 08   4.92633E 03 1.28362E 07   1.58826E 00   1.19689E 11
    BEST SPLIT ON PREDICTOR 2002 =  1.196887E 11 AFTER CLASS  2

PREDICTOR  LRGST PLAC/SMSA PSU31:66
   6 NON-EMPTY CLASSES    1  2  3  4  5  6
 PARTITION       N       WEIGHT         Y-MEAN  Y-VARIANCE         X-MEAN  X-VARIANCE           SLOPE             BSS
 BETWEEN  1    633 2.37010E 04   2.58308E 04 2.11870E 08   1.26270E 04 3.72024E 07   1.45647E 00
     AND  2   1455 5.78740E 04   1.77156E 04 1.24519E 08   9.34238E 03 3.13371E 07   1.09374E 00   3.47037E 11
 BETWEEN  2   1076 4.11590E 04   2.40121E 04 1.89791E 08   1.21277E 04 3.44958E 07   1.39794E 00
     AND  3   1012 4.04160E 04   1.60623E 04 1.04747E 08   8.43205E 03 2.91535E 07   9.60458E-01   3.50862E 11
 BETWEEN  3   1298 5.03480E 04   2.30145E 04 1.79150E 08   1.17912E 04 3.43781E 07   1.34357E 00
     AND  4    790 3.12270E 04   1.53315E 04 1.01702E 08   7.88714E 03 2.72810E 07   9.52956E-01   2.36592E 11
 BETWEEN  4   1442 5.65140E 04   2.25180E 04 1.73994E 08   1.15397E 04 3.50440E 07   1.30042E 00
     AND  5    646 2.50610E 04   1.45608E 04 9.57874E 07   7.49371E 03 2.44100E 07   9.49607E-01   2.01075E 11
 BETWEEN  5   1642 6.49450E 04   2.16429E 04 1.69051E 08   1.11399E 04 3.50309E 07   1.30540E 00
     AND  6    446 1.66300E 04   1.39443E 04 9.43410E 07   7.00371E 03 2.25341E 07   8.63730E-01   1.42371E 11
    BEST SPLIT ON PREDICTOR 1506 =  3.508619E 11 AFTER CLASS  2

PREDICTOR  DIST TO CNTR SMSA  31:58
   5 NON-EMPTY CLASSES    1  2  3  4  5
 PARTITION       N       WEIGHT         Y-MEAN  Y-VARIANCE         X-MEAN  X-VARIANCE           SLOPE             BSS
 BETWEEN  1    386 1.41750E 04   1.88454E 04 1.11413E 08   1.06581E 04 3.52740E 07   1.04984E 00
     AND  2   1702 6.74000E 04   2.03317E 04 1.74009E 08   1.02207E 04 3.52307E 07   1.34702E 00   8.56980E 10
 BETWEEN  2    968 3.71490E 04   2.12913E 04 1.43568E 08   1.15528E 04 3.37033E 07   1.21050E 00
```

```
     AND  3  1120 4.44260E 04  1.90551E 04 1.77832E 08  9.24638E 03 3.41488E 07  1.38168E 00  3.16754E 10
 BETWEEN  3  1304 5.07440E 04  2.22962E 04 1.71517E 08  1.16082E 04 3.41871E 07  1.31504E 00
     AND  4   784 3.08310E 04  1.64149E 04 1.28679E 08  8.13816E 03 2.95526E 07  1.14509E 00  5.79988E 10
 BETWEEN  4  1549 6.04550E 04  2.15046E 04 1.65281E 08  1.11735E 04 3.43420E 07  1.31448E 00
     AND  5   539 2.10800E 04  1.55661E 04 1.35481E 08  7.78065E 03 2.93706E 07  1.11621E 00  4.09532E 10
    BEST SPLIT ON PREDICTOR 1498 =  8.569802E 10 AFTER CLASS  1

PREDICTOR  CURRENT REGION 0V472
   4 NON-EMPTY CLASSES   4  2  3  1
 PARTITION       N       WEIGHT          Y-MEAN  Y-VARIANCE         X-MEAN  X-VARIANCE         SLOPE          BSS
 BETWEEN  4   317 1.32710E 04  2.13324E 04 1.22307E 08  1.06194E 04 3.13668E 07  9.40384E-01
     AND  2  1771 6.83040E 04  1.98288E 04 1.71086E 08  1.02340E 04 3.59993E 07  1.34933E 00  7.05985E 10
 BETWEEN  2   934 4.01C80E 04  2.03634E 04 1.31437E 08  1.06382E 04 3.36420E 07  1.08504E 00
     AND  3  1154 4.14670E 04  1.57929E 04 1.94232E 08  9.96641E 03 3.66160E 07  1.47810E 00  1.12055E 11
 BETWEEN  3  1724 6.38510E 04  1.87222E 04 1.36095E 08  9.86245E 03 3.44896E 07  1.14924E 00
     AND  1   364 1.77240E 04  2.49414E 04 2.32022E 08  1.18611E 04 3.49511E 07  1.63622E 00  3.01520E 11
    BEST SPLIT ON PREDICTOR 1572 =  3.015201E 11 AFTER CLASS  3

PREDICTOR  3-YR AVE $ INC
   7 NON-EMPTY CLASSES   1  2  3  4  5  6  7
 PARTITION       N       WEIGHT          Y-MEAN  Y-VARIANCE         X-MEAN  X-VARIANCE         SLOPE          BSS
 BETWEEN  1   256 8.80500E 03  1.05296E 04 4.23378E 07  1.92335E 03 4.23077E 05  1.85793E 00
     AND  2  1832 7.27700E 04  2.12282E 04 1.65708E 08  1.13099E 04 2.99512E 07  1.33907E 00  2.20117E 10
 BETWEEN  2   513 1.65410E 04  1.17944E 04 5.73436E 07  2.85686E 03 1.36197E 06  1.23784E 00
     AND  3  1575 6.50340E 04  2.21791E 04 1.68534E 08  1.21890E 04 2.62059E 07  1.41314E 00  6.19079E 10
 BETWEEN  3   899 2.89780E 04  1.27768E 04 6.00404E 07  4.33335E 03 3.91099E 06  7.05368E-01
     AND  4  1189 5.25970E 04  2.40934E 04 1.74904E 08  1.35822E 04 2.21234E 07  1.44257E 00  7.27460E 10
 BETWEEN  4  1261 4.30130E 04  1.42346E 04 7.00466E 07  5.79847E 03 7.23897E 06  9.30808E-01
     AND  5   827 3.85620E 04  2.65862E 04 1.87193E 08  1.53142E 04 1.87425E 07  1.43132E 00  5.46434E 10
 BETWEEN  5  1758 6.54110E 04  1.70967E 04 1.02472E 08  8.02113E 03 1.49982E 07  1.17493E 00
     AND  6   330 1.61640E 04  3.21195E 04 2.29491E 08  1.95053E 04 1.14318E 07  1.76156E 00  5.44756E 10
 BETWEEN  6  1966 7.55320E 04  1.85559E 04 1.18597E 08  9.24882E 03 2.29945E 07  1.18059E 00
     AND  7   122 6.04300E 03  3.90410E 04 3.37204E 08  2.33944E 04 3.09720E 06  2.25368E 00  6.59848E 10
    BEST SPLIT ON PREDICTOR 2001 =  7.274601E 10 AFTER CLASS  3

PREDICTOR  RACE                   31:48
   3 NON-EMPTY CLASSES   2  3  1
 PARTITION       N       WEIGHT          Y=MEAN  Y-VARIANCE         X-MEAN  X-VARIANCE         SLOPE          BSS
 BETWEEN  2   406 5.49300E 03  1.17827E 04 5.52670E 07  6.96276E 03 2.54679E 07  8.87249E-01
     AND  3  1682 7.60820E 04  2.06720E 04 1.65903E 08  1.05374E 04 3.51028E 07  1.28359E 00  1.16451E 11
 BETWEEN  3   462 7.32200E 03  1.38035E 04 8.43736E 07  7.61994E 03 2.60314E 07  1.07415E 00
     AND  1  1626 7.42530E 04  2.06917E 04 1.66961E 08  1.05607E 04 3.53912E 07  1.28445E 00  7.27292E 10
    BEST SPLIT ON PREDICTOR 1490 =  1.164507E 11 AFTER CLASS  2

LRGST PLAC/SMSA PSU31:66 YIELDS MAXIMUM BSS =  3.508619E 11


0-STEP LOOKAHEAD TO SPLIT GROUP   1, TOTAL BSS =  3.508619E 11
        1. SPLIT GROUP   1 ON PREDICTOR 1506, BSS =  3.508619E 11
```

```
                 ***** PARTITION CF GROUP    1 *****
FROM ELIGIBLE PREDICTORS AROUND THE CURRENT RANK  1,     0 UP AND  0 DOWN

        MAXIMUM ELIGIBLE BSS AT EACH STEP          MAXIMUM TOTAL BSS (LOOKAHEAD)
    1.SPLIT    1 UN V15C6 BSS= 3.5C862E 11     SPLIT    1 CN V1506 BSS = 3.50862E 11
    PE(1)= 6.661E 10, TUTAL= 3.5C862E 11                         TOTAL= 3.50862E 11
        PREDICTOR 1506 HAS RANK   1

SPLIT GROUP    1 ON LRGST PLAC/SMSA PSU31:66 V1506
    GROUP    2 WITH  1012 OBSERVATIONS FROM  4 CLASSES =  3  4  5  6
               W= 4.04160E 04 Y= 6.491740 08 YSQ= 1.465650 13 X= 3.407900 08
             XSQ= 4.050660 12 Z= 6.604440 12
    GROUP    3 WITH  1076 OBSERVATIONS FROM  2 CLASSES =  1  2
               W= 4.11590E 04 Y= 9.883150 08 YSQ= 3.153590 13 X= 4.991640 08
             XSQ= 7.472210 12 Z= 1.396900 13

              2 CANDICATES -     GROUP       SS
                                    2    4.229269E 12
                                    3    7.804341E 12

                 ATTEMPT SPLIT CN GROUP    3 WITH SS = 7.804341E 12
```

*We omit the next 36 pages*
*tracing the split attempts.*

```
                  ***** PARTITION OF GROUP  12 *****
FROM ELIGIBLE PREDICTORS AROUND THE CURRENT RANK  1,    0 UP AND  0 DOWN

      MAXIMUM ELIGIBLE BSS AT EACH STEP        MAXIMUM TOTAL BSS (LOOKAHEAD)

GROUP  12 COULD NOT BE SPLIT


END OF STAGE 1 OF THE ANALYSIS.   7 FINAL GROUPS.   7 INELIGIBLE FOR SPLITTING.

                   VARIATION EXPLAINED (BSS(0)/TSS) =   7.4%


                   1-WAY ANALYSIS OF VARIANCE ON FINAL GROUPS

                   SOURCE             SUM OF SQUARES        DF     MEAN SQUARE

TOTAL                                  1.332240E 13    81536.    1.633931E 08
POOLED REGRESSION ON SAMPLE            4.795855E 12       1.    4.795855E 12

BETWEEN INDIV SLOPES           S(1)    3.565814E 11       6.    5.949690E 10
OBSNS BETWEEN INDIV LINES      S(2)    7.539118E 12    81523.    9.247854E 07
GROUP MEANS REGRESSION LINE    S(3)    5.690066E 11       5.    1.138013E 11
MEAN SLOPE VS MEANS REGR SLOPE S(4)    6.144236E 10       1.    6.144236E 10
```

*A single regression of house value on income over the whole sample accounts for 36% of the variance:*

$$\frac{480 \times 10^{10}}{1332 \times 10^{10}} = 36\%$$

*This is the unexplained variance around subgroup regression lines.*

| | |
|---|---|
| *Total SS (around mean) =* | $1332 \times 10^{10}$ |
| *- Expl. by overall regression* | $-480 \times 10^{10}$ |
| | $852 \times 10^{10}$ |
| *- Residual "error"* | $-754 \times 10^{10}$ |
| *= Marginal expl. by subgroup regressions (diff. means and diff. slopes)* | $98 \times 10^{10}$ |

$$\frac{98 \times 10^{10}}{1332 \times 10^{10}} = 7.4\% \text{ of variance}$$

*Around mean, explained by different subgroup regressions.*

OR:

$$\frac{98 \times 10^{10}}{852 \times 10^{10}} = 11.5\% \text{ of variance around overall regression, explained.}$$

*For decomposition of the 98 x $10^{10}$, see text ($S_1 + S_3 + S_4 = 98 \times 10^{10}$).*

$\frac{36}{98}$ *of it is differences in regression slopes.*

NOTE: *Mean squares and F-Tests are inappropriate with weighted data.*

```
                                                GROUP SUMMARY TABLE
                                         13 GROUPS OF WHICH  7 ARE FINAL

GROUP   1 , N =     2088, SUM W =  8.157500E 04
    Y MEAN= 2.007342E 04, VARIANCE= 1.633931E 08,    SS(L)/TSS= 1.000, BSS/TSS= 0.026
    X          1.029671E 04              3.524965E 07, CORRELATION= 0.600, B= 1.29176E 00, A= 6.77255E 03, A(NORM)= 2.00734E 04
    SPLIT ON LRGST PLAC/SMSA PSU31:66, BSS(L) =  3.508619E 11  INTO
            2 WITH CLASSES   3  4  5 , 6
            3 WITH CLASSES   1  2

GROUP   3 , N =     1076, SUM W =  4.115900E 04
    Y MEAN= 2.401213E 04, VARIANCE= 1.897908E 08,    SS(L)/TSS= 0.586, BSS/TSS= 0.020
    X          1.212770E 04              3.449597E 07, CORRELATION= 0.596, B= 1.39794E 00, A= 7.05830E 03, A(NORM)= 2.14525E 04
    SPLIT ON DIST TO CNTR SMSA  31:58, BSS(L) =  2.635197E 11  INTO
            4 WITH CLASSES   1  2
            5 WITH CLASSES   3  4  5

GROUP   4 , N =      791, SUM W =  2.952100E 04
    Y MEAN= 2.233470E 04, VARIANCE= 1.579482E 08,    SS(L)/TSS= 0.350, BSS/TSS= 0.008
    X          1.195866E 04              3.376765E 07, CORRELATION= 0.584, B= 1.26253E 00, A= 7.23593E 03, A(NORM)= 2.02364E 04
    SPLIT ON BKT AGE HEAD 9V1009         , BSS(L) =  1.090970E 11  INTO
            6 WITH CLASSES   2  3  4  5
            7 WITH CLASSES   6 ,

GROUP   2*, N =     1012, SUM W =  4.041600E 04
    Y MEAN= 1.606229E 04, VARIANCE= 1.047471E 08,    SS(L)/TSS= 0.317, BSS/TSS= 0.0
    X          8.432055E 03              2.915349E 07, CORRELATION= 0.507, B= 9.60458E-01, A= 7.96366E 03, A(NORM)= 1.78532E 04

GROUP   6 , N =      718, SUM W =  2.583700E 04
    Y MEAN= 2.280399E 04, VARIANCE= 1.520838E 08,    SS(L)/TSS= 0.295, BSS/TSS= 0.007
    X          1.281530E 04              3.008705E 07, CORRELATION= 0.600, B= 1.34883E 00, A= 5.51837E 03, A(NORM)= 1.94068E 04
    SPLIT ON BKT AGE HEAD 9V1009         , BSS(L) =  9.535016E 10  INTO
            8 WITH CLASSES   4  5
            9 WITH CLASSES   2  3

GROUP   5 , N =      285, SUM W =  1.163800E 04
    Y MEAN= 2.826706E 04, VARIANCE= 2.460673E 08,    SS(L)/TSS= 0.214, BSS/TSS= 0.006
    X          1.255647E 04              3.620957E 07, CORRELATION= 0.636, B= 1.65825E 00, A= 7.44527E 03, A(NORM)= 2.45198E 04
    SPLIT ON CURRENT REGION 0V472        , BSS(L) =  7.965927E 10  INTO
           10 WITH CLASSES   4  3  2
           11 WITH CLASSES   1

GROUP   9*, N =      383, SUM W =  1.348000E 04
    Y MEAN= 2.349883E 04, VARIANCE= 1.657862E 08,    SS(L)/TSS= 0.167, BSS/TSS= 0.0
    X          1.257920E 04              2.579814E 07, CORRELATION= 0.664, B= 1.68385E 00, A= 2.30735E 03, A(NORM)= 1.96455E 04

GROUP   8*, N =      335, SUM W =  1.235700E 04
    Y MEAN= 2.205689E 04, VARIANCE= 1.365040E 08,    SS(L)/TSS= 0.126, BSS/TSS= 0.0
    X          1.307288E 04              3.471758E 07, CORRELATION= 0.551, B= 1.09275E 00, A= 7.77146E 03, A(NORM)= 1.90232E 04

GROUP  11*, N =       97, SUM W =  5.111000E 03
    Y MEAN= 3.080915E 04, VARIANCE= 3.124820E 08,    SS(L)/TSS= 0.119, BSS/TSS= 0.0
    X          1.279541E 04              3.968642E 07, CORRELATION= 0.698, B= 1.95977E 00, A= 5.73309E 03, A(NORM)= 2.59123E 04

GROUP  10*, N =      188, SUM W =  6.527000E 03
    Y MEAN= 2.627646E 04, VARIANCE= 1.870159E 08,    SS(L)/TSS= 0.091, BSS/TSS= 0.0
    X          1.236937E 04              3.368021E 07, CORRELATION= 0.577, B= 1.36007E 00, A= 9.45323E 03, A(NORM)= 2.34575E 04

GROUP   7 , N =       73, SUM W =  3.684000E 03
    Y MEAN= 1.904338E 04, VARIANCE= 1.891791E 08,    SS(L)/TSS= 0.052, BSS/TSS= 0.007
    X          5.950719E 03              1.858064E 07, CORRELATION= 0.589, B= 1.87960E 00, A= 7.85839E 03, A(NORM)= 2.72121E 04
    SPLIT ON CURRENT REGION OV472     , BSS(I) =  8.890666E 10  INTO
          12 WITH CLASSES    4  3
          13 WITH CLASSES    2  1

GROUP  13*, N =       41, SUM W =  2.145000E 03
    Y MEAN= 2.171527E 04, VARIANCE= 2.569154E 08,    SS(L)/TSS= 0.040, BSS/TSS= 0.0
    X          5.968941E 03            2.050798E 07, CORRELATION= 0.729, B= 2.54792E 00, A= 6.50690E 03, A(NORM)= 3.27421E 04

GROUP  12*, N =       32, SUM W =  1.539000E 03
    Y MEAN= 1.531939E 04, VARIANCE= 7.585294E 07,    SS(L)/TSS= 0.008, BSS/TSS= 0.0
    X          5.925215E 03            1.649662E 07, CORRELATION= 0.328, B= 7.03581E-01, A= 1.11504E 04, A(NORM)= 1.83950E 04
```

*A (NORM) is the value the subgroup regression one predicts for V when X=grand mean of X. It is useful for comparing levels free from X-effects.*

```
                                   100*BSS/TSS TABLE FOR 0-STEP LOOK-AHEAD
                                          13 GROUPS,   8 PREDICTORS
                                   < INDICATES LESS THAN 1 SPLITS WERE MADE

PREDICTOR                          GROUP NUMBER (* INDICATES THE GROUP IS FINAL)
               1      3      4      2*     6      5      9*     8*    11*    10*     7     13*    12*
  1168       0.9    0.5    0.2    0.3    0.1    0.5    0.1    0.2    0.4    0.2    0.1    0.1    0.2
  1009       1.7    1.3    0.8    0.4    0.7    0.4    0.1    0.0    0.4    0.1  *****  *****  *****
  2002       0.9    0.4    0.3    0.3    0.2    0.1    0.1    0.1    0.1    0.1    0.2    0.2    0.1
  1506       2.6    0.6    0.2    0.2    0.2    0.0    0.2    0.1    0.0    0.0    0.1    0.0    0.0
  1498       0.6    2.0    0.2    0.1    0.1    0.1    0.2    0.0    0.3    0.3    0.4    0.2    0.0
  1572       2.3    1.6    0.6    0.2    0.6    0.6    0.2    0.3  *****   0.4    0.7    0.1    0.1
  2001       0.5    0.6    0.7    0.1    0.4    0.2    0.2    0.4    0.2    0.3    0.3    0.2    0.1
  1490       0.9    0.9    0.7    0.2    0.5    0.3    0.5    0.1    0.0    0.1    0.1    0.0    0.1
```

*Note how close some competing predictors came - indicating that another sample might give quite a different set of splits.*

*Most close runners-up, however, were used in later splits, e.g., V1572 in groups 5 and 7.*

```
                                  100*BSS/TSS TABLE FOR 0-STEP LOOK-AHEAD
                                         13 GROUPS,  8 PREDICTORS

                                   MAXIMUM BSS REGARDLESS OF ELIGIBILITY

PREDICTOR                             GROUP NUMBER (* INDICATES THE GROUP IS FINAL)
               1      3      4     2*      6      5     9*     8*    11*    10*     7     13*    12*
   1168       0.9    0.5    0.2    0.3    0.1    0.5    0.1    0.2    0.4    0.2    0.2    0.1    0.3
   1009       1.7    1.3    0.8    0.4    0.7    0.4    0.1    0.0    0.4    0.1******************
   2002       0.9    0.4    0.3    0.3    0.2    0.1    0.1    0.1    0.1    0.1    0.2    0.2    0.1
   1506       2.6    0.6    0.2    0.2    0.2    0.0    0.2    0.1    0.0    0.0    0.1    0.0    0.0
   1498       0.6    2.0    0.2    0.1    0.1    0.1    0.2    0.0    0.3    0.3    0.4    0.2    0.0
   1572       2.3    1.6    0.6    0.2    0.6    0.6    0.2    0.3******    0.4    0.7    0.1    0.1
   2001       0.5    0.6    0.7    0.1    0.4    0.2    0.2    0.4    0.2    0.3    0.3    0.2    0.1
   1490       0.9    0.9    0.7    0.2    0.5    0.3    0.5    0.1    0.2    0.1    0.2    0.0    0.1
```

```
                                             PROFILE OF CLASS MEANS AND SLOPES


                                     * REQUIRED ROOMS 23:12   , ETA = 0.C24
                                                         GROUP
CLASS           1            3            4            2*           6            5            9*           8*           11*          10*
  2   N        661          270          201          391          143           69           37          106           18           51
      YMEAN  1.762E 04    2.135E C4    2.029F 04    1.503E 04    2.124E 04    2.425F 04    2.328E 04    2.055E 04    2.473E 04    2.402E 04
      XMEAN  7.956E C3    9.928E 03    9.990E 03    6.588E 03    1.216E 04    9.760E 03    1.250E 04    1.205E 04    9.215E 03    1.002E 04
      SLOPE  1.068E 00    1.119E CO    1.054E 00    8.767E-01    1.169E 00    1.297E 00    2.230E 00    8.915E-01    1.519E 00    1.173E 00
  3   N        557          292          225          265          214           67          128           86           33           34
      YMEAN  2.122E 04    2.532E 04    2.300E 04    1.644E 04    2.301E 04    3.252F 04    2.338E 04    2.246E 04    3.439E 04    3.007E 04
      XMEAN  1.090E 04    1.243E C4    1.218E 04    9.124E 03    1.243E 04    1.318E 04    1.713E 04    1.288E 04    1.301E 04    1.341E 04
      SLOPE  1.457E 00    1.588E CO    1.415E CC    9.424E-01    1.396E 00    1.879E 00    1.483E 00    1.338E 00    2.071E 00    1.710E 00
  4   N        486          283          191          203          189           92          113           76           32           60
      YMEAN  2.256E 04    2.579E C4    2.304E 04    1.811E 04    2.321E 04    3.080F 04    2.419E 04    2.186E 04    3.377E 04    2.828E 04
      XMEAN  1.239F 04    1.363E C4    1.312E 04    1.068E 04    1.326E 04    1.456E 04    1.284F 04    1.384E 04    1.497E 04    1.424E 04
      SLOPE  1.499E 00    1.619E 00    1.361E 00    1.138E 00    1.371E 00    1.920F 00    1.821E 00    7.899E-01    2.103E 00    1.549E 00
  5   N        225          126           93           99           91           33           57           34            7           26
      YMEAN  1.999E 04    2.398E C4    2.440E C4    1.595E 04    2.444E 04    2.287E 04    2.435E 04    2.462E 04    2.732E 04    2.041E 04
      XMEAN  1.156E 04    1.322F 04    1.328E 04    9.836E 03    1.365E 04    1.307E 04    1.358E 04    1.379E 04    1.241E 04    1.343E 04
      SLOPE  1.589E 00    1.560F 00    1.552F CO    1.442E 00    1.689E 00    1.593F 00    1.601E 00    1.873E 00    3.803E CC    6.452F-01
  6   N        159          105           81           54           81           24           48           33            7           17
      YMEAN  1.998E 04    2.252E C4    2.313E 04    1.494E 04    2.313E 04    2.082E 04    2.063E 04    2.703E 04    1.807F 04    2.357E 04
      XMEAN  1.228E 04    1.330E C4    1.409E 04    1.026E 04    1.409E 04    1.111E 04    1.258E 04    1.645E 04    1.235F 04    9.883E 03
      SLOPE  1.377E 00    1.462E CO    1.348E 00    1.066E 00    1.348F 00    2.033F 00    1.213E 00    1.388F 00    1.527E 00    2.772E 00

      SLOPE  1.292E 00    1.398E 00    1.263E 00    9.605E-01    1.349E 00    1.658E 00    1.644E 00    1.093E 00    1.960E 00    1.360E 00


                                     * REQUIRED ROOMS 23:12   , ETA = 0.C24
                                                         GROUP
CLASS           7            13*          12*
  2   N         58           34           24
      YMEAN  1.940E 04    2.170F C4    1.318F 04
      XMEAN  5.687E C3    5.640F 03    5.761F 03
      SLOPF  1.688E 00    2.386E CO    7.064E-01
  3   N         11.           6            5
      YMEAN  2.296E C4    2.294F C4    2.310E 04
      XMEAN  8.036E 03    8.412F 03    7.579E 03
      SLOPE  2.183E 0C    2.C46E 00    2.632F-C1
  4   N          2            1            1
      YMEAN  1.143E C4    1.50CE C4    8.00CF 03
      XMEAN  3.298E 03    2.852E C3    3.726E 03
      SLOPE -8.341E 00    C.0          0.C
  5   N          2            0            2
      YMEAN  2.349E 04    C.0          2.349E 04
      XMEAN  4.866F 03    C.0          4.866F 03
      SLOPE  6.701E 01    C.0          6.692F 01

      SLOPE  1.880E 00    2.548E C0    7.036E-C1


                                     BKT AGE HEAD SV1009       , ETA = 0.C49
                                                         GROUP
CLASS           1            3            4            2*           6            5            9*           8*           11*          10*
  2   N        372          209          164          163          164           45          164            0           16           29
      YMEAN  1.987E 04    2.274F C4    2.072F 04    1.638E 04    2.072F 04    2.987E 04    2.072F 04    0.0          3.539E 04    2.525E 04
      XMEAN  1.083E 04    1.180F C4    1.148F C4    9.636E 03    1.148F 04    1.292E 04    1.148E 04    0.0          1.362E 04    1.234E 04
      SLOPE  1.778E 00    2.C29E C0    1.680F 00    1.112F 00    1.680E 00    2.605E 0C    1.680E 00    0.0          3.201E 00    1.786E 00
  3   N        530          319          219          211          219          100          219            0           37           62
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      YMEAN  2.370E 04     2.761E 04     2.577E 04     1.813E 04     2.577E 04     3.110E 04     2.577E 04     0.0           3.433E 04     2.795E 04
      XMEAN  1.257E 04     1.370E 04     1.348E 04     1.096E 04     1.348E 04     1.410E 04     1.348E 04     0.0           1.401E 04     1.419E 04
      SLOPE  1.675E 00     1.708E 00     1.635E 00     1.293E 00     1.635E 00     1.801E 00     1.635E 00     0.0           2.122E 00     1.322E 00
  4   N         479           268           195           211           195            73             0           195            22            51
      YMEAN  2.134E 04     2.476E 04     2.334E 04     1.677E 04     2.334E 04     2.805E 04     0.0           2.334E 04     3.064E 04     2.631E 04
      XMEAN  1.273E 04     1.415E 04     1.427E 04     1.083E 04     1.427E 04     1.386E 04     0.0           1.427E 04     1.500E 04     1.310E 04
      SLOPE  1.224E 00     1.200E 00     1.128E 00     9.988E-01     1.128E 00     1.401E 00     0.0           1.128E 00     1.719E 00     1.126E 00
  5   N         378           185           140           193           140            45             0           140            15            30
      YMEAN  1.908E 04     2.219E 04     2.037E 04     1.620E 04     2.037E 04     2.738E 04     0.0           2.037E 04     2.567E 04     2.862E 04
      XMEAN  9.781E 03     1.156E 04     1.150E 04     8.134E 03     1.150E 04     1.172E 04     0.0           1.150E 04     1.066E 04     1.250E 04
      SLOPE  1.124E 00     1.217E 00     1.061E 00     9.073E-01     1.061E 00     1.603E 00     0.0           1.061E 00     1.451E 00     1.664E 00
  6   N         329            95            73           234             0            22             0             0             7            15
      YMEAN  1.530E 04     1.877E 04     1.904E 04     1.376E 04     0.0           1.788E 04     0.0           0.0           1.489E 04     1.948E 04
      XMEAN  4.764E 03     5.591E 03     5.951E 03     4.396E 03     0.0           4.397E 03     0.0           0.0           2.492E 03     5.414E 03
      SLOPE  1.507E 00     1.888E 00     1.880E 00     1.220E 00     0.0           2.166E 00     0.0           0.0           9.892E 00     1.994E 00

      SLOPE  1.292E 00     1.398E 00     1.263E 00     9.605E-01     1.349E 00     1.658E 00     1.684E 00     1.093E 00     1.960E 00     1.360E 00


                                        BKT AGE HEAD 9V1009        , ETA = 0.049
                                                                    GROUP
CLASS            7            13*           12*
  6   N          73            41            32
      YMEAN  1.904E 04     2.172E 04     1.532E 04
      XMEAN  5.951E 03     5.969E 03     5.925E 03
      SLOPE  1.880E 00     2.548E 00     7.036E-01

      SLOPE  1.880E 00     2.548E 00     7.036E-01


                                        SEX & MAR STATUS           , ETA = 0.029
                                                                    GROUP
CLASS            1             3             4             2*            6             5             9*            8*           11*           10*
  1   N        1660           846           607           914           566           239           326           240            84           155
      YMEAN  2.109E 04     2.524E 04     2.340E 04     1.670E 04     2.364E 04     2.973E 04     2.381E 04     2.343E 04     3.262E 04     2.741E 04
      XMEAN  1.143E 04     1.327E 04     1.312E 04     9.475E 03     1.366E 04     1.363E 04     1.294E 04     1.456E 04     1.386E 04     1.345E 04
      SLOPE  1.369E 00     1.468E 00     1.339E 00     1.019E 00     1.400E 00     1.692E 00     1.708E 00     1.140E 00     1.994E 00     1.385E 00
  2   N          87            37            32            45            24             5             5            19             2             3
      YMEAN  1.443E 04     1.630E 04     1.583E 04     1.325E 04     1.844E 04     1.869E 04     3.447E 04     1.540E 04     6.600E 03     3.164E 04
      XMEAN  6.390E 03     7.890E 03     7.940E 03     5.450E 03     1.051E 04     7.638E 03     1.333E 04     9.973E 03     4.035E 03     1.150E 04
      SLOPE  1.083E 00     1.505E 00     1.325E 00     7.990E-01     1.739E 00     3.730E 00     2.750E 00     9.414E-01     7.552E-01     7.661E 00
  3   N         346           193           152           153           128            41            57            76            11            30
      YMEAN  1.580E 04     1.819E 04     1.773E 04     1.360E 04     1.747E 04     1.950E 04     1.796E 04     1.725E 04     2.089E 04     1.863E 04
      XMEAN  4.926E 03     6.098E 03     6.191E 03     3.847E 03     7.049E 03     5.827E 03     7.547E 03     6.829E 03     5.957E 03     5.745E 03
      SLOPE  1.588E 00     1.598E 00     1.459E 00     1.436E 00     1.521E 00     2.153E 00     1.775E 00     1.448E 00     2.302E 00     1.987E 00

      SLOPE  1.292E 00     1.398E 00     1.263E 00     9.605E-01     1.349E 00     1.658E 00     1.684E 00     1.093E 00     1.960E 00     1.360E 00


                                        SEX & MAR STATUS           , ETA = 0.029
                                                                    GROUP
CLASS            7            13*           12*
  1   N          41            22            19
      YMEAN  2.080E 04     2.534E 04     1.523E 04
      XMEAN  7.391E 03     7.519E 03     7.235E 03
      SLOPE  2.018E 00     2.626E 00     7.455E-01
  2   N           8             6             2
      YMEAN  1.202E 04     1.391E 04     6.000E 03
      XMEAN  4.209E 03     4.654E 03     2.785E 03
      SLOPE  1.215E-01    -1.769E-01    -5.539E-01
  3   N          24            13            11
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     YMEAN   1.841E 04      1.919E 04      1.737E 04
     XMEAN   3.395E 03      3.921E 03      4.082E 03
     SLOPE   2.683E 00      4.503E 00      1.101E 00

     SLOPE   1.880E 00      2.548E 00      7.036E-01


                                        LRGST PLAC/SMSA PSU31:66, ETA = 0.123
                                                      GROUP
CLASS            1              3              4              2*             6              5              9*             8*             11*            10*
  1  N         633            633            438              0            398            195            211            187             81            114
     YMEAN   2.583E 04      2.583E 04      2.351E 04      0.0            2.405E 04      3.014E 04      2.536E 04      2.267E 04      3.273E 04      2.732E 04
     XMEAN   1.263E 04      1.263E 04      1.213E 04      0.0            1.308E 04      1.354E 04      1.327E 04      1.288E 04      1.365E 04      1.343E 04
     SLOPE   1.456E 00      1.456E 00      1.300E 00      0.0            1.369E 00      1.632E 00      1.853E 00      9.675E-01      1.924E 00      1.258E 00
  2  N         443            443            353              0            320             90            172            148             16             74
     YMEAN   2.154E 04      2.154E 04      2.105E 04      0.0            2.147E 04      2.363E 04      2.155E 04      2.137E 04      2.069E 04      2.461E 04
     XMEAN   1.145E 04      1.145E 04      1.177E 04      0.0            1.253E 04      1.011E 04      1.187E 04      1.329E 04      8.425E 03      1.067E 04
     SLOPE   1.240E 00      1.240E 00      1.201E 00      0.0            1.305E 00      1.672E 00      1.423E 00      1.260E 00      1.910E 00      1.580E 00
  3  N         222              0              0            222              0              0              0              0              0              0
     YMEAN   1.855E 04      0.0            0.0            1.855E 04      0.0            0.0            0.0            0.0            0.0            0.0
     XMEAN   1.028E 04      0.0            0.0            1.028E 04      0.0            0.0            0.0            0.0            0.0            0.0
     SLOPE   9.284E-01      0.0            0.0            9.284E-01      0.0            0.0            0.0            0.0            0.0            0.0
  4  N         144              0              0            144              0              0              0              0              0              0
     YMEAN   1.846E 04      0.0            0.0            1.846E 04      0.0            0.0            0.0            0.0            0.0            0.0
     XMEAN   9.486E 03      0.0            0.0            9.486E 03      0.0            0.0            0.0            0.0            0.0            0.0
     SLOPE   8.726E-01      0.0            0.0            8.726E-01      0.0            0.0            0.0            0.0            0.0            0.0
  5  N         200              0              0            200              0              0              0              0              0              0
     YMEAN   1.578E 04      0.0            0.0            1.578E 04      0.0            0.0            0.0            0.0            0.0            0.0
     XMEAN   8.460E 03      0.0            0.0            8.460E 03      0.0            0.0            0.0            0.0            0.0            0.0
     SLOPE   1.076E 00      0.0            0.0            1.076E 00      0.0            0.0            0.0            0.0            0.0            0.0
  6  N         446              0              0            446              0              0              0              0              0              0
     YMEAN   1.394E 04      0.0            0.0            1.394E 04      0.0            0.0            0.0            0.0            0.0            0.0
     XMEAN   7.004E 03      0.0            0.0            7.004E 03      0.0            0.0            0.0            0.0            0.0            0.0
     SLOPE   8.637E-01      0.0            0.0            8.637E-01      0.0            0.0            0.0            0.0            0.0            0.0

     SLOPE   1.292E 00      1.398E 00      1.263E 00      9.605E-01      1.349E 00      1.658E 00      1.684E 00      1.093E 00      1.360E 00      1.360E 00


                                        LRGST PLAC/SMSA PSU31:66, ETA = 0.123
                                                      GROUP
CLASS            7              13*            12*
  1  N          40             21             19
     YMEAN   2.000E 04      2.375E 04      1.519E 04
     XMEAN   6.010E 03      6.646E 03      5.193E 03
     SLOPE   2.043E 00      2.582E 00      9.031E-01
  2  N          32             20             13
     YMEAN   1.782E 04      1.932E 04      1.550E 04
     XMEAN   5.875E 03      5.171E 03      6.964E 03
     SLOPE   1.475E 00      2.427E 00      2.262E-01

     SLOPE   1.880E 00      2.548E 00      7.036E-01


                                        DIST TO CNTR SMSA  31:58, ETA = 0.072
                                                      GROUP
CLASS            1              3              4              2*             6              5              9*             8*             11*            10*
  1  N         386            282            282            104            250              0            117            133              0              0
     YMEAN   1.885E 04      2.003E 04      2.003E 04      1.643E 04      2.123E 04      0.0            2.186E 04      2.065E 04      0.0            0.0
     XMEAN   1.066E 04      1.117E 04      1.117E 04      9.618E 03      1.248E 04      0.0            1.238E 04      1.257E 04      0.0            0.0
     SLOPE   1.050E 00      1.132E 00      1.132E 00      8.008E-01      1.235E 00      0.0            1.360E 00      1.142E 00      0.0            0.0
  2  N         582            509            509             73            468              0            266            202              0              0
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      YMEAN   2.280E 04   2.343E 04   2.343E 04   1.855E 04   2.350E 04   0.0         2.417E 04   2.276E 04   0.0         0.0
      XMEAN   1.210E 04   1.233E 04   1.233E 04   1.056E 04   1.296E 04   0.0         1.765E 04   1.333E 04   0.0         0.0
      SLOPE   1.282E 00   1.309E 00   1.309E 00   9.077E-01   1.393E 00   0.0         1.823E 00   1.061E 00   0.0         0.0
   3  N          336         212          0          124          0          212          0           0           59         153
      YMEAN   2.504E 04   2.975E 04   0.0         1.729E 04   0.0         2.975E 04   0.0         0.0         3.451E 04   2.700E 04
      XMEAN   1.176E 04   1.312E 04   0.0         9.522E 03   0.0         1.312E 04   0.0         0.0         1.367E 04   1.279E 04
      SLOPE   1.571E 00   1.592E 00   0.0         1.033E 00   0.0         1.592E 00   0.0         0.0         2.049E 00   1.212E 00
   4  N          245          55          0          190          0           55          0           0           27          28
      YMEAN   1.739E 04   2.147E 04   0.0         1.610E 04   0.0         2.147E 04   0.0         0.0         2.185E 04   2.091E 04
      XMEAN   9.911E 03   1.045E 04   0.0         8.429E 03   0.0         1.045E 04   0.0         0.0         1.079E 04   9.956E 03
      SLOPE   1.205E 00   1.604E 00   0.0         9.709E-01   0.0         1.604E 00   0.0         0.0         1.699E 00   1.497E 00
   5  N          539          18          0          521          0           18          0           0           11           7
      YMEAN   1.597E 04   2.217E 04   0.0         1.529E 04   0.0         2.217E 04   0.0         0.0         3.237E 04   3.162E 04
      XMEAN   7.781E 03   1.277E 04   0.0         7.573E 03   0.0         1.277E 04   0.0         0.0         1.293E 04   1.253E 04
      SLOPE   1.116E 00   1.772E 00   0.0         9.897E-01   0.0         1.772E 00   0.0         0.0         1.394E 00   3.367E 00

      SLOPE   1.292E 00   1.398E 00   1.263E 00   9.605E-01   1.349E 00   1.658E 00   1.684E 00   1.093E 00   1.960E 00   1.360E 00


                                     DIST TO CNTR SMSA  31:58, ETA = 0.072
                                                      GROUP
CLASS             7           13*          12*
   1  N          32           15           17
      YMEAN   1.427E 04   1.485E 04   1.375E 04
      XMEAN   4.833E 03   4.616E 03   5.026E 03
      SLOPE   7.088E-01   1.002E 00   5.502E-01
   2  N          41           26           15
      YMEAN   2.283E 04   2.554E 04   1.732E 04
      XMEAN   6.836E 03   6.722E 03   7.067E 03
      SLOPE   2.148E 00   2.695E 00   7.078E-01

      SLOPE   1.890E 00   2.548E 00   7.036E-01


                                     CURRENT REGION OV472     , ETA = 0.063
                                                      GROUP
CLASS             1            3            4            2*           6            5            9*           8*           11*          10*
   1  N          364          253          156          111          143          97           75           68           97            0
      YMEAN   2.494E 04   2.796E 04   2.597E 04   1.786E 04   2.608E 04   3.081E 04   2.700E 04   2.514E 04   3.081E 04   0.0
      XMEAN   1.186E 04   1.288E 04   1.295E 04   9.464E 03   1.375E 04   1.280E 04   1.358E 04   1.392E 04   1.280E 04   0.0
      SLOPE   1.636E 00   1.696E 00   1.487E 00   9.931E-01   1.712E 00   1.960E 00   2.028E 00   1.458E 00   1.960E 00   0.0
   2  N          617          330          265          287          237          65          123          114           0           65
      YMEAN   1.988E 04   2.277E 04   2.153E 04   1.695E 04   2.176E 04   2.798E 04   2.327E 04   2.035E 04   0.0         2.798E 04
      XMEAN   1.065E 04   1.253E 04   1.219E 04   8.735E 03   1.310E 04   1.395E 04   1.266E 04   1.350E 04   0.0         1.395E 04
      SLOPE   1.150E 00   1.255E 00   1.138E 00   9.159E-01   1.171E 00   1.652E 00   1.517E 00   9.587E-01   0.0         1.652E 00
   3  N          790          296          218          494          199          78          110           89           0           78
      YMEAN   1.595E 04   1.936E 04   1.804E 04   1.425E 04   1.889E 04   2.296E 04   1.933E 04   1.813E 04   0.0         2.296E 04
      XMEAN   8.552E 03   1.076E 04   1.074E 04   7.451E 03   1.159E 04   1.083E 04   1.165E 04   1.150E 04   0.0         1.083E 04
      SLOPE   1.179E 00   1.411E 00   1.323E 00   1.007E 00   1.394E 00   1.615E 00   1.688E 00   1.085E 00   0.0         1.615E 00
   4  N          317          197          152          120          139          45           75           64           0           45
      YMEAN   2.133E 04   2.462E 04   2.361E 04   1.729E 04   2.437E 04   2.771E 04   2.439E 04   2.434E 04   0.0         2.771E 04
      XMEAN   1.062E 04   1.157E 04   1.147E 04   9.448E 03   1.223E 04   1.189E 04   1.226E 04   1.220E 04   0.0         1.189E 04
      SLOPE   9.404E-01   8.984E-01   9.806E-01   7.850E-01   1.045E 00   6.205E-01   1.272E 00   8.874E-01   0.0         6.205E-01

      SLOPE   1.292E 00   1.398E 00   1.263E 00   9.605E-01   1.349E 00   1.658E 00   1.684E 00   1.093E 00   1.960E 00   1.360E 00


                                     CURRENT REGION OV472     , ETA = 0.063
                                                      GROUP
CLASS             7           13*          12*
   1  N          13           13            0
```

```
     YMEAN    2.497E 04   2.497E 04   0.0
     XMEAN    5.911E 03   5.911E 03   0.0
     SLOPE    2.704E 00   2.704E 00   0.0
  2  N           28          28            0
     YMEAN    1.995E 04   1.995E 04   0.0
     XMEAN    6.001E 03   6.001E 03   0.0
     SLOPE    2.504E 00   2.504E 00   0.0
  3  N           19           0           19
     YMEAN    1.336E 04   0.0         1.336E 04
     XMEAN    6.029E 03   0.0         6.029E 03
     SLOPE    1.133E 00   0.0         1.133E 00
  4  N           13           0           13
     YMEAN    1.799E 04   0.0         1.799E 04
     XMEAN    5.785E 03   0.0         5.785E 03
     SLOPE    9.983E-02   0.0         9.988E-02

     SLOPE    1.880E 00   2.548E 00   7.036E-01


                                3-YR AVE $ INC            , ETA = 0.351
                                                           GROUP
CLASS             1           3           4          2*           6           5           9*          8*         11*         10*
  1  N          256          69          52         187          32          17           8          24           6          11
     YMEAN    1.053E 04   1.162E 04   1.078E 04   1.020E 04   9.618E 03   1.346E 04   6.161E 03   1.020E 04   1.020E 04   1.687E 04
     XMEAN    1.923E 03   2.049E 03   2.041E 03   1.886E 03   1.917E 03   2.066E 03   2.554E 03   1.811E 03   2.113E 03   2.018E 03
     SLOPE    1.859E 00   4.292E 00   2.234E 00   1.051E 00  -3.399E-02   9.278E 00   3.335E 00   6.982E-01  -2.374E 00   1.362E 01
  2  N          257          99          79         158          60          20          27          33           3          17
     YMEAN    1.373E 04   1.569E 04   1.469E 04   1.215E 04   1.478E 04   1.966E 04   1.403E 04   1.518E 04   2.865E 04   1.536E 04
     XMEAN    3.919E 03   3.908E 03   3.845E 03   3.925E 03   3.931E 03   4.156E 03   3.920E 03   3.937E 03   4.625E 03   4.027E 03
     SLOPE   -5.459E-01   2.504E 00   9.737E-01  -1.507E 00   2.293E 00   4.815E 00   2.848E 00   1.991E 00  -1.104E 01   4.945E-01
  3  N          386         182         127         204         115          55          68          47          14          39
     YMEAN    1.408E 04   1.558E 04   1.597E 04   1.310E 04   1.489E 04   1.469E 04   1.464E 04   1.522E 04   1.161E 04   1.590E 04
     XMEAN    6.297E 03   6.317E 03   6.317E 03   6.284E 03   6.434E 03   6.317E 03   6.442E 03   6.423E 03   6.346E 03   6.152E 03
     SLOPE   -5.791E-01  -3.003E 00  -3.314E 00   8.110E-01  -1.075E 00  -2.365E 00  -1.896E 00   4.508E-01  -6.548E-01  -2.537E 00
  4  N          362         188         145         174         137          43          78          59          16          27
     YMEAN    1.724E 04   1.857E 04   1.791E 04   1.571E 04   1.698E 04   2.233E 04   1.677E 04   1.702E 04   2.329E 04   2.131E 04
     XMEAN    8.824E 03   8.834E 03   8.869E 03   8.814E 03   8.895E 03   8.723E 03   8.963E 03   8.805E 03   8.650E 03   8.797E 03
     SLOPE    2.755E 00   3.295E 00   2.575E 00   2.340E 00   7.081E 00   6.403E 00   1.588E 00   2.839E 00   5.941E 00   7.543E 00
  5  N          497         295         217         202         205          78         126          79          27          51
     YMEAN    2.259E 04   2.387E 04   2.218E 04   2.078E 04   2.203E 04   2.822E 04   2.226E 04   2.171E 04   3.070E 04   2.654E 04
     XMEAN    1.229E 04   1.232E 04   1.223E 04   1.225E 04   1.225E 04   1.295E 04   1.211E 04   1.244E 04   1.367E 04   1.247E 04
     SLOPE    1.297E 00   1.459E 00   1.323E 00   1.034E 00   1.452E 00   1.199E 00   1.951E 00   8.324E-01   7.066E-01   1.374E 00
  6  N          208         157         111          51         110          46          50          60          14          30
     YMEAN    2.799E 04   2.940E 04   2.678E 04   2.346E 04   2.675E 04   3.528E 04   2.957E 04   2.432E 04   4.061E 04   3.184E 04
     XMEAN    1.718E 04   1.717E 04   1.709E 04   1.722E 04   1.710E 04   1.736E 04   1.699E 04   1.720E 04   1.724E 04   1.745E 04
     SLOPE    1.163E 00   1.351E 00   1.066E 00   4.069E-01   1.106E 00   1.232E 00   1.299E 00   1.194E 00   4.964E-01   1.708E 00
  7  N          127          86          60          36          59          26          26          33          11          13
     YMEAN    3.904E 04   4.244E 04   3.995E 04   3.029E 04   3.337E 04   4.783E 04   4.423E 04   3.542E 04   5.721E 04   4.260E 04
     XMEAN    2.339E 04   2.339E 04   2.344E 04   2.341E 04   2.341E 04   2.329E 04   2.336E 04   2.345E 04   2.345E 04   2.309E 04
     SLOPE    2.255E 00   2.370E 00   2.968E 00   1.953E 00   2.703E 00   1.507E 00   1.911E 00   3.541E 00  -1.064E 00   4.516E 00

     SLOPE    1.292E 00   1.398E 00   1.263E 00   9.605E-01   1.349E 00   1.658E 00   1.684E 00   1.033E 00   1.360E 00   1.340E 00


                                3-YR AVE $ INC            , ETA = 0.351
                                                           GROUP
CLASS             7          13*         12*
  1  N           20          12           8
     YMEAN    1.130E 04   1.172E 04   1.053E 04
     XMEAN    2.096E 03   2.254E 03   1.805E 03
     SLOPE    3.618E 00   1.942E 00   4.657E 00
  2  N           19           9           10
     YMEAN   1.461E 04   1.702E 04   1.219E 04
     XMEAN   3.770E 03   3.771E 03   3.769E 03
     SLOPE  -7.157E-01   5.199E 00  -3.598E 00
  3  N           12           8            4
     YMEAN   2.110E 04   1.867E 04   2.601E 04
     XMEAN   5.762E 03   5.656E 03   5.976E 03
     SLOPE  -6.846E 00  -1.304E 00  -1.609E 01
  4  N            8           4            4
     YMEAN   2.955E 04   4.126E 04   1.725E 04
     XMEAN   8.571E 03   8.917E 03   8.208E 03
     SLOPE   1.601E 01   1.033E 01   3.623E 00
  5  N           12           7            5
     YMEAN   2.442E 04   3.079E 04   1.524E 04
     XMEAN   1.190E 04   1.169E 04   1.221E 04
     SLOPE  -2.274E-01  -7.225E-01   1.953E 00
  6  N            1           0            1
     YMEAN   3.000E 04   0.0         3.000E 04
     XMEAN   1.539E 04   0.0         1.539E 04
     SLOPE   0.0         0.0         0.0
  7  N            1           1            0
     YMEAN   7.500E 04   7.500E 04   0.0
     XMEAN   2.500E 04   2.500E 04   0.0
     SLOPE   0.0         0.0         0.0

     SLOPE   1.880E 00   2.548E 00   7.036E-01


                             RACE                 31:48, ETA = 0.030
                                                          GROUP
CLASS            1           3           4           2*          6           5           9*          8*          11*         10*
  1  N          1626        775         537         851         476         238         243         233          94         144
     YMEAN   2.069E 04   2.498E 04   2.341E 04   1.649E 04   2.376E 04   2.862E 04   2.476E 04   2.276E 04   3.062E 04   2.699E 04
     XMEAN   1.056E 04   1.254E 04   1.243E 04   8.624E 03   1.324E 04   1.279E 04   1.295E 04   1.353E 04   1.288E 04   1.272E 04
     SLOPE   1.284E 00   1.390E 00   1.255E 00   9.332E-01   1.361E 00   1.641E 00   1.730E 00   1.103E 00   1.974E 00   1.310E 00
  2  N           406         256         216         150         209          40         113          92           2          38
     YMEAN   1.178E 04   1.354E 04   1.313E 04   9.660E 03   1.441E 04   1.679E 04   1.471E 04   1.392E 04   2.388E 04   1.505E 04
     XMEAN   6.963E 03   8.400E 03   8.510E 03   5.728E 03   9.985E 03   7.543E 03   1.090E 04   8.539E 03   7.336E 03   7.594E 03
     SLOPE   8.872E-01   7.978E-01   7.580E-01   9.619E-01   7.158E-01   2.135E 00   9.324E-01   4.265E-01   1.670E 01   2.154E 00
  3  N            56          45          38          11          37           7          27          10           1           6
     YMEAN   1.987E 04   2.109E 04   1.981E 04   1.553E 04   1.980E 04   2.827E 04   2.019E 04   1.873E 04   5.500E 04   1.828E 04
     XMEAN   9.594E 03   9.376E 03   9.541E 03   1.037E 04   9.842E 03   8.456E 03   1.027E 04   8.667E 03   1.189E 04   7.171E 03
     SLOPE   1.246E 00   1.385E 00   1.204E 00   1.163E 00   1.335E 00   7.530E 00   1.488E 00   7.994E-01   3.750E 00   3.716E 00

     SLOPE   1.292E 00   1.398E 00   1.263E 00   9.605E-01   1.349E 00   1.658E 00   1.684E 00   1.093E 00   1.960E 00   1.360E 00


                             RACE                 31:48, ETA = 0.030
                                                          GROUP
CLASS            7           13*         12*
  1  N            61          37          24
     YMEAN   2.088E 04   2.316E 04   1.714E 04
     XMEAN   6.599E 03   6.352E 03   7.003E 03
     SLOPE   1.750E 00   2.454E 00   3.778E-01
  2  N            11           4           7
     YMEAN   7.707E 03   8.232E 03   7.348E 03
     XMEAN   2.268E 03   2.382E 03   2.190E 03
     SLOPE   2.779E-01   6.028E 00  -2.058E-01
  3  N             1           0           1
     YMEAN   2.000E 04   0.0         2.000E 04
     XMEAN   3.337E 03   0.0         3.337E 03
     SLOPE   0.0         C.0         0.0
     SLOPE   1.880E CO   2.548E CC   7.036F-01
..... END OF ANALYSIS .....
```

Appendix VI

# A Note on Partitioning for Maximum between Sum of Squares

11/10/62

by W. A. Ericson

## 1. The Problem

This note presents some results, both positive and negative, concerned with analysis of the following problem:

One is given $k>2$ sets of observations, where

$$\bar{x}_i \ , \quad i = 1, 2, \ldots, k$$

is the mean of the observations within the i'th set and

$$N_i \ , \quad i = 1, 2, \ldots, k$$

is the number of observations in that set. The problem is to partition these $k$ sets of observations into two nonempty classes such that "between class sum of squares" is maximized. In other words, to find $I$, a set of any $m$ $(1 \leq m \leq k)$ of the $k$ indices $i = 1, 2, \ldots, k$, such that

$$N_I(\bar{x}_I - \bar{x})^2 + N_{\bar{I}}(\bar{x}_{\bar{I}} - \bar{x})^2 \tag{1}$$

is maximized, where

$$N_I = \sum_{i \varepsilon I} N_i \ , \qquad N_{\bar{I}} = \sum_{i \not\varepsilon I} N_i \ ,$$

$$\bar{x}_I = \frac{1}{N_I} \sum_{i \varepsilon I} N_i \bar{x}_i \ , \qquad \bar{x}_{\bar{I}} = \frac{1}{N_{\bar{I}}} \sum_{i \not\varepsilon I} N_i \bar{x}_i \ ,$$

and $\bar{x}$ is the overall mean, i.e.,

$$\bar{x} = \frac{N_I \bar{x}_I + N_{\bar{I}} \bar{x}_{\bar{I}}}{N_I + N_{\bar{I}}} \ .$$

## 2. Previous Results

No literature search having been made, it is not known whether this problem has been researched by other investigators. This remains a point for further study.

## 3. Restatement and Assumptions

It is well-known that the problem outlined above is basically unchanged by the addition of the same arbitrary constant to each $\bar{x}_i$. It may thus be assumed without loss of generality that

$$\bar{x}_1 \geq \bar{x}_2 \geq \ldots \geq \bar{x}_k > 0 \tag{2}$$

Furthermore, it is easily seen that maximizing (1) by choice of I is equivalent to maximizing

$$f(I) \equiv \frac{(N_I \bar{x}_I)^2}{N_I} + \frac{(N_{\bar{I}} \bar{x}_{\bar{I}})^2}{N_{\bar{I}}} \tag{3}$$

## 4. A Negative Result

The following algorithm was suggested for finding I and its complement, $\bar{I}$, which maximizes (3):

a) Compute f(I) for I taken, in turn to be {1} , {2}, ..., {k} .

b) Pick the maximum f(I) over these I's. Suppose, e.g., I = {a} maximizes f(I) over the I's considered in (a).

c) Compute f(I) for I taken in turn to be {a,1} , ..., {a, a - 1} , {a, a + 1} , ... {a, k} .

d) Choose that I, among those considered in (c) which maximizes f(I), say I = {a,b} . If f( {a} ) > f( {a,b} ), stop and assert I = {a} yields maximum value of (3), otherwise continue the process, looking next at f(I) for I's of the form {a,b,i} , $i \neq a$, $i \neq b$, repeating steps (c) and (d) above.

This procedure does <u>not</u> lead invariably to the optimum or maximizing partition, I. That this is so is demonstrated by the following counterexample:

Suppose k = 5 and the data are as shown below:

| i: | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\bar{x}_i$: | 3.1 | 3.0 | 2.0 | 2.0 | 1.0 |
| $N_i$: | 1 | 2 | 3 | 1 | 3 |

It is easily verified that

| I | $\bar{I}$ | f(I) |
|---|---|---|
| {1} | {2,3,4,5} | 41.72111 |
| {2} | {1,3,4,5} | 42.85125 |
| {3} | {1,2,4,5} | 40.40142 |
| {4} | {1,2,3,5} | 39.31764 |
| {5} | {1,2,3,4} | 44.77285 |

Following the suggested algorithm we next look at $I = (5,i)$ , $i = 1,2,3,4$, and obtain the following:

| I | $\bar{I}$ | f(I) |
|---|---|---|
| {5,1} | {2,3,4} | 41.96916 |
| {5,2} | {1,3,4} | 40.84200 |
| {5,3} | {1,2,4} | 44.30250 |
| {5,4} | {1,2,3} | 44.25166 |

Each of these values of $f(I)$ being less than $f(\{5\})$, we conclude, as per the suggested algorithm, that $I = \{5\}$ maximizes (3). This is not true since it is easily shown that

$$f(\{1,2\}) = 44.88904 > f(\{5\}) = 44.77285$$

## 5. The Basic Result

It will be proved in this section that (3) is maximized over all possible I's by $I^*$ where $I^*$ is that set $I_m \equiv \{1,2, \ldots, m\}$ , $1 \leq m < k$ for which $f(I^*) \geq f(I_m)$ for all m. Thus to find the maximizing partition one need only compute f(I) for the $k-1$ sets $I_m$ and choose the maximum. Furthermore, $I^*$, obtained in this fashion, maximizes (3) over any partition of the $N = \sum_1^k N_i$ individual observations into two sets (assuming each individual observation within any set equals the set mean $\bar{x}_i$ say).

The present proof of these assertions, while straightforward, involves considerable tedious algebra. Further study may yield more succinct and more tidy demonstrations. The present proof is given in two parts. We first state and prove the theoretical results, in some degree of generality and then make the necessary identifications to the problem stated in § 1 by which the assertions stated above become established.

We adopt the following notation: let

$$a_1 \geq a_2 \geq a_3 \geq \ldots \geq a_N \tag{4}$$

be any nonincreasing sequence of real positive numbers. Let $P_m$ and $P_n$ be <u>any</u> partition of the N $a_i$'s, i.e., $P_m$ is any set of m of the $a_i$'s and $P_n$ is the set of the remaining $n = N - m$ $a_i$'s. Further, let $H_m$, $L_m$ and M be respectively the set of the largest m $a_i$'s, the set of the smallest m $a_i$'s, and the $n - m$ middle $a_i$'s. (It is assumed that $n \geq m$, hence M is null if $n = m$, otherwise not.) Thus

$$H_m = \{a_1, \ldots, a_m\}$$

$$L_m = \{a_{N-m+1}, \ldots, a_N\}$$

$$M = \{a_{m+1}, \ldots, a_{N-m}\}$$

The first result may then be stated as

<u>Theorem A:</u> At least one of the following is true:

$$\text{a)} \quad \frac{(\Sigma(H_m))^2}{m} + \frac{(\Sigma(M) + \Sigma(L_m))^2}{n} \geq \frac{(\Sigma(P_m))^2}{m} + \frac{(\Sigma(P_n))^2}{n}$$

$$\text{b)} \quad \frac{(\Sigma(L_m))^2}{m} + \frac{(\Sigma(M) + \Sigma(H_m))^2}{n} \geq \frac{(\Sigma(P_m))^2}{m} + \frac{(\Sigma(P_n))^2}{n} \quad ,$$

where $\Sigma(H_m) \equiv \sum_{a_i \in H_m} a_i$ , etc.

<u>Proof</u>: The theorem is obviously true if either $\Sigma(L_m) = \Sigma(P_m)$ or $\Sigma(H_m) = \Sigma(P_m)$. We then consider the other cases, i.e., $\Sigma(H_m) > \Sigma(P_m) > \Sigma(L_m)$, and show that if (a) fails then (b) holds. Straightforward algebra[1] shows that if (a) is false, then

$$[m\Sigma(P_n) + m(\Sigma(L_m) + \Sigma(M)) - n(\Sigma(H_m) + \Sigma(P_m))] > 0. \tag{5}$$

[1] The major hint needed in going from (a) and (b) to (5) and (6) is to replace

$$[\Sigma(m) + \Sigma(L_m)]^2 \text{ by } [\Sigma(m) + \Sigma(L_m)] \; [\Sigma(P_n) + \Sigma(P_n) - \Sigma(H_m)]$$

and to replace

$$[\Sigma(P_n)]^2 \text{ by } [\Sigma(P_n)] \; [\Sigma(m) + \Sigma(L_m) + \Sigma(H_m) - \Sigma(P_m)] \quad \text{etc.}$$

Similarly, (b) is true if

$$[m\Sigma(P_n) + m(\Sigma(M) + \Sigma(H_m)) - n(\Sigma(L_m) + \Sigma(P_m))] \geq 0. \qquad (6)$$

That (5) implies (6) is obvious, since the left side of (6) is greater than or equal to the left side of (5). ▲

The second main result is given by the following:

<u>Theorem B:</u> Suppose

$$a_1 \geq \cdots \geq a_m > a_{m+1} = \cdots = a_{m+n}$$

$$= a_{m+n+1} = \cdots = a_{m+n+\ell} > a_{m+n+\ell+1} \geq \cdots \geq a_{m+n+\ell+r}$$

where $m + n + \ell + r = N$, $m \geq 0$, $n > 0$, $\ell > 0$, $r \geq 0$, and $m + r \geq 1$. Then at least one of the following statements is defined and true:

d) $$\frac{1}{m}(\Sigma_m)^2 + \frac{1}{n+\ell+r}((n+\ell)a + \Sigma_r)^2 \geq \frac{1}{m+n}(\Sigma_m + na)^2 + \frac{1}{\ell+r}(\ell a + \Sigma_r)^2$$

or

d) $$\frac{1}{m+n+\ell}(\Sigma_m + (n+\ell)a)^2 + \frac{1}{r}(\Sigma_r)^2 \geq \frac{1}{m+n}(\Sigma_m + na)^2 + \frac{1}{\ell+r}(\ell a + \Sigma_r)^2 ,$$

where $a \equiv a_i$, $i = m + 1, \ldots, m + n + \ell$, $\Sigma_m \equiv \sum_{i=1}^{m} a_i$ ,

$\Sigma_r \equiv \sum_{i=1}^{r} a_{m+n+\ell+i}$ .

▼ <u>Proof</u>: If $m = 0$, it is immediately verifiable that (d) is true. Likewise, if $r = 0$, then (c) is true. Suppose then that m, n, r, and $\ell$ are all positive. Straightforward algebra shows that (c) is equivalent to

c') $$A \equiv (\Sigma_m)^2 - 2ma\,\Sigma_m \geq \frac{m(m+n)}{(n+\ell+r)\ (\ell+r)}(\Sigma_r)^2 - \frac{2mr\ (m+n)}{(n+\ell+r)\ (\ell+r)}a\Sigma_r$$

$$+ \frac{m\ [(m+n)\ell - (\ell+r)n]^2 - m[(m+n)\ell^2 + (\ell+r)n^2]}{n(n+\ell+r)\ (\ell+r)}a^2 \equiv B$$

and (d) is equivalent to:

d') $$A \equiv (\Sigma_m)^2 - 2ma \leq \frac{(m+n+\ell)\ (m+n)}{r(\ell+r)}(\Sigma_r)^2 - \frac{2(m+n+\ell)\ (m+n)}{(\ell+r)}a\Sigma_r$$

$$- \frac{\{[(m+n)\ell - (\ell+r)n]^2 - r[(m+n)\ell^2 + (\ell+r)n^2]\}}{\ell(\ell+r)}a^2 \equiv C .$$

To show that either (c) or (d) is true (or both) it suffices then to show that if (c') is false then (d') must be true. This is clearly established if the right side of the inequality in (c') is less than or equal to the right side of the inequality in (d'), i.e., if $C - B \geq 0$. But some simple but tedious algebra shows that

$$C - B = \frac{(m+n)\,[(n+\ell+r)\,(m+n+\ell) - mr]}{r(n+\ell+r)\,(\ell+r)}\,[\Sigma_r - ra]^2 \quad .$$

which is obviously nonnegative. ▲

To use these results for the problem stated in § 1 above and to establish the assertions at the beginning of the present section one need only identify the following nonincreasing sequence with those sequences of $a_i$'s referred to above:

$$\underbrace{\bar{x}_1, \ldots, \bar{x}_1}_{N_1}, \quad \underbrace{\bar{x}_2, \ldots, \bar{x}_2}_{N_2}, \quad \underbrace{\bar{x}_3, \ldots, \bar{x}_3}_{N_3}, \ \ldots, \ \underbrace{\bar{x}_k, \ldots, \bar{x}_k}_{N_k} \quad .$$

Then it is clear that Theorem A establishes the fact that for <u>any</u> partition of these $N = \sum_1^k N_i$ $\bar{x}_i$ 's into two sets of $m$ and $n = N - m$ elements respectively will yield a value of "between sum of squares," (3), no larger than that for either the partition consisting of the $m$ largest $\bar{x}_i$'s and the $N-m$ remaining or the $m$ smallest $\bar{x}_i$'s and the $N-m$ remaining. This result clearly includes the case where for every $i = 1, \ldots, k$ all $N_i$ $\bar{x}_i$'s are put in the same one of the two sets forming the partition, i.e., the case where the partition is of the $k$ sets of means rather than of the $N$ individual means.

Theorem B then closes the remaining loophole, viz., it may be that some partition, $I$, $\bar{I}$, of the $k$ sets of means into $N_I = \sum_{i \in I} N_i$ and $N_{\bar{I}} = N - N_I$ observations, respectively, has a sum of squares, (3), which is no larger than that for the partition consisting, say, of the largest $N_I$ individual $\bar{x}_i$'s and the $N_{\bar{I}}$ remaining $\bar{x}_i$'s. However, this latter partition may very easily split one set of $N_i$ identical $\bar{x}_i$'s. Theorem B then says that for any partition of the N individual $\bar{x}_i$'s into the $m$ largest and $N - m$ remaining and where the partioning point occurs within one of the $k$ sets of observations then there is another partition into largest and smallest $\bar{x}_i$'s where the partitioning point occurs <u>between</u> two of the $k$ sets of $\bar{x}_i$'s and which has a between sum of squares no smaller than the original partition.

Theorems A and B then together demonstrate that to find the partition which maximizes (3) one need only look at the $k - 1$ partitions, $I_m$, where $I_m = \{1,2, \ldots, m\}$ , $1 \leq m < k$, and choose that one yielding the largest value of (3).

## 6. A Final Negative Result

It was further conjectured that perhaps (3), $f(\{I_m\})$, $m = 1,2, \ldots, k - 1$, treated as a function of $m$ was well-behaved in the sense of say concavity and that, e.g., if $f(\{I_1\}) > f(\{I_2\})$ then one might be able to stop and assert $I^* = I_1$, and thus not look at all $k - 1$ $I_m$'s. This is not the case, however, as witnessed by the following counter example:

| $i$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\bar{x}_i$ | 3.000 | 2.01000 | 2.0010 | 2.0001 | 1.0000 |
| $N_i$ | 1 | 1 | 1 | 1 | 2 |

,

here one finds the following values for $f(\{I_m\})$, $m = 1, 2, 3, 4$:

| $I_m$ | $f(\{I_m\})$ |
|---|---|
| {1} | 21.84 |
| {1,2} | 21.55 |
| {1,2,3} | 21.72 |
| {1,2,3,4} | 22.30 |

## 7. Conclusions

The above results indicate that to find the partition which maximizes the between sum of squares, (3), one need only compute (3) for the $k - 1$ partitions consisting of the first set of size $N_1$ and all the rest, the first two sets of size $N_1 + N_2$ and all the remaining, etc., and choosing that one which maximizes (3). Further the partition found in this manner maximizes (3) over all partitions of the $N = \sum_1^k N_i$ individual observations (assuming each observation within any one of the $k$ sets equals the mean of that set). Finally it does not seem possible to improve on this technique, in the sense of reducing the computational burden.

# PROBLEMS IN THE ANALYSIS OF SURVEY DATA, AND A PROPOSAL

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Appendix VII

Most of the problems of analyzing survey data have been reasonably well handled, except those revolving around the existence of interaction effects. Indeed, increased efficiency in handling multivariate analyses even with non-numerical variables, has been achieved largely by assuming additivity. An approach to survey data is proposed which imposes no restrictions on interaction effects, focuses on importance in reducing predictive error, operates sequentially, and is independent of the extent of linearity in the classifications or the order in which the explanatory factors are introduced.

## A. NATURE OF THE DATA AND THE WORLD FROM WHICH THEY COME

THE increasing availability of rich data from cross section surveys calls for more efficient methods of data scanning and data reduction in the process of analysis. The purpose of this paper is to spell out some of the problems arising from the nature of the data and the nature of the theories which are being tested with the data, to show that present methods of dealing with these problems are often inadequate, and to propose a radical new method for analyzing survey data. There are seven things about the data or about the world from which they come which need to be kept in mind.

First, there is a wide variety of information about each person interviewed in a survey. This is good, because human behavior is motivated by more than one thing. But the very richness of the data creates some problems of how to handle them.

Second, we are dealing not with variables for the most part, but with classifications. These vary all the way from age, which can be thought of as a variable put into classes, to occupation or the answers to attitudinal questions, which may not even have a rank order in any meaningful sense. Even when measures seem to be continuous variables, such as age or income, there is good reason to believe that their effects are not linear. For instance, people earn their highest incomes in the middle age ranges. Expenditures do not change uniformly with changes in income at either extreme of the income scale.

Third, there are errors in all the measures, not just in the dependent variable, and there is little evidence as to the size of these errors, or as to the extent to which they are random.

Fourth, the data come from a sample and generally a complex one at that. Hence, there is sample variability piled on top of measurement error. The fact that almost all survey samples are clustered and stratified leads to problems of the proper application of statistical techniques. Statistical tests usually assume simple random samples rather than probability samples. More ap-

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propriate tests have been developed for simple statistics such as proportions, means, and a few others.

Fifth, and extremely important, there are intercorrelations between many of the explanatory factors to be used in the analysis—high income goes along with middle age, with advanced education, with being white, with not being a farmer, and so forth. This makes it difficult to assess the relative importance of different factors, since their intercorrelations get in the way. Since many of them are classifications rather than continuous variables, it is not even easy to measure the extent of the intercorrelation. Measures of association for cross classification raise notoriously difficult problems which have not really been solved in any satisfactory way.[1]

Sixth, there is the problem of interaction effects. Particularly in the social sciences, there are two powerful reasons for believing that it is a mistake to assume that the various influences are additive. In the first place, there are already many instances known of powerful interaction effects—advanced education helps a man more than it does a woman when it comes to making money; and it does a white man more good than a Negro. The effect of a decline in income on spending depends on whether the family has any liquid assets which it can use up. Women have their hospitalizations at different ages than men. Second, the measured classifications are only proxy variables for other things and are frequently proxies for more than one construct. Several of the measured factors may jointly represent a theoretical construct. We may have interaction effects not because the world is full of interactions, but because our variables have to interact to produce the theoretical constructs that really matter. The idea of a family life cycle, unless arbitrarily created out of its components in advance, is a set of interactions between age, marital status, presence, and age of children.[2] It is therefore often misleading to look at the over-all gross effects of age or level of education. Where interaction effects exist, the concept of a main effect is meaningless, and it is our belief that in human behavior there are so many interaction effects that we must change our approach to the problems of analysis.

Another example of interaction effects appeared in the attempt to build equivalent adult scales to represent the differences in living expenses of families of different types. After many years of analysis, one of the most recent studies in this field has concluded "when its size changes, families' tastes appear to change in more complicated ways than visualized by our hypothesis."[3] More

[1] One seemingly appropriate measure for two classifications both being used to predict the same variable is one called lambda suggested by Goodman and Kruskal. With many kinds of survey data this measure, which assumes that an absolute prediction has to be made for each individual, is too insensitive to deal with situations where each class on the predicting characteristic has the same modal class on the other characteristic that is to be predicted. An effective and properly stochastic measure would be derived by assigning a one-zero dummy variable to belonging to each class of each of the two characteristics and then computing the canonical correlation between the two sets of dummy variables.

See Leo A. Goodman and William H. Kruskal, "Measures of association for cross classifications," *Journal of the American Statistical Association*, 49 (December, 1954), 732–64.

[2] John B. Lansing and James N. Morgan, "Consumer finances over the life cycle," in *Consumer Behavior*, Volume II, L. Clark (Editor) (New York: New York University Press, 1955).

See also Leslie Kish and John B. Lansing, "Family life cycle as an independent variable," *American Sociological Review*, XXII (October, 1957), 512–9.

[3] In other words family composition had different effects on different expenditures. F. G. Forsythe, "The relationship between family size and family expenditure," *Journal of the Royal Statistical Society*, Series A, vol. 123 (1961), 367–97, quote from p. 386.

recently in analyzing factors affecting spending unit income, it has become obvious that age and education cannot operate additively with race, retired status, and whether the individual is a farmer. The attached table illustrates this with actual average incomes for a set of nonsymmetrical groups. The twenty-one groups account for two-thirds of the variance of individual spending unit incomes, whereas assuming additivity for race and labor force status even with joint age-education variables produces a regression which with 30 variables accounts for only 36 per cent of the variance. A second column in the

TABLE 1. SPENDING UNIT INCOME AND THE NUMBER IN THE UNIT WITHIN VARIOUS SUBGROUPS

| Group | Spending unit average (1958) income | Number in unit | Number of cases |
|---|---|---|---|
| Nonwhite, did not finish high school | $ 2489 | 3.3 | 191 |
| Nonwhite, did finish high school | 5005 | 3.4 | 67 |
| White, retired, did not finish high school | 2217 | 1.7 | 272 |
| White, retired, did finish high school | 4520 | 1.7 | 72 |
| White, nonretired farmers, did not finish high school | 3950 | 3.6 | 87 |
| White nonretired farmers, did finish high school | 6750 | 3.6 | 24 |
| *The Remainder* | | | |
| 0–8 grades of school | | | |
| 18–34 years old | 4150 | 3.8 | 72 |
| 35–54 years old | 4670 | 3.8 | 240 |
| 55 and older—not retired | 4846 | 2.2 | 208 |
| 9–11 grades of school | | | |
| 18–34 years old | 5032 | 3.7 | 112 |
| 35–54 years old | 6223 | 3.4 | 202 |
| 55 and older—not retired | 4720 | 2.1 | 63 |
| 12 grades of school | | | |
| 18–34 years old | 5458 | 3.3 | 193 |
| 35–54 years old | 7765 | 3.8 | 291 |
| 55 and older—not retired | 6850 | 2.0 | 46 |
| Some college | | | |
| 18–34 years old | 5378 | 3.0 | 102 |
| 35–54 years old | 7930 | 3.8 | 112 |
| 55 and older—not retired | 8530 | 2.0 | 36 |
| College graduates | | | |
| 18–34 years old | 7520 | 3.8 | 80 |
| 35–54 years old | 8866 | 2.9 | 150 |
| 55 and older—not retired | 10879 | 1.8 | 34 |

Source: 1959 Survey of Consumer Finances.

table gives the average number of people in the unit, and it can be seen that this particular breakdown is not particularly useful for analyzing the number of people in a unit. On the other hand, if each group were to be used to analyze expenditure behavior, income, and family size are likely to operate jointly rather than additively.

In view of the fact that intercorrelation among the predictors on the one hand and interaction effects on the other are frequently confused, it seems useful to give a pictorial example indicating both the differences between them and the way in which they operate when both are present. Our concern is not with statistical tests to distinguish between them, but with the effects of ignoring their presence.

Chart I shows pictorially three cases, real but exaggerated. First, there is a case where the two explanatory factors, income and education, are correlated with one another, but do not interact. Second, a case where income and being self-employed interact with one another but are not correlated, and third, a situation where income and asset holdings are correlated with one another and also interact in their effect on saving. The ellipsoids represent the area where most of the dots on a scatter diagram would appear. In the first case, it is clear that a simple relation between income and saving would exaggerate the effect of income on saving by failing to allow for the fact that high income people have more education, and that highly educational people also save more. An ordinary multiple regression, however, using a dummy variable representing high education would adequately handle this difficulty. In the second case there is no particular correlation, we assume, between income and being self-employed, but the self-employed have a much higher marginal propensity to save than other people. Here, the simple relationship between income and saving becomes a weighted compromise between the two different effects that really exist. A multiple correlation would show no effect of being self-employed and the same compromise effect of income. Only a separate analysis for the self-employed and the others would reveal the real state of the world. In the third case, not only do the high-asset people have a higher marginal propensity to save, but they also tend to have a higher income. Multiple correlation clearly will not take care of this situation in any adequate way. It *will* produce an "income effect" which can be added to an "asset effect" to produce an estimate of saving. Here the income effect is an average of two different income effects. The estimated asset effect is likely to come out closer to zero than if income had been ignored. Of course, where interactions exist, there is little use in attempting to measure separate effects.

Finally, there are logical priorities and chains of causation in the real world. Some of the predicting characteristics are logically prior to others in the sense that they can cause them but cannot be affected by them. For instance, where a man grows up may affect how much education he gets, but his education cannot change where he grew up. We are not discussing here the quite different analysis problem where the purpose is not to explain one dependent variable but to untangle the essential connections in a network of relations.

In dealing with a single dependent variable representing some human behavior, we might end up with at least three stages in the causal process—early childhood and parental factors, actions and events during the lifetime, and current situational and attitudinal variables. If this were the end of the problem we could simply run three separate analyses. The first would analyze the effects of early childhood and parental factors. The second would take the residuals from this analysis and analyze them against events during a man's lifetime up until the present, and the third would take the residuals from the

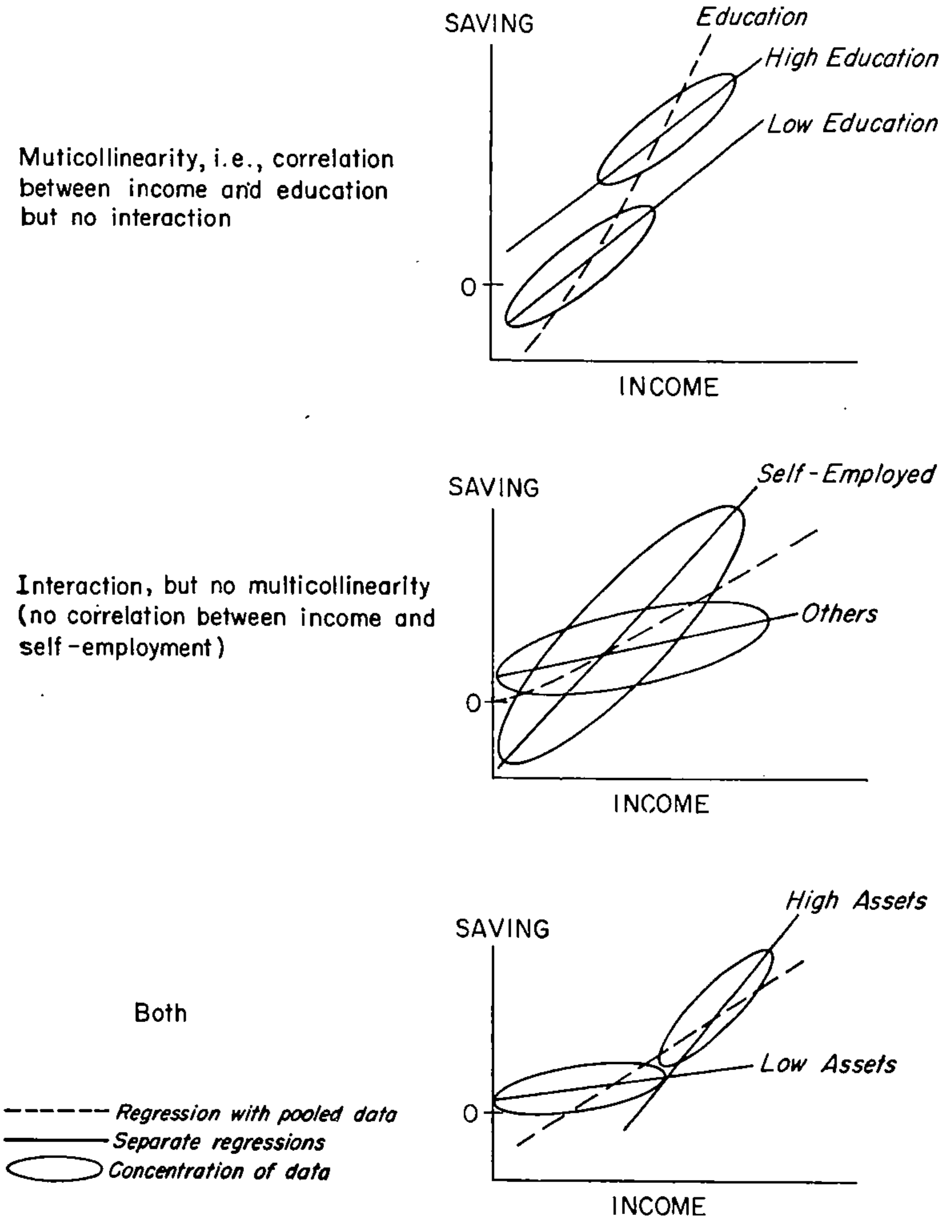
SAVING
Education
High Education
Low Education
Muticollinearity, i.e., correlation between income and education but no interaction
0
INCOME
SAVING
Self-Employed
Interaction, but no multicollinearity (no correlation between income and self-employment)
Others
0
INCOME
High Assets
SAVING
Both
Low Assets
0
Regression with pooled data
Separate regressions
Concentration of data
INCOME

CHART I. Combinations of Multicollinearity and Interaction and Their Effects.

second analysis and analyze them against current situational and attitudinal variables. But the real world is not even that simple, because some of the same variables which are logically prior in their direct effects may also tend to mediate the effect of later variables. For instance, a man's race has a kind of logical priority to it, but at the same time it may affect the way other things such as the level of his education operate to determine his income.

This is an impressive array of problems. Before we turn to a discussion of current attempts to solve these problems and to our own suggestions, it is essential to ask first what kind of theoretical structure is being applied and what the purposes of analysis are.

### B. NATURE OF THE THEORY AND PURPOSES OF ANALYSIS

Perhaps the most important thing to keep in mind about survey data in the social sciences is that the theoretical constructs in most theory are not identical with the factors we can measure in the survey. The simple economic idea of ability to pay for any particular commodity is certainly a function not only of income but of family size, other resources, expected future income, economic security, and even extended family obligations. A man's expectations about his own economic future, which we may theorize will affect his current behavior, might be measured by a battery of attitudinal and expectational questions or by looking at his education, occupation, age, and the experience of others in the same occupation and education group who are already older. The fact that the theoretical constructs in which we are interested are not the same as the factors we can measure, nor even simply related to them, should affect our analysis techniques and focus attention on creating or locating important interaction effects to represent these constructs.

Second, there are numerous hypotheses among which a selection is to be made. Even if the researcher preferred to restrict himself to a single hypothesis and test it, the intercorrelations among the various explanatory factors mean that the same result might support any one of several hypotheses.[4] Hence, comparisons of relative importance of predictors, and selecting those which reduce predictive errors most, are required.

When we remember that there are also variable errors of measurement, the problem of selecting between alternative hypotheses becomes doubly difficult, and ultimately requires the use of discretion on the part of the researcher. Better measurement of a factor might increase its revealed importance.

Finally, researchers may have different reasons why they wish to predict individual behavior. Most will want to predict behavior of individuals in the population, not just in the sample, which makes the statistical problem somewhat more complicated. But some may also want to focus on the behavior of some crucial individuals by assigning more weight to the behavior of some rather than others. Others may want to test some explanatory factors, however small their apparent effect, because they are important. They may be important because they are subject to public policy influences or because they

[4] For an excellent statement of the application of this problem to the economists' concern with the permanent income hypothesis versus the relative income hypothesis, see Jean Crockett, "Liquid assets and the theory of consumption" (New York: National Bureau of Economic Research, 1962) (mimeographed).

are likely to change over time, or because they are crucial to some larger theoretical edifice. The nature of these research purposes thus combines with the nature of the data and their characteristics to make up the problem of how to analyze the data.

## C. THE STRATEGY CHOICE IN ANALYSIS

One can think of a series of strategies ranging from taking account of only the main effects of each explanatory classification separately or jointly, to trying to take account of all possible combinations of all the classifications at once. Even if there were enough data to allow the last, however, it would not be of much use. The essence of research strategy then consists of putting some restrictions on the process in order to make it manageable. One possibility is to cut the number of explanatory factors utilized, and another is to restrict the freedom with which we allow them to operate.[5] One might assume away most or all interaction effects, for instance, and keep a very large number of explanatory classifications. Still further reduction in the number of variables is possible, if one assumes linearity for measured variables or, what amounts to the same thing builds arbitrary scales, incestuously derived out of the same data in order to convert each classification into a numerical variable. Clearly, the more theoretical or statistical assumptions one is willing to impose on the data, the more he can reduce the complexity of the analysis. A difficulty is that restrictions imposed in advance cannot be tested. There seems some reason to argue that it would be better to use an approach which developed its restrictions as it went along. In any case keeping these problems in mind we turn now to a summary of how analysis problems in using survey data are currently being handled and some of the difficulties that present methods still leave unsolved.

## D. HOW PROBLEMS IN ANALYSIS ARE CURRENTLY BEING HANDLED—AN APPRAISAL

We take the seven problems in section A in the same order in which they are presented there plus the major problem in section B, that of theoretical constructs not measured directly by the factors on which we have data. The first problem was the existence of many factors. The simplest procedure has been to look at them one at a time always keeping in mind the extent to which one factors is intercorrelated with others. Another technique, particularly with attitudes, has been to build indexes or combinations of factors either arbitrarily or with the use of some sort of factor analysis technique.[6] The difficulty is that the first of these is quite arbitrary, and the second is arbitrary in a different sense, in that most mechanical methods of combining factors are based on the intercorrelations between the factors themselves and not in the way in which they may affect the dependent variable. It is quite possible for two highly correlated factors to influence the dependent variable in opposite ways. Building a combination of the two only on the basis of their intercorrelation would create a factor which would have no correlation at all with the dependent

[5] For a discussion of alternative strategies made while commenting on a series of papers, see James Morgan, "Comments," in *Consumption and Saving*, Volume I, I. Friend and R. Jones (Editors.) (Philadelphia: University of Pennsylvania Press, 1960), pp. 276–84.

[6] Charles Westoff and others, *Family Planning in Metropolitan America* (Princeton: Princeton University Press, 1961).

variable. With highly correlated attitudes, however, some such reduction to a few factors may be required and meaningful.

With the advent of better computing machinery, the problem of multiple factors has frequently been handled by using multiple correlation techniques. The use of these techniques, of course, required solving the second problem, that arising from the fact that in many cases we have classifications rather than continuous variables. This has been done in two ways, first, by building arbitrary scales. For instance, one could assign the numbers one, two, three, four, five, and six to the six age groups in order. Or if age were being used to predict income, one could assign a set of numbers representing the average income of people in those age groups.[7] But unless machine capacity is extremely limited, a far more flexible method which is coming into favor is to use what have been called dummy variables.[8] The essence of this technique is to assign a dummy variable to each class of a characteristic except one. It is called a dummy variable because it takes the value one if the individual belongs in that subclass or a zero if he does not. If ordinary regression procedures are to be used, of course, dummy variables cannot be assigned to every subclass of any characteristic, since this would overdetermine the system. However, at the Survey Research Center we have developed an iterative program for the IBM 7090, the output of which consists of coefficients for each subclass of each characteristic, the set for each characteristic having a weighted mean of zero. This means that the predicting equation has the over-all mean as its constant term, and an additive adjustment for each characteristic, depending on the subclass into which the individual falls on that characteristic. This is the standard analysis of variance formulation when all interactions are assumed to be zero. Of course, the coefficients of dummy variables using a regular matrix inversion routine can easily be converted into sets of this sort. There remain two difficulties with this technique. One is the problem of interaction effects, which are either assumed away or have to be built in at the beginning in the creation of the classes. A second arises from the nature of the classifications frequently used in survey data. Even though association between, say, occupation and the incidence of unemployment faced by an individual is not terribly high, the occupation code generally includes one or two categories such as the farmers and the retired who, by definition, cannot be unemployed at all. When dummy variables are assigned to these classes, it may easily occur that there is a perfect association between a dummy variable representing one of these peculiar (not applicable) groups in one code and a dummy variable representing something else in another classification (not unemployed). If the researcher omits one of each such pair of dummy variables in a regression routine, he is all right.

A third problem, that of errors in the data, is generally handled by not re-

[7] For an example see Jerry Miner, "Consumer Personal Debt—An Intertemporal Analysis," in *Consumption and Saving*, Volume II, I. Friend and R. Jones (Editors) (Philadelphia: University of Pennsylvania Press, 1960), 400–61.

[8] Daniel Suits, "The Use of Dummy Variables in Regression Equations," *Journal of the American Statistical Association*, 52 (December, 1957), 548–51.

T. P. Hill, "An Analysis of the Distribution of Wages and Salaries in Great Britain," *Econometrica*, 27 (July, 1959), 355–81.

jecting hypotheses too easily and by attempting to use some judgment in the assessment of relative importance of different factors or different hypotheses keeping in mind the accuracy with which the variables have probably been measured.

The fact that the data come from a sample has frequently been ignored. As the analysis techniques become more complicated, it becomes almost impossible to keep the structure of the sample in mind too. However, there is some reason to believe that the clustering and stratification of the sample become less and less important the more complex and more multivariate the analysis being undertaken.[9]

What about intercorrelations among the predictors? The main advantage of multivariate techniques like multiple regression is that they take care of these intercorrelations among the predictors, at least in a crude sense. Indeed, if one compares an ordinary subclass mean with the multivariate coefficient of the dummy variable associated with belonging to that subclass, the difference between the two is the result of adjustments for intercorrelations. Where these differences seem likely to be the result of a few major interrelations, some statement as to the factors correlated with the one in question (and responsible for the attenuation of its effect on the multivariate analysis) are often given to the reader. It is, of course, true that where intercorrelations between two predictors are too high, no analysis can handle this problem, and it becomes necessary to remove one of them from the analysis.

Perhaps the most neglected of the problems of analysis has been the problem of interaction effects. The reason is very simple. The assumption that no interactions exist generally leads to an extremely efficient analysis procedure and a great reduction in the complexity of the computing problem. Those of us who have looked closely at the nature of survey data, however, have become increasingly impressed with the importance of interaction effects and the useful way in which allowing for interactions between measured factors gets us closer to the effects of more basic theoretical constructs. Where interaction effects have not been ignored entirely, they have been handled in a number of ways. They can be handled by building combination predictors in the first place, such as combinations of age and education or the combination of age, marital status, and children known as the family life cycle.[10] Sometimes where almost all the interactions involve the same dichotomy, two separate analyses are called for.[11] Interactions are also handled by rerunning the analysis for

[9] Actually there are no formulas available for sampling errors of many of the statistics from complex probability samples. Properly selected part-samples can be used to estimate them by a kind of hammer-and-tongs procedure, but this is expensive. See Leslie Kish, "Confidence intervals for clustered samples," *American Sociological Review*, 22 (April, 1957), 154–65. So long as the samples are representative of a whole population the basic statistical model is presumably the "fixed" one, see M. B. Wilk and O. Kempthorne, "Fixed, mixed, and random models," *Journal of the American Statistical Association*, 50 (December, 1955), 1144–67.

See also L. Klein and J. Morgan, "Results of alternative statistical treatments of sample survey data," *Journal of the American Statistical Association*, 46 (December, 1951), 442–60.

[10] Guy Orcutt and others, *Microanalysis of Socioeconomic Systems* (New York: Harper and Brothers, 1961).

[11] For instance, hospital utilization was studied separately for men and women in Grover Wirick, Robin Barlow, and James Morgan, "Population survey: Health care and its financing," *Hospital and Medical Economics*, Volume I, Walter McNerney (Editor) (Chicago: American Hospital Association, 1962).

Participation in recreation was studied separately for those with and without paid vacations; see Eva Mueller and Gerald Gurin, *Participation in Outdoor Recreation: Factors Affecting Demand Among American Adults* (U.S. (U.S.G.P.O., ORRRC Study Report 20, 1962.)

some subgroup of the population. In a recent study of factors affecting hourly earnings, for instance, the analysis was rerun for the white, nonfarmer males only, to test the hypothesis that some of the effects like that of education were different for the non-whites, women, and farmers.[12] A difficulty with this technique, of course, is that if one merely wants to see whether the interaction biases the estimates for the whole population seriously, one reruns the analysis with the group that makes up the largest part of the sample. But if one wants to know whether there are different patterns of effects for some small subgroup, the analysis must be run for that small subgroup.

Another method of dealing with interaction effects is to look at two- and three-way tables of residuals from an additive multivariate analysis. This requires the process, often rather complicated and expensive, of creating the residuals from the multivariate analysis and then analyzing them separately.[13] Where some particular interaction is under investigation, an effective alternative is to isolate some subgroup on a combination of characteristics such as the young, white, college graduates. It is then possible to derive an estimate of the expected average of that subgroup on the dependent variable by summing the multivariate coefficients multiplied by the subgroup distributions over each of the predictors. Comparing this expected value with the actual average for that subgroup indicates whether there is something more than additive effect. It is only feasible to do this with a few interactions, just as it is possible to put in cross product terms in multiple regressions in only a few of the total possible cases. Consequently, most of these methods of dealing with interaction effects are either limited, or expensive and time-consuming.

Still another technique for finding interactions is to restrict the total number of predictors, use cell means as basic data, and use a variance analysis looking directly for interaction effects.[14] Aside from the various statistical assumptions that have to be made, this turns out to be a relatively cumbersome method of dealing with the data. It requires a good deal of judgment in the selecting of the classes to avoid getting empty cells or cells with very small numbers of cases,

[12] James Morgan, Martin David, Wilbur Cohen, and Harvey Brazer, *Income and Welfare in the United States* (New York: McGraw-Hill, 1962).

Malcolm R. Fisher, "Exploration in savings behavior," *Bulletin of the Oxford University Institute of Statistics*, 18 (August, 1956), 201–77.

[13] James Morgan, "An analysis of residuals from 'normal' regressions," in *Contributions of Survey Methods to Economics*, L. Klein (Editor) (New York: Columbia University Press, 1954).

[14] F. Gerald Adams, *Some Aspects of the Income Size Distribution* (unpublished Ph.D. dissertation, The University of Michigan, 1956); and a summary, "The size of individual incomes: Socio-economic variables and chance variation," *Review of Economics and Statistics*, XL (November, 1958), 394–8.

James Morgan, "Factors related to consumer savings" in *Contributions of Survey Methods to Economics*, L. Klein (Editor) (New York: Columbia University Press, 1954).

Mordechai Kreinin, "Factors associated with stock ownership," *Review of Economics and Statistics*, XLI (February, 1959), 12–23; "Analysis of liquid asset ownership," *Review of Economics and Statistics*, XLIII (February, 1961), 76–80.

M. Kreinin, J. Lansing, J. Morgan, "Analysis of life insurance premiums," *Review of Economics and Statistics*, XXXIX (February, 1957), 46–54.

Robert Ferber has pointed out that using the highest order interaction as "error" may hide significant main effects or lower-order interaction effects, and that the heteroscedasticity of means based on subcells of different sizes may make the tests nonconservative. He has made use of the more complex method of fitting constants which provides an exact test for interactions but assumes that the individual observations are all independent. Since this assumption is not correct for most multistage samples the results of this method are also nonconservative. See Robert Ferber, "Service expenditures at mid-century," in *Consumption and Saving*, Volume I, I. Friend and R. Jones (Editors) (Philadelphia: University of Pennsylvania Press, 1960), pp. 436–60.

and the unequal cell frequencies lead to heterogeneity of variances which makes the *F*-test nonconservative. Sometimes interaction effects are considered important only when they involve one extremely important variable. In the case of much economic behavior, current income appears to be such a variable. In this case one can rely on covariance techniques, but these techniques tend to become far too complex when a large number of other factors are involved. Also, as more and more questions arise about the meaning of current income as a measure of ability to pay, the separation of current income for special treatment becomes more doubtful.

Finally, it is also true that if we restrict the number of variables, multiple regression techniques, particularly using dummy variables, can build in almost all feasible interaction effects. One way to restrict the number of variables is to make an analysis with an initial set and run the residuals against a second set of variables. However, unless there is some logical reason why one set takes precedence over another, this is treacherous since the explanatory classifications used in the second set will have a downward bias in their coefficients if they are at all associated with the explanatory classifications used in the first set.[15]

All these methods for dealing with interaction effects require building them in somehow without knowing how many cases there are for which each interaction effect could be relevant. The more complex the interaction, the more difficult it is to tell, of course.

The problem of logical priorities in the data and chains of causation can be handled either by restricting the analysis to one level or by conducting the analysis sequentially, always keeping in mind that the logically prior variables may have to be reintroduced in later analyses on the chance that they may mediate the effects of other variables. In practice, very little analysis of survey data has paid much attention to this problem. Perhaps the reason is that only recently has anyone been able to handle the other problems so that a truly multivariate analysis was possible. And it is only when many variables begin to be used simultaneously that the problem of their position in a causal structure becomes crucial.

Finally, there is the problem remaining from section B that the constructs of theories do not have any one-to-one correspondence with the measures from the survey. Sometimes this problem is handled by building complex variables that hopefully represent the theoretical construct. The life cycle concept, for instance, has been used this way. In a recent study, a series of questions that seemed to be asking evaluations of occupations were translated into a measure which was (hopefully) an index measure of achievement motivation.[16] More commonly, the analyst has been constrained to interpret each of the measured characteristics in terms of some theoretical meaning which it hopefully has. This is often not very satisfactory. In the case of liquid assets, the amount of

[15] James Morgan, "Consumer investment expenditures," *American Economic Review*, XLVIII (December, 1958), 874–902, Appendix, 898–901.

Arthur S. Goldberger and D. B. Jochems, "A note on stepwise least squares," *Journal of the American Statistical Association*, 56 (March, 1961), 105–11.

[16] Morgan, David, Cohen, and Brazer, *Income and Welfare in the United States*. (New York: McGraw-Hill Book Company, Inc., 1962).

these assets a man has represents both his past propensity to save and his present ability to dissave, two effects which could be expected to operate in opposite directions. In general, the analysis of survey data has been much better than this summary of problems would indicate. Varied approaches have been ingeniously used, and cautiously interpreted.

### E. PROPOSAL FOR A PROCESS FOR ANALYZING DATA

One way to focus on the problems of analyzing data is to propose a better procedure. The proposal made here is essentially a formalization of what a good researcher does slowly and ineffectively, but insightfully on an IBM sorter. With large masses of data, weighted samples, and a desire for estimates of the reduction in error, however, we need to be able to simulate this process on large scale computing equipment. The basic idea is the sequential identification and segregation of subgroups one at a time, nonsymmetrically, so as to select the set of subgroups which will reduce the error in predicting the dependent variable as much as possible relative to the number of groups. A subgroup may be defined as membership in one or more subclasses of one or more characteristics. If more than one characteristic is used, the membership is joint, not alternative.

It is assumed that where the problem of chains of causation and logical priority of one variable over another exists, that this problem will be handled by dividing the explanatory variables or predictors into sets. One then takes the pooled residuals from an analysis using the first set of predictors and analyses these residuals against the second set of predictors. The residuals from the analysis using this second set could then be run against a third set. In practice, we might easily end up with three states—early childhood or parental factors, actions and events during the lifetime, and current situational and attitudinal variables.

The possibilities of interactions between variables in different stages can be handled by reintroducing in the second or third analyses, factors whose simple effects have already been removed, but which may also mediate the effects of factors at one of the later stages, that is, nonwhites may have their income affected by education differently from whites.

Temporarily setting aside these complications, we turn now to a description of the process of analysis using the variables from any *one* stage of the causal process. Since even the best measured variable may actually have nonlinear effects on the dependent variable, we treat each of the explanatory factors as a set of classifications. As we said, our purpose is to identify and segregate a set of subgroups which are the best we can find for maximizing our ability to predict the dependent variable. We mean maximum relative to the number of groups used, since an indefinitely large number of subgroups would "explain" everything in the sample. To be more sophisticated, if we use a model based on the assumption that we want to predict back to the population, there is an optimal number of subgroups. However, as an approximation we propose that with samples of two to three thousand we arbitrarily segregate only those groups, the separation of which will reduce the total error sum of squares by at least one per cent and do not even attempt further subdivision unless the group to be divided has a residual error (within group sum of squares) of at least two per cent of the total sum of squares. This restricts us to a *maximum* of fifty-one groups. It is just as arbitrary as the use of the 5 per cent level in significance tests and perhaps should be subject to later revision on the basis of experience.

We now describe the process of analysis in the form of a series of decision rules and instructions. We think of the sample in the beginning as a single group. The first decision is what single division of the parent group into two will do the most good. A second decision has then to be made: Which of the two groups we now have has the largest remaining error sum of squares, and hence should be investigated next for possible further subdivision? Whenever a further subdivision of a group will not reduce the unexplained sum of squares by at least one per cent of the total original sum of squares, we pay no further attention to that subgroup. Whenever there is no subgroup accounting for at least two per cent of the original sum of squares, we have finished our job. We turn now to a more orderly description of this process.

1) Considering all feasible divisions of the group of observations on the basis of each explanatory factor to be included (but not combinations of factors) find the division of the classes of any characteristic such that the partitioning of this group into two subgroups on this basis provides the largest reduction in the unexplained sum of squares.

Starting with any given group, and considering the various possible ways of splitting it into two groups, it turns out that a quick examination of any possible subgroup provides a rapid estimate of how much the error variance would be reduced by segregating it:

The reduction in error sum of squares is the same size (opposite sign) as the increase in the explained sum of squares.

For the group as a whole, the sum of squares explained by the mean is

$$N\overline{X}^2 = \frac{(\sum X)^2}{N} \tag{1}$$

and the total sum of squares (unexplained by the mean) is

$$\sum (X - \overline{X})^2 = \sum X^2 - \frac{(\sum X)^2}{N} \ . \tag{2}$$

If we now divide the group into two groups of size $N_1$ and $N_2$ and means $\overline{X}_1$ and $\overline{X}_2$, what happens to the explained sum of squares?

$$\text{Explained sum of squares} = N_1\overline{X}_1^2 + N_2\overline{X}_2^2. \tag{3}$$

The division which increases this expression most over $N\overline{X}^2$ clearly does us the most good in improving our ability to predict individuals in the sample.

Fortunately we do not even need to calculate anything more than a term involving the subgroup under inspection, since $N$ and $\sum X$ remain known and constant throughout this search process.

$$N_2 = N - N_1 \tag{4}$$

$$\sum X_2 = \sum X - \sum X_1 \tag{5}$$

$$\therefore \text{ explained sum of squares} = N_1\left(\frac{\sum X_1}{N_1}\right)^2 + (N - N_1)\left(\frac{\sum X_2}{N - N_1}\right)^2$$

$$= \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X - \sum X_1)^2}{N - N_1}.$$

The number of cases (or proportion of sample) and the sum of the dependent variable for any subgroup are enough to estimate how much reduction in error sum of squares would result from separating it from the parent group.

If it seems desirable, a variance components model which takes account of the fact that we really want optimal prediction of members of the population not merely of the sample, can be used. Indeed, the expression for thc estimate of the explained, or "between" component of variance in the population turns out to be

$$\hat{\sigma}_B^2 = \frac{\left(\frac{N-1}{N-2}\left[\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X - \sum X_1)^2}{N - N_1}\right] - \frac{\sum X^2}{N-2}\right) - \frac{(\Sigma x)^2}{N}}{N - \frac{N_1^2 + N_2^2}{N}} \tag{7}$$

which, though it looks formidable, contains only one new element and that is a term from the total sum of squares of the original group which is constant and can be ignored in selecting the best split. The expression in the brackets is the explained sum of squares already derived. $N$, $\sum X$, and $\sum X^2$ are known and constant. The denominator is an adjustment developed by Ganguli for a bias arising from unequal $N$'s. Where $N_1$ equals $N_2$, the denominator becomes equal to $N_1$. The more unequal the $N$'s, the smaller the denominator, relative to an arithmetic mean of the $N$'s. The ratio of the explained component of variance to the total is *rho*, the intraclass correlation coefficient. Hence, in using a population model, we are searching for the particular division of a group into two that will provide the largest *rho*.[17] Computing formulas for weighted data or a dummy (one or zero) dependent variable can be derived easily.

(2) Make sure that the actual reduction in error sum of squares is larger than one per cent of the total sum of squares for the whole sample, i.e., $> .01$ ($\sum X^2$, $-N\bar{X}^2$) (If not select the next most promising group for search for possible subdivision, etc.)

(3) Among the groups so segregated, including the parent, or bereft ones, we now select a group for a further search for another subgroup to be split off. The selection of the group to try is on the basis of the size of the unexplained

[17] R. L. Anderson and T. A. Bancroft, *Statistical Theory in Research* (New York: McGraw-Hill Book Company, 1952).

M. Ganguli, "A note on nested sampling," *Sankhya* 5 (1941), 449–52.

For an example of the use of rho in analysis see Leslie Kish and John Lansing, "The family life cycle as an independent variable," *American Sociological Review*, XXII (October, 1957), 512–4.

sum of squares within the group, or the heterogeneity of the group times its size, which comes to the same thing. It may well *not* be the group with the most deviant mean.

In other words, among the groups, select the one where

$$\sum X_{ij}^2 - N_i\overline{X}_i^2 \text{ is largest.}$$

If it is less than two per cent of the total sum of squares for the whole sample, stop, because no further subdivision could reduce the error sum of squares by more than two per cent. If it is more than two per cent, repeat Step 1.

Note that the process stops when no group accounts for more than two per cent of the error sum of squares. If a group being searched allows no further segregation that will account for one per cent, the next most promising group is searched, because it may still be possible that another group with a smaller sum of squares within it can be profitably subdivided.

Since only a single group is split off at a time, the order of scanning to select that one should not affect the results. Since an independent scanning is done each time, the order in which groups are selected for further investigation should not matter either, hence our criterion is a pure efficiency one.

Chart II shows how the process suggested might arrive at a set of groups approaching those given earlier in Table 1. The numbers are rough estimates from Table 1.

*Note on Amount of Detail in the Codes*

The search for the best single subgroup .which can be split off involves a complete scanning at each stage of each of the explanatory classifications, and within each classification of all the feasible splits. This is not so difficult as it seems, for within any classification not all possible combinations of codes are feasible. If one orders the subclasses in ascending sequence according to their means (on the dependent variable), then it can be shown that the best single division—the one which maximizes the explained sum of squares—will never combine noncontiguous groups.

Hence, starting at either end of the ordered subgroups, the computer will sequentially add one subgroup after another to that side and subtract it from the other side, always recomputing the explained sum of squares. By "explained" we mean that the means of the two halves are used for predicting rather than the over-all mean. Whenever the new division has a higher explained sum of squares, it is retained, otherwise the previous division is remembered. But in any case, the process is continued until there is only one subgroup left on the other side, to allow for the possibility of "local maxima."

The machine then remembers the best split, and the explained sum of squares associated with it, and proceeds to the next explanatory characteristic. If upon repeating this procedure with the subclasses of that characteristic, a still larger explained sum of squares is discovered, the new split on the new characteristic is retained and the less adequate one dropped.

The final result will thus be the best single split, allowing any reasonable

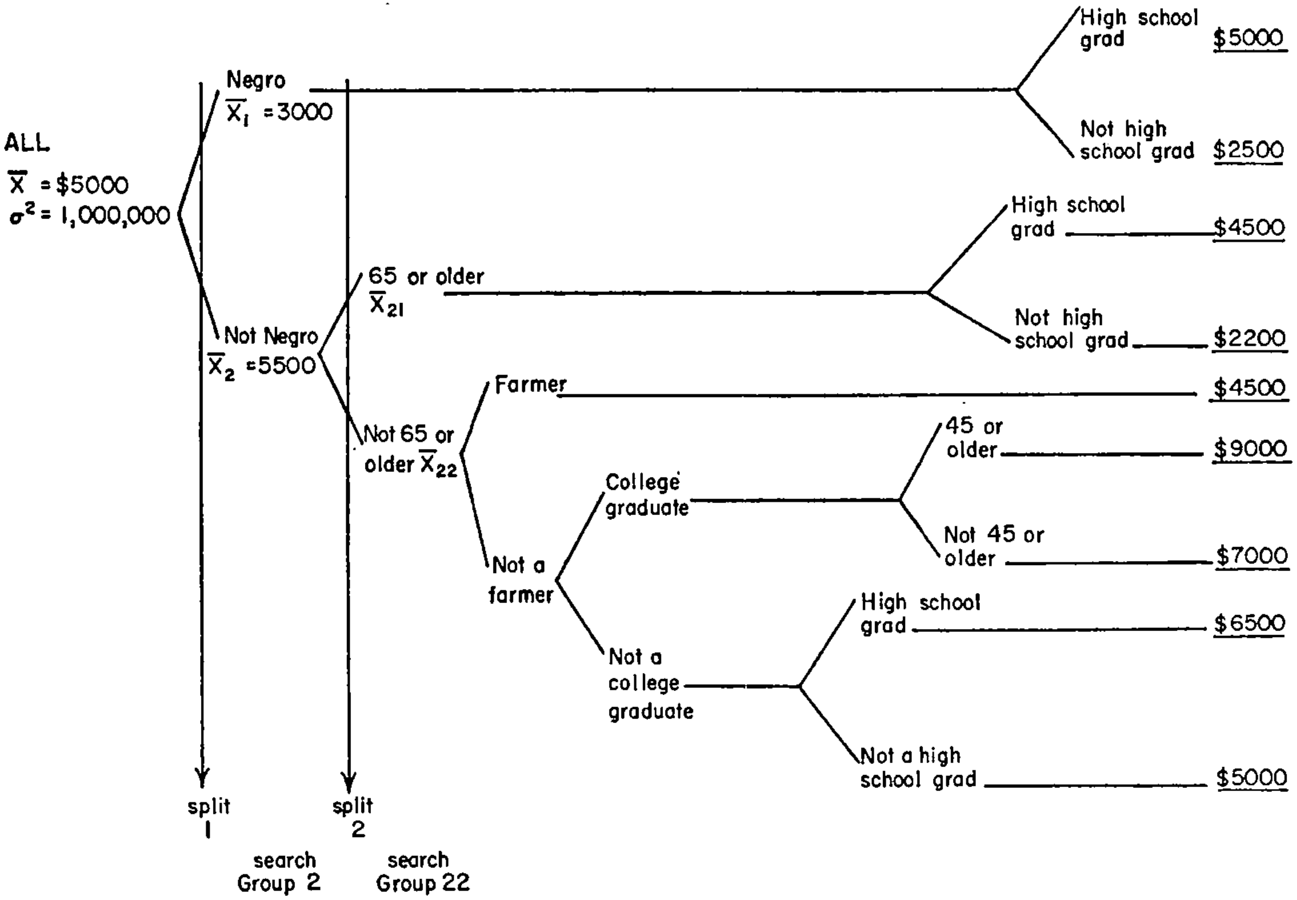

Chart II. Annual Earnings.

combination of subclasses of a single category, to maximize the explained sum of squares. It is easy to see that this choice will not depend on the order in which factors are entered, but may depend on the amount of detail with which they are coded. The number of subclasses probably should not vary too much from one factor to the next.

The authors are planning to try out such a program under a grant from the National Science Foundation. Data which have already been analyzed using dummy variable multiple regressions will be re-analyzed to see whether the new program provides new insights.

## DISCUSSION

What is the theoretical model behind this process? Instead of simplifying the analysis by arbitrary or theoretical assumptions that restrict the number of variables or the way in which they operate, this process essentially restricts the complexity of the analysis by insisting that there be a large enough sample of any particular subgroup so that we can be sure it matters, and by handling problems one at a time. This is essentially what a researcher does when first investigating a sample using a sorter and his own judgment. It is assumed that the sample being used in a situation like this is a representative probability sample of a large important population. It is possible that there may be subgroups of the population whose behavior is of more importance than that of other subgroups, in which case it would be easily possible to weight the data to take account of this fact. It may be that there are certain crucial characteristics, the importance of which must be investigated. In this case, either lower admission criteria could be used or an initial arbitrary division of the sample according to this characteristic could be made before starting.

Why not take all possible subsets, in other words, all possible combinations of characteristics, and then start combining subcells where the means are close to one another? The simple reason is that there are far too many possible subsets, and since this is a sample, the means of these subsets are unstable and unreliable estimates. It is true, however, that this is the only way one would avoid all possibility of failing to discover interaction effects. Let us take a simple example of a stituation where the method we propose would fail to discover interaction effects. Suppose we have males and females, old and young, in the following proportions who go to the hospital each year, young females eight per cent, young males two per cent, old females two per cent, old males eight per cent. Assuming half the population is male and half the population is old, the old-young split would give means of five and five per cent, and the male-female split would give means of five and five per cent. Thus we would never discover that it is young females and old males who go to the hospital. One way out of this difficulty which would also vastly increase the efficiency of the machine processes would be to set up a relatively arbitrary division of the sample into perhaps ten groups to start with, groups which are known to be important and suspected to be different in their behavior. The only problem with this is that the remaining procedures will not be invariant with respect to which initial groups were selected.

*Previous Work of a Similar Nature*

One can never be sure that there does not exist previous work relevant to any "new" idea. William Belson has suggested a sequential, nonsymmetrical division of the sample which he calls "biological classification," for a different purpose, that of matching two groups on other characteristics used as controls so that they can be compared.[18] His procedure is restricted to the case where the criterion can be converted to a one-zero division, and the criterion for subdivision is the best improvement in discrimination. The method takes account of the number of cases, i.e., focuses on improvement in prediction, not on levels of significance. We have proposed this same focus. No rules are provided as to when to stop, or in what order to keep searching, though an intelligent researcher would intuitively follow the rules suggested here.

Another approach to the problem as been suggested and tried by André Danière and Elizabeth Gilboy. Their approach attempts to keep numerical variables whenever there appears to be linearity, at least within ranges, and to repool groups whenever there does not appear any substantial nonlinearity or interaction effect. The method is feasible only where the number of factors is limited. The pooling both of groups and of ranges of "variables" makes it complicated.[19] In practice, they found it useful to restrict the number of allowable interaction effects.

There are also studies going on in the selection of test items to get the best prediction with a limited set of predictors. But the prediction equation in these analyses always seems to be multiple regression without any interaction effects.[20] Group-screening methods have been suggested whereby a set of factors is lumped and tested and the individual components checked only if the group seems to have an effect. These procedures, however, require knowledge of the direction of each effect and again assume no interaction effects.[21] These group-screening methods are largely used in experimental designs and quality control procedures. It is interesting, however, that they usually end up with two-level designs, and our suggested procedure of isolating one subgroup at a time has some similarity to this search for simplicity.

The approach suggested here bears a striking resemblance to Sewall Wright's path coefficients, and to procedures informally called "pattern analysis." The justification for it, however, comes not fron any complicated statistical theory, nor from some enticing title, but from a calculated belief that for a large range of problems, the real world is such that the proposed procedure will facilitate understanding it, and foster the development of better connections between theoretical constructs and the things we can measure.

One possible outcome, for those who want precise measurement and testing,

[18] William A. Belson, "Matching and prediction on the principle of biological classification," *Applied Statistics*, VIII (1959), 65–75.

[19] André Danière and Elizabeth Gilboy, "The specification of empirical consumption structures, in *Consumption and Saving*, Volume I, I. Friend and R. Jones (Editors) (Philadelphia: University of Pennsylvania Press, 1960), pp. 93–136.

[20] Paul Horst and Charlotte MacEwan, "Optimal test-length for multiple prediction, the general case," *Psychometrika*, 22 (December, 1957), 311–24 and references cited therein.

[21] G. S. Watson, "A Study of the group-screening method," *Technometrics*, 3 (August, 1961), 371–88.

G. E. P. Box, "Integration of techniques for process control," *Transactions of the Eleventh Annual Convention of the American Society for Quality Control, 1958.*

is the development of new constructs, as combinations of the measured "variables," which are then created immediately in new studies and used in the analysis. The family life cycle was partly theoretical, partly empirical in its development. Other such constructs may appear from our analysis, and then acquire theoretical interpretation.

### F. WHAT NEEDS TO BE DONE?

It may seem that the procedure proposed here is actually relatively simple. Each stage involves a simple search of groups defined as a subclass of any one classification and a selection of one with a maximum of a certain expression which is easily computed. It turns out, however, that the computer implications of this approach are dramatic. The approach, if it is to use the computer efficiently requires a large amount of immediate access storage which does not exist on many present-day computers. Our traditional procedures for multivariate analysis involve storing information in the computer in the form of a series of two-way tables, or cross-product moments. This throws away most of the interesting and potentially fruitful interconnectedness of survey data, and we only recapture part of it by multivariate processes which assume additivity. The implications of the proposed procedure are that we need to be able to keep track of all the relevant information about each individual in the computer as we proceed with the analysis.

Only an examination of the pedigree of the groups selected by the machine will tell whether they reveal things about the real world, or lead to intuitively meaningful theoretical constructs, which had not already come out of earlier "multivariate" analyses of the same data.

It may prove necessary to add constraints to induce more symmetry, such as giving priority to seriatim splits on the same characteristic, since this might make the interpretation easier. Or we may want to introduce an arbitrary first split, say on sex, to see whether offsetting interactions previously hidden could be uncovered in this way.

Most statistical estimates carry with them procedures for estimating their sampling variability. Sampling stability with the proposed program would mean that using a different sample, one would end up with the same complex groups segregated. No simple quantitative measure of similarity seems possible, nor any way of deriving its sampling properties. The only practical solution would seem to be to try the program out on some properly designed half-samples, taking account of the original sample stratification and controls, and to describe the extent of similarity of the pedigrees of the groups so isolated. Since the program "tries" an almost unlimited number of things, no significance tests are appropriate, and in any case the concern is with discovering a limited number of "indexes" or complex constructs which will explain more than other possible sets.

It seems clear that the procedure takes care of most of the problems discussed earlier in this paper. It takes care of any number of explanatory factors, giving them all an equal chance to come in. It uses classifications, and indeed only those sets of subclasses which it actually proves important to distinguish. The results still depend on the detail with which the original data were coded.

Differential quality of the measures used remains a problem. Sample complexities are relatively unimportant since measures of importance in reducing predictive error are involved rather than tests of significance, and one can restrict the objective to predicting the sample rather than the population. Intercorrelations among the predictors are adequately handled, and logical priorities in causation can be.

Most important, however, the interaction effects which would otherwise be ignored, or specified in advance arbitrarily from among a large possible set, are allowed to appear if they are important.

There is theory built into this apparently empiristic process, partly in the selection of the explanatory characteristics introduced, but more so in the rules of the procedures. Where there is one factor of supreme theoretical interest, it can be held back and used to explain the differences remaining within the homogeneous groups developed by the program. This is a severe test both for the effect of this factor and for possible first-order interaction effects between it and any of the other factors used in defining the groups.

Finally, where it is desired to create an index of several related measures, such as attitudinal questions in the same general area, the program can be restricted to these factors and to five or ten groups, and will create a complex index with maximal predictive power.

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